

AULA DE EXERCÍCIOS:L2.

1. Em cada item a seguir, determine o domínio $\Omega \subset \mathbb{R}^2$ de $f: \Omega \rightarrow \mathbb{R}$ e faça um esboço do domínio:

(a) $f(x, y) = \frac{1}{x^2 + y^2 - 1}$

(b) $f(x, y) = \sqrt{1 - x^2 - y^2}$

(c) $f(x, y) = \sqrt{x^2 - 4y^2 + 16}$

(d) $f(x, y) = \frac{1}{\sqrt{1 - x^2 - y^2}}$

(e) $f(x, y) = \ln(x^2 + y)$

(f) $f(x, y) = \arcsen(x + y)$

(g) $f(x, y) = \arcsen \frac{x}{x + y}$

(h) $f(x, y) = \arctan \frac{x + y}{x - y}$

(d) $f(x, y) = \frac{1}{\sqrt{1 - x^2 - y^2}}$

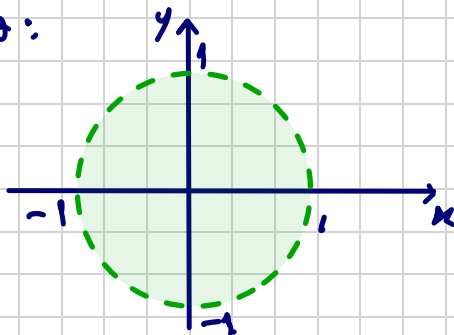
condição de existência: $1 - x^2 - y^2 > 0$

$-x^2 - y^2 > -1 \quad (x-1)$

$x^2 + y^2 < 1$

$D(f) = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

gráfico do domínio:



$$(g) f(x, y) = \arcsin \frac{x}{x+y}$$

condição de existência:

$$x+y \neq 0$$

$$x \neq -y$$

$$\left[\begin{array}{l} \underline{\text{obs:}} \text{ Do cálculo 1: } y = \arcsin u \Leftrightarrow \\ x = \sin y \\ -1 \leq x \leq 1. \end{array} \right]$$

No novo caso; devemos impor também que

$$\overbrace{-1 \leq \frac{x}{x+y} \leq 1}^{\text{II}}$$

I

$$(I): \frac{x}{x+y} \leq 1 \Leftrightarrow \frac{x}{x+y} - 1 \leq 0$$

$$\Leftrightarrow \frac{x - x - y}{x+y} \leq 0 \Leftrightarrow \frac{-y}{x+y} \leq 0 \quad (x-1)$$

$$\Leftrightarrow \frac{y}{x+y} \geq 0 \quad (\text{ou seja precisamos analisar onde este quociente é } \geq 0)$$

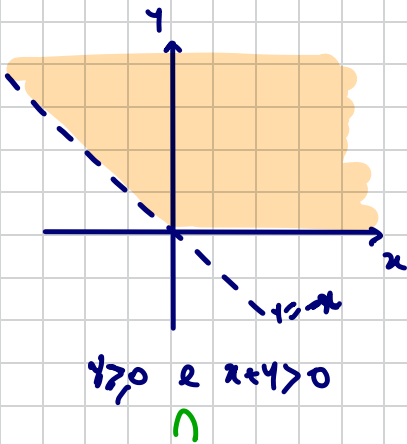
Então:

$$\frac{y}{x+y} \geq 0 \Leftrightarrow$$

$$\left\{ \begin{array}{l} y \geq 0 \text{ e } x+y > 0 \\ \text{ou} \\ y \leq 0 \text{ e } x+y < 0 \end{array} \right.$$

$$\rightarrow y > -x$$

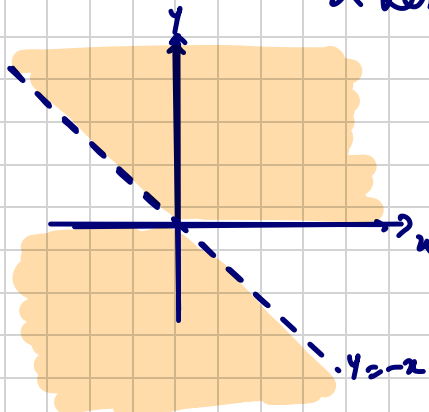
$$\rightarrow y < -x$$



ou

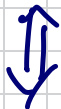
|| U "ou"
obtemos a região para a desigualdade (I):

