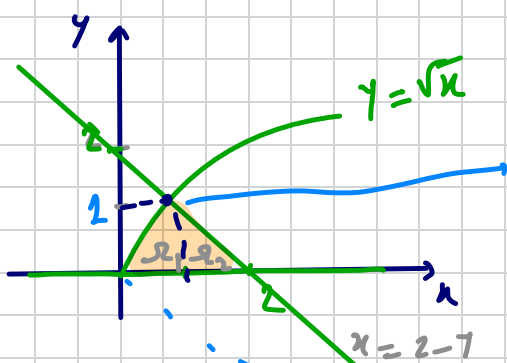


02) $\iint_{\Omega} x\sqrt{y} \, dx \, dy$; sendo Ω a região formada pelas retas $y=0$, $x+y=2$ e pela parábola $x=y^2$, no 1º quadrante.

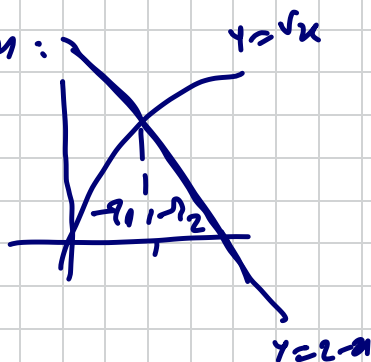
solução:



$$\begin{aligned} x &= 2-y = y^2 \\ y^2 + y - 2 &= 0 \\ \Rightarrow \boxed{y=1} \\ y &\neq -2 \end{aligned}$$

Podemos resolver de duas formas:

$$\underline{1.ª}: \iint_{\Omega} f = \iint_{\Omega_1} f + \iint_{\Omega_2} f$$



$$\underline{2.ª}: \iint_{\Omega} f = \int_{y=0}^{y=1} \left(\int_{x=y^2}^{x=2-y} x\sqrt{y} \, dx \right) dy =$$

$$y \Rightarrow x = 2 - y$$

$$= \int_{y=0}^1 \sqrt{y} \left(\int_{x=y^2}^{x=2-y} x \, dx \right) dy =$$

$\uparrow$
 $g(y)$

ESTA INTEGRAL MAIS EXTERNA "ESPERA" O RESULTADO DA INTEGRAL MAIS INTERNA, POIS A MAIS INTERNA SERÁ UMA FUNÇÃO DE y .

$$= \int_{y=0}^{y=1} \sqrt{y} \cdot \frac{x^2}{2} \Big|_{x=y^2}^{x=2-y} dy = \int_0^1 \sqrt{y} \cdot \left(\frac{(2-y)^2}{2} - \frac{y^4}{2} \right) dy =$$

$$= \frac{1}{2} \int_0^1 \sqrt{y} \cdot (4 - 4y + y^2 - y^4) dy =$$

$$= \frac{1}{2} \int_0^1 (4y^{\frac{1}{2}} - 4y \cdot y^{\frac{1}{2}} + y^2 \cdot y^{\frac{1}{2}} - y^4 \cdot y^{\frac{1}{2}}) dy =$$

$$= \frac{1}{2} \int_0^1 (4y^{\frac{1}{2}} - 4y^{\frac{3}{2}} + y^{\frac{5}{2}} - y^{\frac{9}{2}}) dy =$$

$$= \frac{1}{2} \left(4 \cdot \frac{y^{\frac{3}{2}}}{\frac{3}{2}} - 4 \frac{y^{\frac{5}{2}}}{\frac{5}{2}} + \frac{y^{\frac{7}{2}}}{\frac{7}{2}} - \frac{y^{\frac{11}{2}}}{\frac{11}{2}} \right) \Big|_0^1 =$$

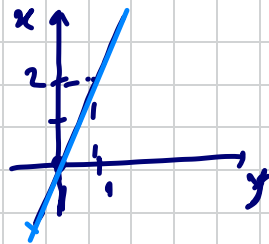
$$= \left(\frac{4}{3} y^{\frac{3}{2}} - \frac{4}{5} y^{\frac{5}{2}} + \frac{1}{7} y^{\frac{7}{2}} - \frac{1}{11} y^{\frac{9}{2}} \right) \Big|_0^1 =$$

$$= \frac{4}{3} - \frac{4}{5} + \frac{1}{7} - \frac{1}{11} - 0 = \dots = \frac{676}{1155}$$

03) Obtenha o volume do tetraedro limitado pelos planos $x+2y+z=2$, $x=2y$, $x=0$ e $z=0$.

