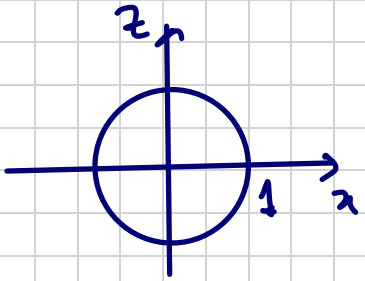
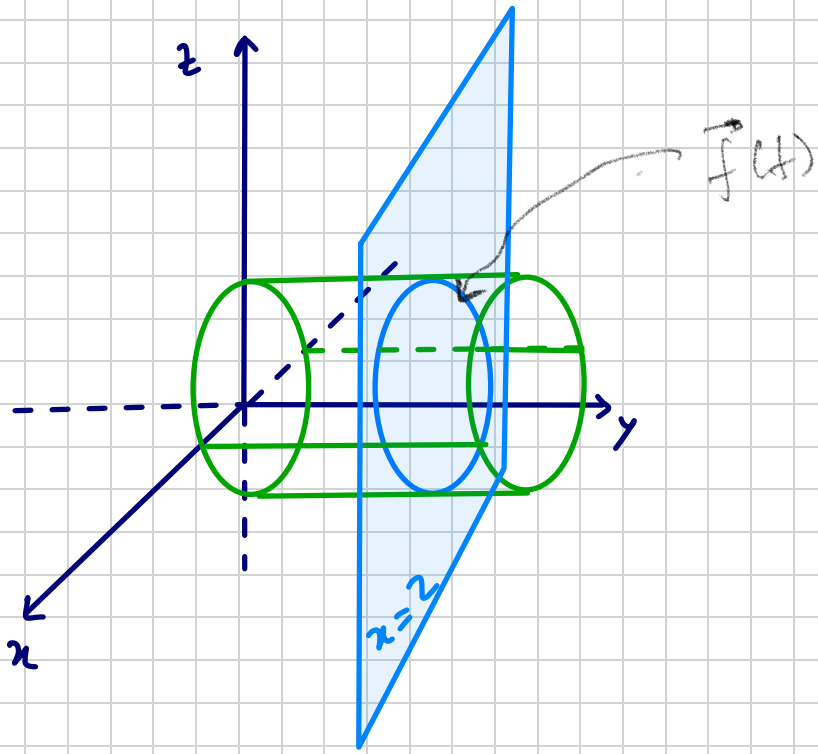
01)

