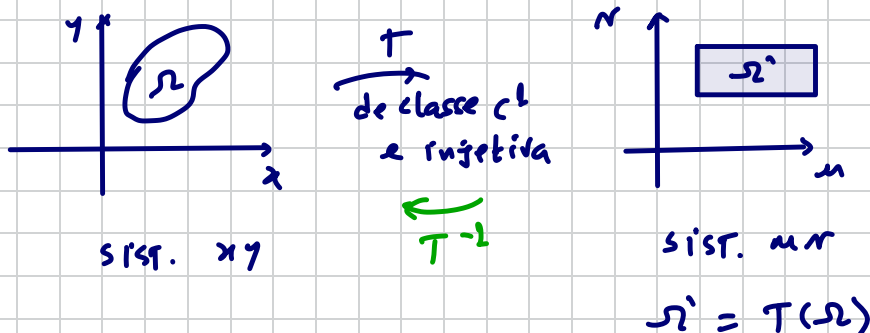


Nesta aula veremos deduzirmos a importante fórmula de mudança de coordenadas para integrais duplas:



Dessa forma, sendo $f: \Omega \subset \mathbb{R}^2 \rightarrow \mathbb{R}$ integrável, então

$$\iint_{\Omega} f(x, y) dx dy = \iint_{\Omega'} f(x(u, v), y(u, v)) \cdot |\det J(T)(u, v)| du dv$$

Vimos também um exemplo simples e importante: a mudança para o sist de coordenadas polares.

$$x = x(u, v)$$

$$y = y(u, v)$$

$$j(T)(u,v) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{bmatrix}.$$

No caso da dist. polar, temos:

$$T: \begin{cases} x = \rho \cos \theta \\ y = \rho \sin \theta \end{cases}$$

$(u,v) \rightarrow (\rho, \theta)$. Neste caso, vemos que:

$$j(T)(\rho, \theta) = \begin{bmatrix} \frac{\partial x}{\partial \rho} & \frac{\partial y}{\partial \rho} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\rho \sin \theta & \rho \cos \theta \end{bmatrix}$$

$$\Rightarrow \text{det } j(T)(\rho, \theta) = \begin{vmatrix} \cos \theta & \sin \theta \\ -\rho \sin \theta & \rho \cos \theta \end{vmatrix} =$$

$$= \rho \cos^2 \theta + \rho \sin^2 \theta = \rho (\underbrace{\cos^2 \theta + \sin^2 \theta}_{=1}) = \rho$$

Então:

$$\underbrace{\iint_R f(x,y) dx dy}_{= \iint} = \iint_{R'} f(\underbrace{x(\rho, \theta)}_{x = \rho \cos \theta}, \underbrace{y(\rho, \theta)}_{y = \rho \sin \theta}) \cdot \underbrace{|\rho|}_{>0} \cdot d\rho d\theta =$$

$$= \underbrace{\iint_{\Sigma'} f(\rho \cos \theta, \rho \sin \theta) \cdot \rho \, d\rho \, d\theta}$$

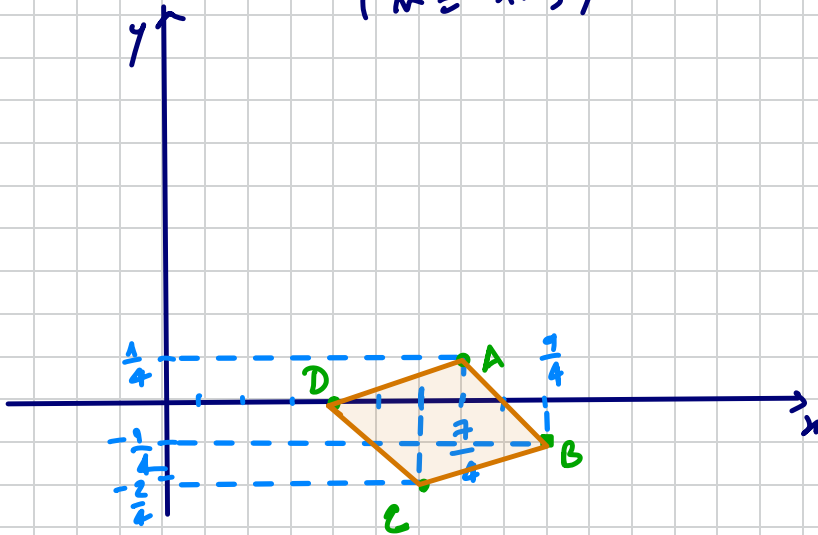
02) (LISTA 03 - EXERCÍCIO 09)

9. Calcule a integral dupla

$$\iint_{\Omega} \sqrt{x+y} \ln(x-3y) \, dx \, dy,$$

onde Ω é o quadrilátero $ABCD$ de vértices $A(\frac{7}{4}, \frac{1}{4})$, $B(\frac{9}{4}, -\frac{1}{4})$, $C(\frac{3}{2}, -\frac{1}{2})$ e $D(1, 0)$.

