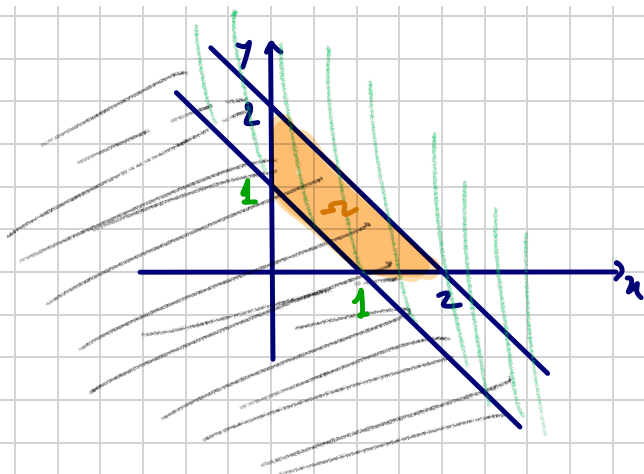


AULA DE EXERCÍCIOS.

LISTA 03

10. Calcule $\iint_{\Omega} \frac{\cos(x-y)}{\sin(x+y)} dx dy$, onde Ω é o trapézio formado por $1 \leq x+y \leq 2$, $x \geq 0$ e $y \geq 0$.



$$x+y \leq 2$$

$$y \leq -x+2$$

$$x+y \geq 1$$

$$y \geq -x+1$$

$$\begin{array}{l} \text{Então } \left\{ \begin{array}{l} u = x-y \\ + \quad v = x+y \end{array} \right. \end{array}$$

$$u+v = 2x$$

$$\Rightarrow \boxed{x = \frac{1}{2}u + \frac{1}{2}v}$$

$$y = v - x = v - \frac{1}{2}u - \frac{1}{2}v$$

$$\Rightarrow \boxed{y = -\frac{1}{2}u + \frac{1}{2}v}$$

$$T: \begin{cases} x = \frac{1}{2}u + \frac{1}{2}v \\ y = -\frac{1}{2}u + \frac{1}{2}v \end{cases}$$

$$\iint_{\Omega} \frac{\cos(x-y)}{\sin(x+y)} dx dy = \iint_{\Omega'} \frac{\cos u}{\sin v} \cdot |\det J(T)(u,v)| \cdot du dv,$$

onde:

$$J(T) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\Rightarrow \det J(T) = \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{2}$$

Resta encontrar a região Ω' a partir de Ω e

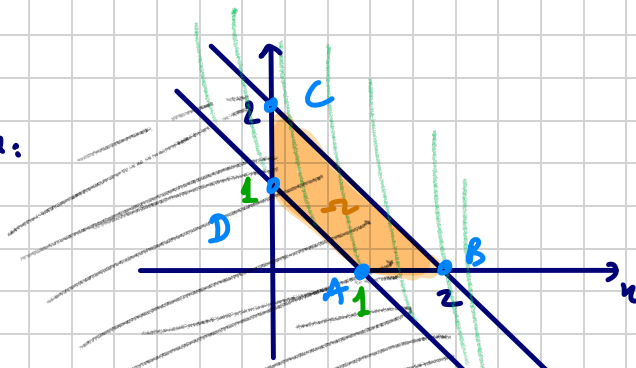
de $T(x,y) = (u,v)$, onde $\begin{cases} u = x-y \\ v = x+y \end{cases}$

Se mos:

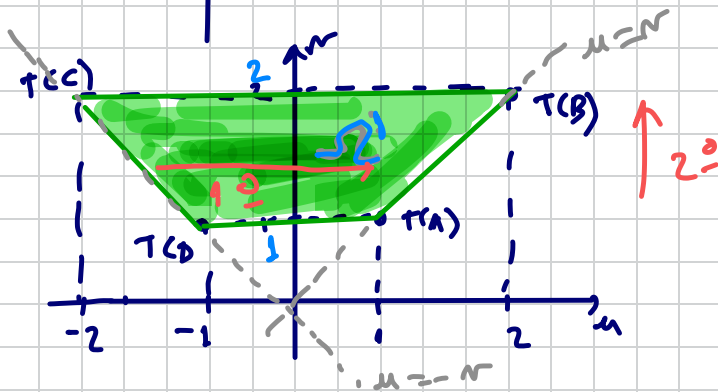
$$1 \leq \underbrace{x+y}_v \leq 2, \text{ com } x \geq 0 \text{ e } y \geq 0$$

$$1 \leq v \leq 2.$$

Note que:



(x, y)	$(u, v) = (x-y, x+y)$
$A(1, 0)$	$T(A) = (1-0, 1+0) = (1, 1)$
$B(2, 0)$	$T(B) = (2-0, 2+0) = (2, 2)$
$C(0, 2)$	$T(C) = (0-2, 0+2) = (-2, 2)$
$D(0, 1)$	$T(D) = (0-1, 0+1) = (-1, 1)$