(a) $\boxed{x = 2}$

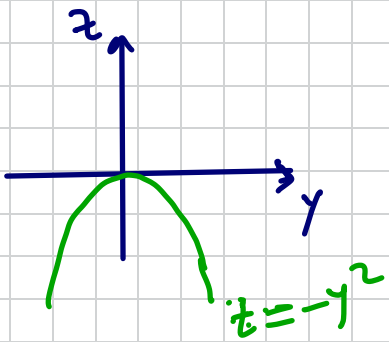
$y = \operatorname{sen} t$

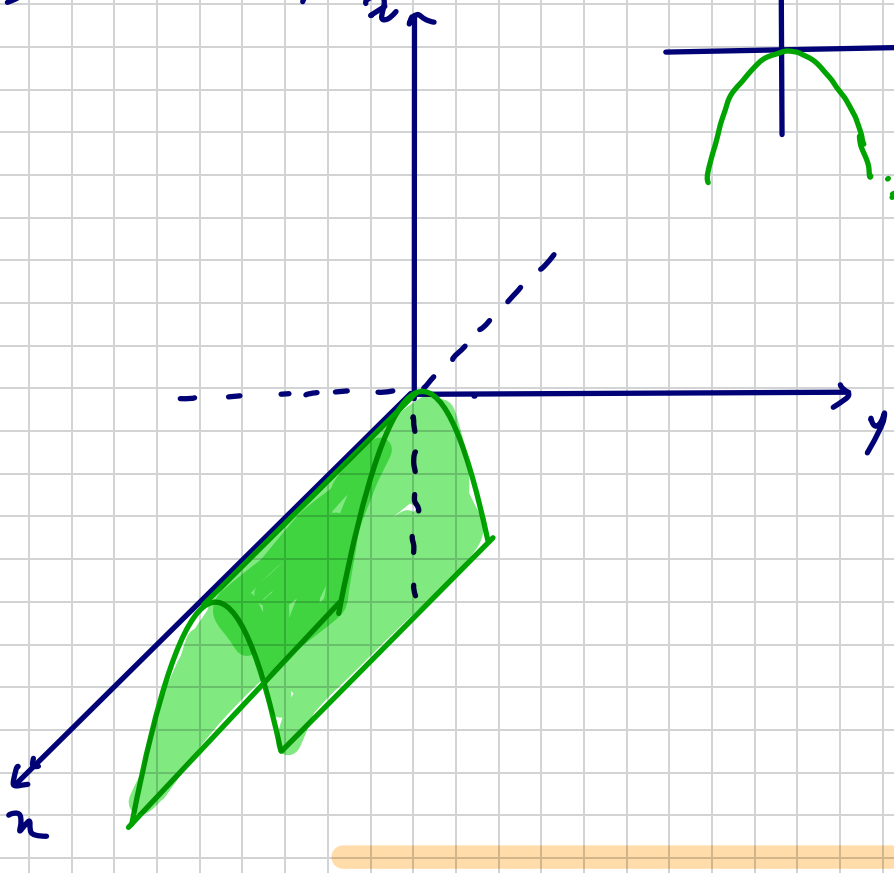
$z = \cos t$ $\Big\}$ $\rightarrow$ $x^2 + z^2 = 1$.





(b) $z = -y^2$





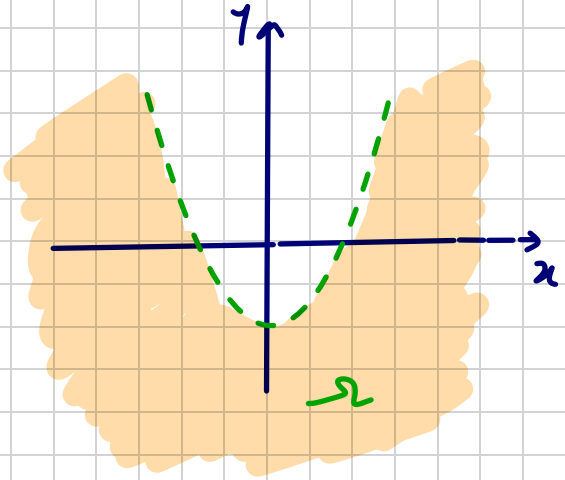
02) $f: \Omega \subset \mathbb{R}^2 \to \mathbb{R}$; $f(x,y) = \ln(\underbrace{x^2-y-2}_{>0})$

(a) $x^2-y-2>0 \Leftrightarrow -y > 2-x^2$

$\Leftrightarrow y < x^2-2$

$\Omega = D(f) = \{(x,y) \in \mathbb{R}^2 : y < x^2-2\}$

região do domínio:



(b) Ω é um aberto de $\mathbb{R}^2$ pois todo ponto $a \in \Omega$ é interior a Ω, ou seja, $\forall a \in \Omega$, $\exists \delta > 0$ tal que $B_\delta(a) \subset \Omega$.

FRONTEIRA DE Ω:

$$\partial\Omega = \{(x,y) \in \mathbb{R}^2 : y = x^2-2\}$$

FECHO: $\overline{\Omega}$:

$$\overline{\Omega} = \text{int}(\Omega) \cup \partial\Omega = \{(x,y) \in \mathbb{R}^2 : y \leq x^2-2\}.$$

(c) $\frac{\partial f}{\partial \vec{u}}$; onde $\vec{u} = \left(\frac{3}{5}, -\frac{4}{5}\right)$.

Note que $\|\vec{u}\| = \sqrt{\frac{9}{25} + \frac{16}{25}} = 1$.