(I):



$$(II): \frac{x}{x+y} \geq -1 \Leftrightarrow \frac{x}{x+y} + 1 \geq 0 \Leftrightarrow \frac{x+x+y}{x+y} \geq 0$$

$$\Leftrightarrow \frac{2x+y}{x+y} \geq 0$$

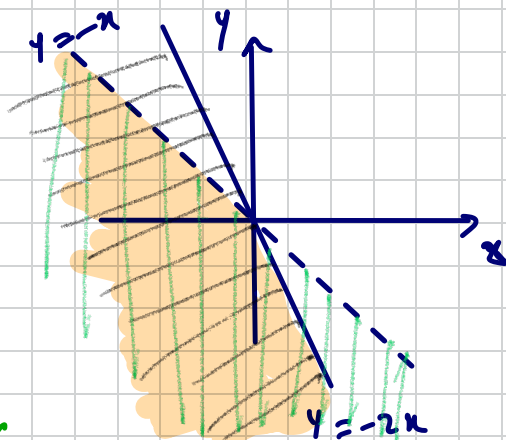
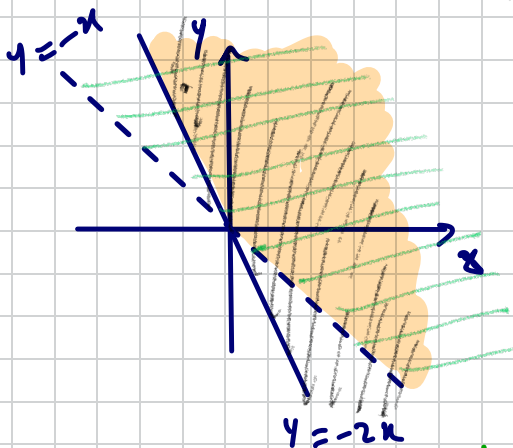


$y \geq -2x$

$$\left\{ \begin{array}{l} 2x+y \geq 0 \text{ e } x+y > 0 \\ \text{ou "U"} \end{array} \right. \rightarrow y > -x$$