Logo, obtenemos:

$$\begin{aligned}
 \iint_{\Omega} \frac{\cos(x-y)}{\sin(x+y)} dx dy &= \iint_{\Omega'} \frac{\cos u}{\sin v} \cdot \left| \det J_T(x, y) \right| du dv = \\
 &= \int_{v=1}^{v=2} \int_{u=-v}^{u=v} \frac{\cos u}{\sin v} \cdot \frac{1}{2} du dv = \frac{1}{2} \int_{v=1}^{v=2} \frac{1}{\sin v} \left(\int_{u=-v}^{u=v} \cos u du \right) dv = \\
 &= \frac{1}{2} \int_{v=1}^{v=2} \frac{1}{\sin v} \cdot \sin u \Big|_{u=-v}^{u=v} dv =
 \end{aligned}$$

$$= \frac{1}{2} \int_{n=1}^{n=2} \frac{1}{\sin n} \cdot (\sin n - (-\sin n)) \, dn =$$

$$= \frac{1}{2} \int_1^2 \frac{1}{\sin n} (+2 \sin n) \, dn = \int_1^2 dn = n \Big|_1^2 =$$

$$= 2 - 1 = 1 //$$

↳ :

13. Calcule as integrais impróprias abaixo.

(a) $\int_0^\infty \int_0^\infty e^{-x-y} \, dx \, dy$

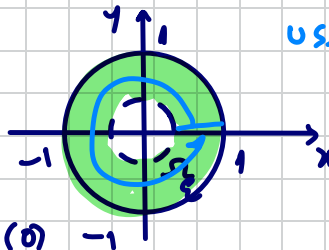
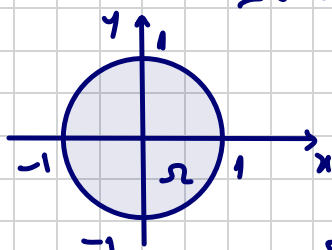
(b) $\int_0^\infty \int_0^\infty x^2 e^{-x^2-y^2} \, dx \, dy$

(c) $\iint_{x^2+y^2 \leq 1} \frac{x^2 \, dx \, dy}{(x^2+y^2)^{\frac{7}{4}}}$

(d) $\iint_{x^2+y^2 \leq 1} \ln \sqrt{x^2+y^2} \, dx \, dy$

$$\iint_{x^2+y^2 \leq 1} \frac{x^2 \, dx \, dy}{(x^2+y^2)^{\frac{7}{4}}} = \lim_{\varepsilon \rightarrow 0^+} \iint_{\Omega_\varepsilon} \frac{x^2 \, dx \, dy}{(x^2+y^2)^{\frac{7}{4}}} \quad \text{Ⓢ}$$

Some $0 < \varepsilon < 1$.



USAREMOS
COORDENADAS
POLARES.

$$\varepsilon < \rho < 1.$$

$$0 \leq \theta \leq 2\pi$$

$$\Omega_\varepsilon = \Omega \setminus B_\varepsilon(0)$$

$$= \lim_{\varepsilon \rightarrow 0^+} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=\varepsilon}^{\rho=1} \frac{(\rho \cos \theta)^2 \rho d\rho d\theta}{(\rho^2)^{\frac{7}{4}}} =$$

$x = \rho \cos \theta$
 $x^2 + y^2 = \rho^2$
 $dx dy = \rho d\rho d\theta$

$$= \lim_{\varepsilon \rightarrow 0^+} \int_{\theta=0}^{\theta=2\pi} \int_{\rho=\varepsilon}^{\rho=1} \frac{\rho^3 \cos^2 \theta d\rho d\theta}{\rho^{\frac{7}{2}}} =$$

$$= \lim_{\varepsilon \rightarrow 0^+} \int_{\theta=0}^{\theta=2\pi} \cos^2 \theta \left(\int_{\rho=\varepsilon}^{\rho=1} \rho^{3-\frac{7}{2}} d\rho \right) d\theta =$$

$$= \lim_{\varepsilon \rightarrow 0^+} \int_{\theta=0}^{\theta=2\pi} \cos^2 \theta d\theta \cdot \int_{\rho=\varepsilon}^{\rho=1} \rho^{-\frac{1}{2}} d\rho =$$

CONST. PARA
O LIMITE

$$= \int_{\theta=0}^{\theta=2\pi} \cos^2 \theta d\theta \cdot \lim_{\varepsilon \rightarrow 0^+} \frac{\rho^{\frac{1}{2}}}{\frac{1}{2}} \bigg|_{\rho=\varepsilon}^{\rho=1} =$$

$\downarrow$
 $\frac{1 + \cos 2\theta}{2}$

$$= \int_{\theta=0}^{\theta=2\pi} \frac{1 + \cos 2\theta}{2} d\theta \cdot 2 \cdot \lim_{\epsilon \rightarrow 0^+} \sqrt{\rho} \Big|_2^1 =$$

$$= \left[\frac{1}{2} \int_0^{2\pi} d\theta + \frac{1}{2} \cdot \frac{1}{2} \int_0^{2\pi} \cos 2\theta (2 d\theta) \right] \cdot 2 \cdot \lim_{\epsilon \rightarrow 0^+} (1 - \sqrt{\epsilon})$$

$$= \left(\frac{1}{2} (2\pi - 0) + \frac{1}{4} \sin 2\theta \Big|_0^{2\pi} \right) \cdot 2 \cdot 1$$

$$= \left[\pi + \frac{1}{4} (\underbrace{\sin 4\pi - \sin 0}_{=0}) \right] \cdot 2 = \underline{\underline{2\pi}}$$