Assim:

$$\frac{\partial f}{\partial \vec{u}} = \nabla f \cdot \vec{u} \quad ; \text{ onde:}$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = \left(\frac{2x}{x^2 - y - 2}, \frac{-1}{x^2 - y - 2}\right)$$

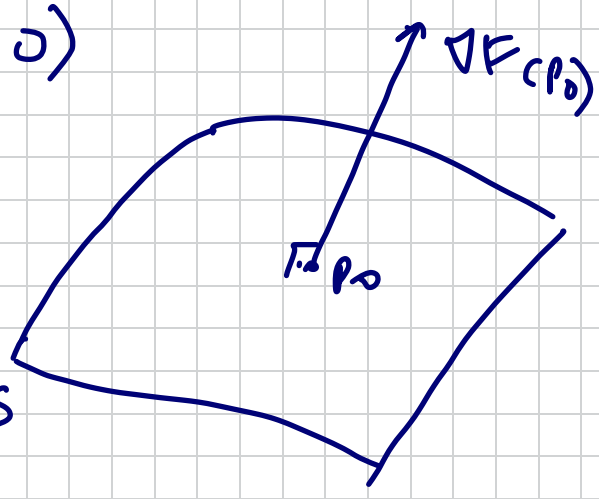
Portanto:

$$\frac{\partial f}{\partial \vec{u}} = \nabla f \cdot \vec{u} = \left(\frac{2x}{x^2 - y - 2}, -\frac{1}{x^2 - y - 2}\right) \cdot \left(\frac{3}{5}, -\frac{4}{5}\right)$$

$$= \frac{6x}{x^2 - y - 2} + \frac{4}{5} \cdot \frac{1}{x^2 - y - 2}$$

$$= \frac{30x + 4}{5(x^2 - y - 2)}$$

(d) $P_0(-2, 1, 0)$



Seja $F(x,y,z) = f(x,y) - z$

$$\Rightarrow F(x,y,z) = \ln(x^2 - y - 2) - z$$

$$\Rightarrow \nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)$$

$$= \left(\frac{2x}{x^2 - y - 2}, \frac{-1}{x^2 - y - 2}, -1 \right)$$

$$\Rightarrow \nabla F(P_0) = \nabla F(-2, 1, 0) =$$

$$= \left(\frac{2 \cdot (-2)}{(-2)^2 - 1 - 2}, \frac{-1}{(-2)^2 - 1 - 2}, -1 \right)$$

$$= (-4, -1, -1)$$

PLANO TANGENTE: Dado $P(x,y,z) \in (\pi)$, então:

$$(\pi): \nabla F(P_0) \cdot \overrightarrow{P_0P} = 0 \quad ; \text{ onde}$$

$$\overrightarrow{P_0P} = P - P_0 = (x, y, z) - (-2, 1, 0)$$

$$\Rightarrow \overrightarrow{P_0P} = (x+2, y-1, z)$$

Assim:

$$(\pi): \nabla F(P_0) \cdot \overrightarrow{P_0P} = 0$$

$$(\pi): (-4, -1, -1) \cdot (x+2, y-1, z) = 0$$

$$(\Pi): -4(x+2) - (y-1) - z = 0$$

$$(\Pi): -4x - 8 - y + 1 - z = 0$$

$$\boxed{(\Pi): -4x - y - z - 7 = 0}$$

---

03) $f(x,y,z) = x^2 + yz$

(a) $\nabla f(2,1,-1)$:

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) = (2x, z, y)$$

$$\Rightarrow \nabla f(2,1,-1) = (2\cdot(2), -1, 1) = (4, -1, 1).$$

(b) $x = r + s$ $\qquad f(r,s) = x^2 + yz$

$y = rs$

$z = r^2 + s^2$

- $\dfrac{\partial f}{\partial r} = \dfrac{\partial f}{\partial x}\cdot\dfrac{\partial x}{\partial r} + \dfrac{\partial f}{\partial y}\cdot\dfrac{\partial y}{\partial r} + \dfrac{\partial f}{\partial z}\cdot\dfrac{\partial z}{\partial r}$

$$= 2x\cdot 1 + z\cdot s + y\cdot 2r$$

$$= 2(r+s) + (r^2+s^2)s + 2\cdot r^2 s.$$

- $\frac{\partial f}{\partial \rho} = \frac{\partial f}{\partial x}\cdot\frac{\partial x}{\partial \rho} + \frac{\partial f}{\partial y}\cdot\frac{\partial y}{\partial \rho} + \frac{\partial f}{\partial z}\cdot\frac{\partial z}{\partial \rho}$

$= 2x\cdot 1 + z\cdot\sigma + y\cdot 2\rho$

$= 2(\sigma+\rho) + (\sigma^2+\rho^2)\cdot\sigma + 2\sigma\rho^2$

---

04) $f(x,y) = \begin{cases} \frac{3xy-y^2}{x^2+y^2}, & \text{se } (x,y)\neq(0,0) \\ 0, & \text{se } (x,y)=(0,0) \end{cases}$

(a) • $\lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} f(x,y) = \lim\limits_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{-y^2}{y^2} = -1.$

• $\lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} f(x,y) = \lim\limits_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{0}{x^2} = 0$

Logo, por caminhos diferentes obtemos limites diferentes, mostrando que $\nexists \lim\limits_{(x,y)\to(0,0)} f(x,y)$, onde segue que f não é contínua na origem.