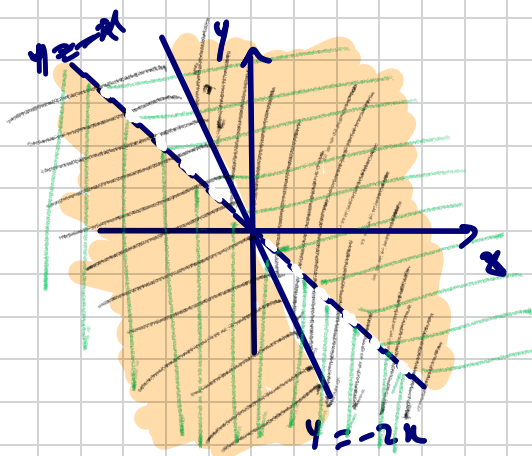
$$2x+y \leq 0 \text{ e } x+y < 0$$

$\rightarrow y \leq -2x$ $y < -x$

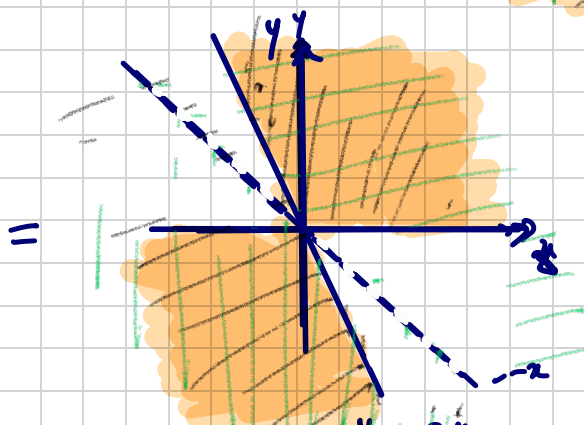
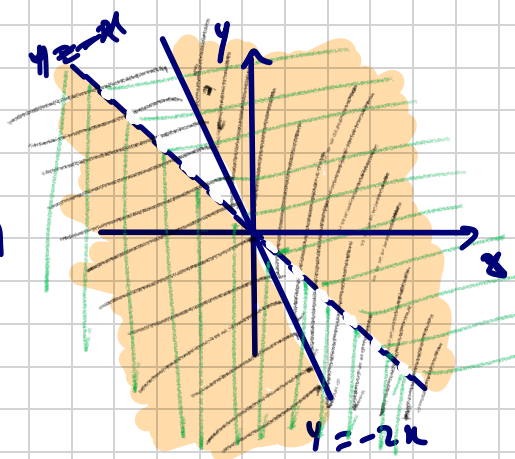
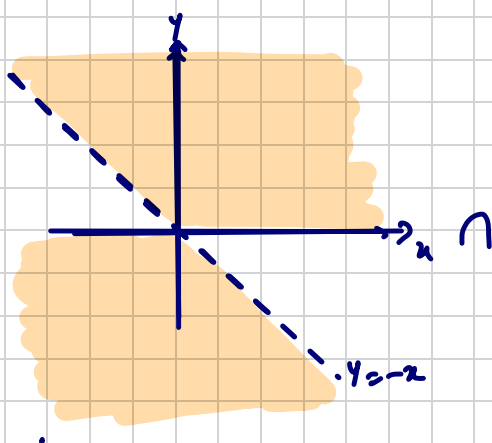


"U"

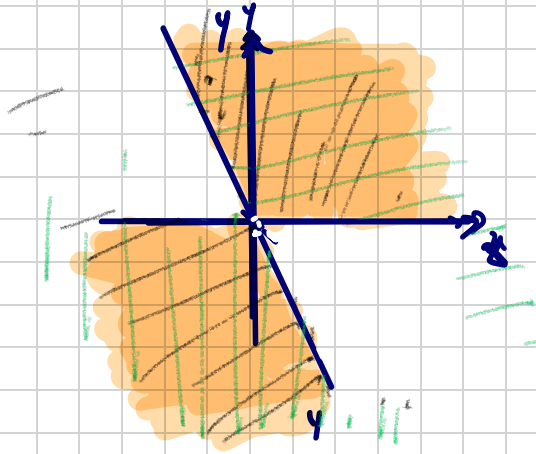
(II):



Escrevendo (I) $\cap$ (II):



Finalmente, combinando com a condição $x \neq -y$ obtemos (no caso já não está na figura acima); temos o gráfico final do domínio:



Gráfico

L2:

7. Desenhe as superfícies definidas parametricamente pelas seguintes funções:

$$(a) f \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \text{ onde } u, v \in \mathbb{R}$$

$$(b) f \begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos u \sin v \\ \sin u \sin v \\ \cos v \end{pmatrix}, \text{ onde } 0 \leq u \leq 2\pi, 0 \leq v \leq \frac{\pi}{2}.$$