solução: Emere $\begin{cases} u = x+y \\ v = x-3y \end{cases}$



$$\begin{cases} u = x+y \\ v = x-3y \end{cases}$$

$$u - v = 0 + 4y$$

$$\Rightarrow y = \frac{1}{4}u - \frac{1}{4}v$$

$$u = x + y$$

$$x = u - y = u - \left(\frac{1}{4}u - \frac{1}{4}v\right)$$

$$\boxed{x = \frac{3}{4}u + \frac{1}{4}v}$$

$$\tau: \begin{cases} x = \frac{3}{4}u + \frac{1}{4}v \\ y = \frac{1}{4}u - \frac{1}{4}v \end{cases}$$

$$\underline{j(\tau)(u,v)} = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\Rightarrow \det j(\tau)(u,v) = \begin{vmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{vmatrix} = \frac{3}{4} \cdot \left(-\frac{1}{4}\right) - \left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$$

$$= -\frac{3}{16} - \frac{1}{16} = -\frac{4}{16} = -\frac{1}{4}$$

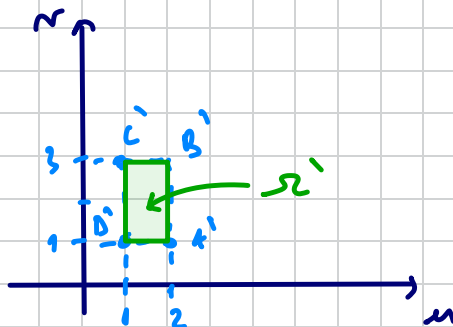
Dersu forma, determinas:

$$\iint_{\Omega} \sqrt{\underline{x+y}} \cdot \ln(\underline{x-y}) \, dx \, dy = \iint_{\Omega'} \sqrt{u} \cdot \ln v \cdot \underbrace{|\det j(\tau)(u,v)|}_{-\frac{1}{4}} \, du \, dv$$

$$= \frac{1}{4} \iint_{\mathcal{R}'} \sqrt{u} \cdot \ln v \cdot du dv$$

Desenhando a região $\mathcal{R}'$, temos:

(x, y)	$(u, v) = (x+y, x-3y)$
$A(\frac{7}{4}, \frac{1}{4})$	$A' = T(A) = (\frac{7}{4} + \frac{1}{4}, \frac{7}{4} - \frac{3}{4}) = (2, 1)$
$B(\frac{9}{4}, -\frac{1}{4})$	$B' = T(B) = (\frac{9}{4} - \frac{1}{4}, \frac{9}{4} + \frac{3}{4}) = (2, 3)$
$C(\frac{3}{2}, -\frac{1}{2})$	$C' = T(C) = (\frac{3}{2} - \frac{1}{2}, \frac{3}{2} + \frac{3}{2}) = (1, 3)$
$D(1, 0)$	$D' = T(D) = (1+0, 1-3 \cdot 0) = (1, 1)$



Então, note que em $\mathcal{R}'$, temos: $1 \leq u \leq 2$
 $1 \leq v \leq 3$

Answer:

$$\iint_{\Omega} f(x,y) dx dy = \frac{1}{4} \iint_{\Omega'} \sqrt{u} \cdot \ln r \, du dr =$$

$$= \frac{1}{4} \int_{r=1}^{r=3} \int_{u=1}^{u=2} \sqrt{u} \cdot \ln r \, du dr =$$

$$= \frac{1}{4} \int_{r=1}^{r=3} \ln r \, dr \cdot \int_{u=1}^{u=2} u^{\frac{1}{2}} du \quad \text{⊕}$$

• $\int \ln r \, dr = \int u \, dw = u \cdot w - \int w \, du$
(INTEGRAÇÃO POR PARTES)

$$\left[\begin{array}{l} u = \ln r \Rightarrow du = \frac{dr}{r} \\ dw = dr \Rightarrow w = r \end{array} \right.$$

$$\Rightarrow \int \ln r \, dr = r \cdot \ln r - \int \cancel{r} \cdot \frac{dr}{\cancel{r}}$$

$$= r \ln r - r + C$$

$$\textcircled{8} \quad \frac{1}{4} \cdot (n \ln n - n) \Big|_{n=1}^{n=3} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} \Big|_{u=1}^{u=2} =$$

$$= \frac{1}{4} \cdot (3 \ln 3 - 3 - [1 \cdot \ln 1 - 1]) \cdot \frac{2}{3} \left(2^{\frac{3}{2}} - 1^{\frac{3}{2}} \right)$$

$$= \frac{1}{4} \cdot (3 \ln 3 - 2) \cdot \frac{2}{3} \cdot (\sqrt{2^3} - 1)$$

$$= \frac{(3 \ln 3 - 2) \cdot (2\sqrt{2} - 1)}{6}$$

03) (QUESTÃO DE UMA PROVA DE 2005)

Calcule $\iint_R \frac{1}{x-y} \cdot \cos(2x+3y) dx dy$, onde R é a

região do plano xy definida pelo quadrilátero $ABCD$

de vértices nos pontos $A(\frac{3}{5}, -\frac{2}{5})$; $B(\frac{6}{5}, -\frac{4}{5})$; $C(\frac{9}{5}, -\frac{1}{5})$
e $D(\frac{6}{5}, \frac{1}{5})$.

(tenha)