(b) $\cdot \frac{\partial f}{\partial x}(0,0) = \lim_{h \to 0} \frac{f(0+h,0) - f(0,0)}{h} =$

$= \lim_{h \to 0} \frac{f(h,0) - 0}{h} = \lim_{h \to 0} \frac{\frac{0}{h^2}}{h} = 0$

$\cdot \frac{\partial f}{\partial y}(0,0) = \lim_{h \to 0} \frac{f(0,0+h) - f(0,0)}{h} =$

$= \lim_{h \to 0} \frac{f(0,h) - 0}{h} = \lim_{h \to 0} \frac{\frac{-h^2}{h^2}}{h}$

$= \lim_{h \to 0} \frac{-1}{h} = \nexists$.

---

05) $\vec{f}(t) = (2\cos t, 4t, 2\operatorname{sen} t)$.

$A(0, 2\pi, 2)$. Qual o valor do parâmetro t?

Basta fazer:

$$\begin{cases} 2 \cdot \cos t = 0 \\ 4t = 2\pi \\ 2\operatorname{sen} t = 2 \end{cases} \Leftrightarrow \boxed{t = \frac{\pi}{2}}$$

Logo, queremos, primeiramente, obter:

$\vec{f}'\left(\frac{\pi}{2}\right)$; onde:

$$\vec{f}'(t) = (-2\operatorname{sen} t,\ 4,\ 2\cos t)$$

$$\Rightarrow \vec{f}'\left(\frac{\pi}{2}\right) = \left(-2\operatorname{sen}\frac{\pi}{2},\ 4,\ 2\cos\frac{\pi}{2}\right) = (-2,\ 4,\ 0)$$

Agora, note que $\left\|\vec{f}'\left(\frac{\pi}{2}\right)\right\| = \sqrt{(-2)^2 + (4)^2 + 0^2}$

$$= \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Assim, o vetor tangente unitário será dado por:

$$\vec{u} = \frac{1}{\left\|\vec{f}'\left(\frac{\pi}{2}\right)\right\|} \cdot \vec{f}'\left(\frac{\pi}{2}\right) =$$

$$= \frac{1}{2\sqrt{5}} \cdot (-2, 4, 0) = \left(-\frac{1}{\sqrt{5}},\ \frac{2}{\sqrt{5}},\ 0\right)$$

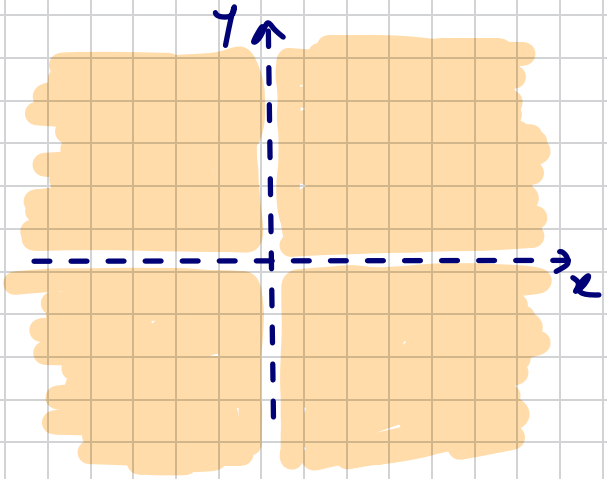
---

06) $f(x,y) = x^3 y^2$; $\vec{g}(x,y) = \left(\frac{1}{x^2 y},\ -\frac{y}{x}\right)$

(a) $D\vec{g} = ?$

$x^2 y \neq 0$ e $x \neq 0 \Leftrightarrow x, y \neq 0$.

$\Omega = D\vec{g} = \{(x,y) \in \mathbb{R}^2 :\ x \neq 0 \text{ e } y \neq 0\}$.