$$(a) f: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

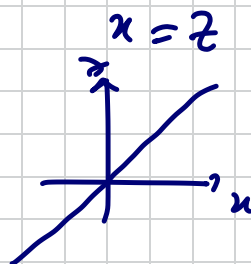
$$f(u, v) = (x, y, z) ; \text{ onde:}$$

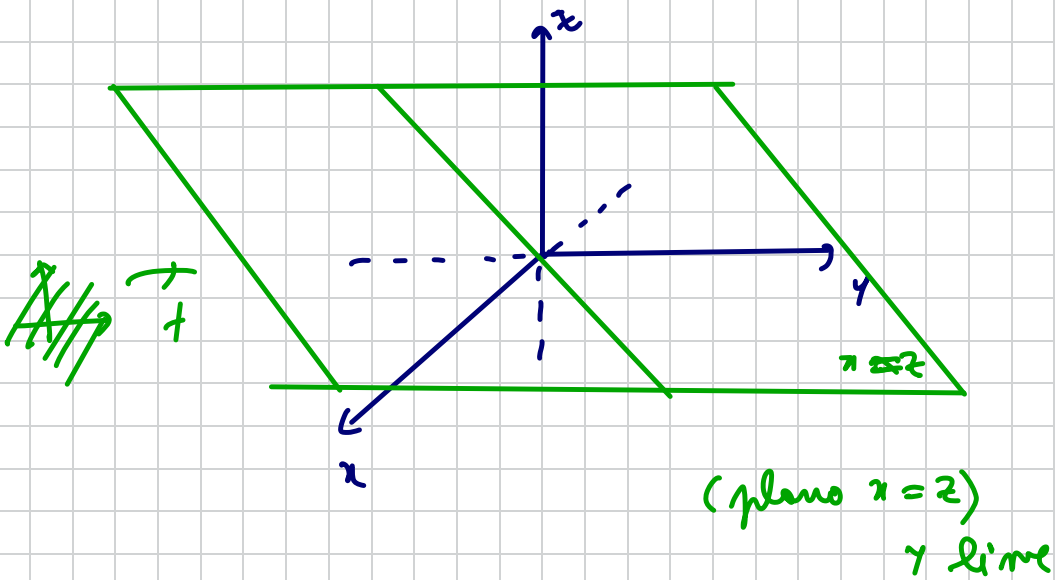
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\Leftrightarrow \begin{cases} x = u + 0 + 1 \\ y = 0 + v + 1 \\ z = u + 0 + 1 \end{cases} \Rightarrow \begin{cases} x = 1 + u \\ y = 1 + v \\ z = 1 + u \end{cases}$$

$\Downarrow$

$$y = 1 + v \in \mathbb{R}.$$



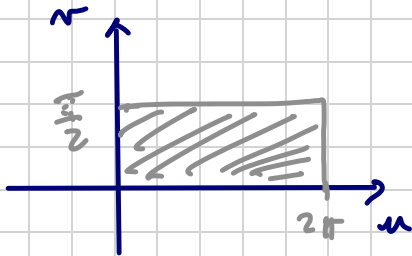


(b) $f: \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}^3$

$$f(u, v) = (\underbrace{\cos u \sin v}_x, \underbrace{\sin u \sin v}_y, \underbrace{\cos v}_z);$$

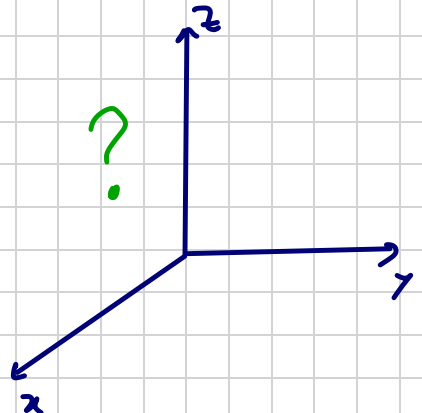
onto $\Omega: [0, 2\pi] \times [0, \frac{\pi}{2}]$

u v



f

?



Temos:

$$\begin{cases} x = \cos u \cdot \sin r \\ y = \sin u \cdot \sin r \\ z = \cos r \end{cases}$$

Elementos ao quadrado e somando, temos:

$$\begin{aligned} x^2 + y^2 + z^2 &= \cos^2 u \cdot \sin^2 r + \sin^2 u \cdot \sin^2 r + \cos^2 r \\ &= \sin^2 r \cdot (\underbrace{\cos^2 u + \sin^2 u}_{=1}) + \cos^2 r \\ &= \sin^2 r + \cos^2 r = 1. \end{aligned}$$

$$\Rightarrow \boxed{x^2 + y^2 + z^2 = 1} \quad (\text{esfera no } \mathbb{R}^3).$$

Também, note que $0 \leq u \leq 2\pi$

$$0 \leq r \leq \frac{\pi}{2}$$

$x = \cos u \cdot \sin r$ — assume menor valor quando

$u = \frac{\pi}{2}$ e $r = 0$
ou 2π
e o menor valor é zero.