(b) $d_a g$; onde $a=(-2,1)$

$$\vec{g}=\left(\frac{1}{x^2y}, -\frac{y}{x}\right)=\left(x^{-2}y^{-1}, -y\cdot x^{-1}\right)=(g_1, g_2)$$

$$d_a g=\begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} \text{ ; onde:}$$

- $g_{11}=\frac{\partial g_1}{\partial x}=-2x^{-3}y^{-1}=-\frac{2}{x^3y}$
- $g_{12}=\frac{\partial g_1}{\partial y}=-x^{-2}\cdot(-1)y^{-2}=+\frac{1}{x^2y^2}$
- $g_{21}=\frac{\partial g_2}{\partial x}=-y\cdot(-1)x^{-2}=+\frac{1}{x^2y}$
- $g_{22}=\frac{\partial g_2}{\partial y}=-1\cdot x^{-1}=-\frac{1}{x}$

$$\Rightarrow d_a g=\begin{pmatrix} -\frac{2}{x^3y} & \frac{1}{x^2y^2} \\ \frac{1}{x^2y} & -\frac{1}{x} \end{pmatrix}(-2,1)=$$

$$=\begin{pmatrix} \frac{-2}{(-2)^3\cdot 1} & \frac{1}{(-2)^2\cdot(1)^2} \\ \frac{1}{(-2)^2\cdot 1} & -\frac{1}{-2} \end{pmatrix}=\begin{pmatrix} -\frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{pmatrix}$$

(c) $u : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$;

$$u(x,y) = \vec{g}(x,y) \cdot \nabla f(x,y) \quad ; \text{ onde}$$

$$\vec{g}(x,y) = \left(\frac{1}{x^2 y}, -\frac{y}{x} \right) \quad ; \quad e$$

$$\nabla f(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right) = \left(3x^2y^2, 2x^3y \right)$$

$$\Rightarrow \quad u(x,y) = \left(\frac{1}{x^2 y}, -\frac{y}{x} \right) \cdot \left(3x^2y^2, 2x^3y \right)$$

$$= \frac{3x^2y^2}{x^2y} - \frac{2x^3y^2}{x} = 3y - 2x^2y^2$$

$$\Rightarrow \boxed{u(x,y) = 3y - 2x^2y^2}$$

$\frac{\partial u}{\partial \vec{w}}$; onde $\vec{w} = \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$.

Note que $\|\vec{w}\| = \sqrt{\frac{2}{4} + \frac{2}{4}} = 1$.

$$\Rightarrow \frac{\partial u}{\partial \vec{w}} = \nabla u \cdot \vec{w} \quad ; \text{ onde:}$$

$$\nabla u = \left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \right) = \left(-4xy^2, 3 - 4x^2y \right)$$

$$\Rightarrow \frac{\partial u}{\partial \vec{w}} = \left(-4xy^2, 3 - 4x^2y \right) \cdot \left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \right)$$

$$= -2\sqrt{2}xy^2 - \frac{3\sqrt{2}}{2} + 2\sqrt{2}x^2y$$

(d)

$$F(x,y,z) = u(x,y) - z = 3y - 2x^2y^2 - z$$

(π): $\nabla F(P) \cdot \overrightarrow{PP_0} = 0$; onde $P_0(x,y,z) \in (\pi)$.

$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right) = \left(-4xy^2, 3 - 4x^2y, -1\right)$$

$$P(-1, 1, u(-1,1)) = (-1, 1, 3.(1) - 2(-1)^2.(1)^2)$$

$$= (-1, 1, 1)$$

$$\Rightarrow \nabla F(P) = (-4.(-1).(1)^2, 3 - 4.(-1)^2.(1), -1)$$

$$= (4, -1, -1)$$

$$\overrightarrow{PP_0} = P_0 - P = (x,y,z) - (-1,1,1)$$

$$= (x+1, y-1, z-1)$$

Disso, obtemos:

(π): $\nabla F(P) \cdot \overrightarrow{PP_0} = 0$

(π): $(4,-1,-1) \cdot (x+1, y-1, z-1) = 0$

(π): $4x + 4 - y + 1 - z + 1 = 0$

$$\boxed{(\pi): 4x - y - z + 6 = 0}$$