Assume maior valor $\cos u = 1$ e $\sin r = 1$.

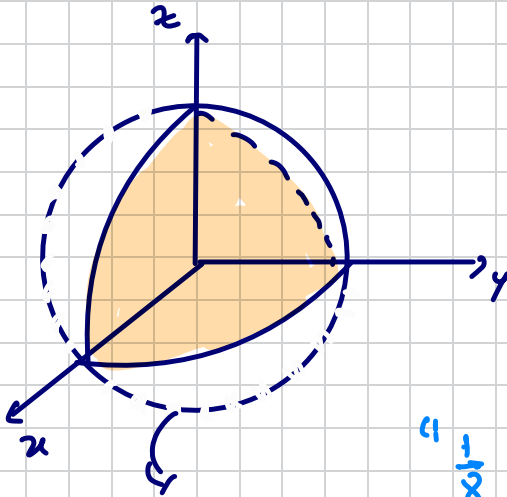
$$\Rightarrow 0 \leq x \leq 1.$$

Do mesmo modo: $0 \leq y \leq 1.$

$$\text{e } z = \cos v \quad ; \quad 0 \leq v \leq \frac{\pi}{2}$$

$$\text{então } 0 \leq z \leq 1$$

Portanto o gráfico fica só no 1º octante



" $\frac{1}{8}$ da esfera"

$$x^2 + y^2 + z^2 = 1.$$



LISTA 03

5. Em cada exercício a seguir, mostre que existe $\lim_{(x,y) \rightarrow (0,0)} f(x,y)$.

(a) $f(x,y) = \frac{x^2y + xy^2}{x^2 + y^2}$ (b) $f(x,y) = \frac{2xy}{\sqrt{x^2 + y^2}}$ (c) $f(x,y) = \frac{x^2 + 2xy}{\sqrt{x^2 + y^2}}$

(a) Note que:

$$|f(x,y)| = \left| \frac{x^2y + xy^2}{x^2 + y^2} \right| = \frac{|x^2y + xy^2|}{|x^2 + y^2|} =$$

$$= \frac{|x^2y + xy^2|}{x^2 + y^2} \leq \frac{x^2 \cdot |y| + |x| \cdot y^2}{x^2 + y^2} \quad \leq$$

$$|a+b| \leq |a| + |b|$$

$$\text{Como: } \begin{cases} |x| = \sqrt{x^2} \leq \sqrt{x^2 + y^2} \\ |y| = \sqrt{y^2} \leq \sqrt{x^2 + y^2} \end{cases} ; \text{ ent \u00e3o.}$$

$$\leq \frac{x^2 \cdot \sqrt{x^2 + y^2} + \sqrt{x^2 + y^2} \cdot y^2}{x^2 + y^2} =$$

$$= \frac{\sqrt{x^2 + y^2} (x^2 + y^2)}{x^2 + y^2} = \sqrt{x^2 + y^2}.$$

Demo, obtenemos:

$$0 \leq |f(x, y)| \leq \sqrt{x^2 + y^2}$$

Teo 7. do L'Hôpital: $\sqrt{x^2 + y^2} \rightarrow 0$
 $(x, y) \rightarrow (0, 0)$

e

$$0 \rightarrow 0$$

$(x, y) \rightarrow (0, 0)$

$$0 \leq |f(x, y)| \leq \sqrt{x^2 + y^2}$$



$$\lim_{(x, y) \rightarrow (0, 0)} |f(x, y)| = 0$$

$$\Rightarrow \lim_{(x, y) \rightarrow (0, 0)} f(x, y) = 0.$$