Solução:

traços: no plano xy : $x=2y$

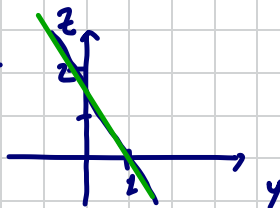


$$\boxed{x+2y+z=2} \text{ (plano no } \mathbb{R}^3 \text{)}$$

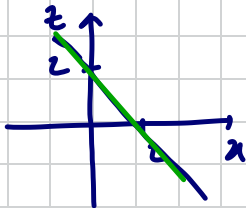
↘ traços: • $x=0$ (plano yz):

$$z=2-x-2y$$

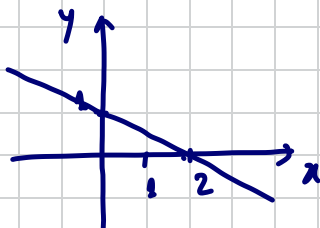
$$2y+z=2$$



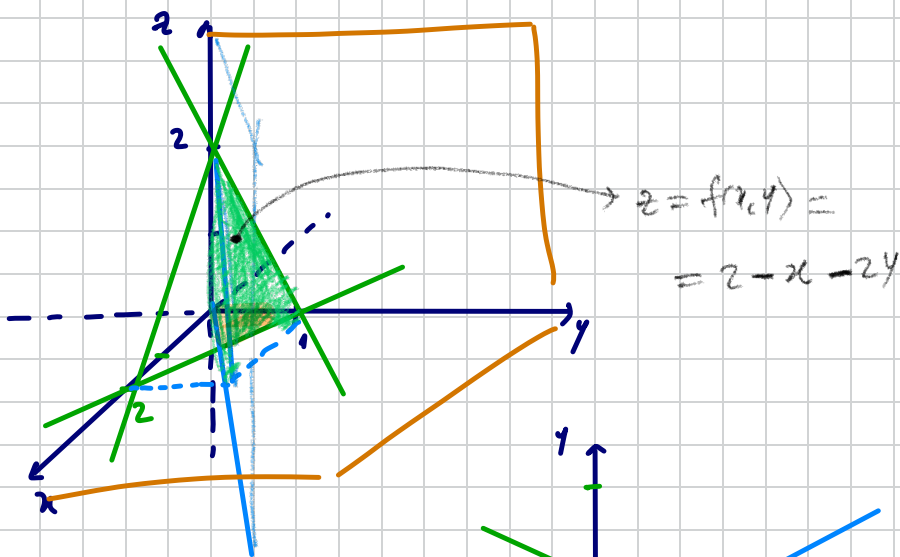
• $y=0$: $x+z=2$ (plano xz)



• $z=0: x+2y=2$ (plano xy)



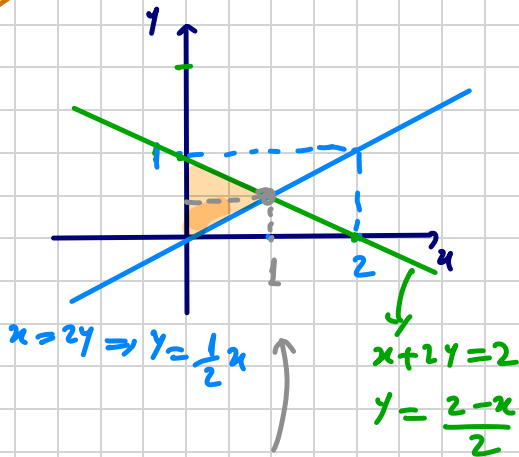
Combinando estes traços do plano $x+2y+z=2$,
obtemos:



$$z = f(x, y) = 2 - x - 2y$$

Agora, o volume do
tetraedro encontrado
será dado por:

$$V = \iint_D f(x, y) \cdot dA ;$$



INTERCEPTO:

$$y = y$$

$$\frac{1}{2}x = \frac{2-x}{2}$$

$$x = 1$$

onde Ω é dado pela região: da reta verde até a reta azul, no intervalo de x de 0 até $x=1$;

ou seja:

$$V = \int_{x=0}^{x=1} \int_{y=\frac{1}{2}x}^{y=\frac{2-x}{2}} (2-x-2y) dy dx =$$

$$= \int_{x=0}^{x=1} \left(2y - xy - y^2 \right) \bigg|_{y=\frac{1}{2}x}^{y=\frac{2-x}{2}} dx =$$

$$= \int_{x=0}^{x=1} \left(\cancel{2} \cdot \left(\frac{2-x}{\cancel{2}} \right) - x \cdot \left(\frac{2-x}{2} \right) - \left(\frac{2-x}{2} \right)^2 - \right. \\ \left. - \left[\cancel{2} \cdot \frac{1}{\cancel{2}} x - x \cdot \frac{1}{2} x - \left(\frac{1}{2} x \right)^2 \right] \right) dx$$

$$= \int_0^1 \left(2-x - \frac{2x-x^2}{2} - \frac{4-4x+x^2}{4} - x + \frac{x^2}{2} + \frac{x^2}{4} \right) dx$$

$$= \int_0^1 \left(\underbrace{2-x}_{\text{green}} - x + \frac{x^2}{2} - \underbrace{1}_{\text{green}} + \cancel{x} - \cancel{\frac{x^2}{4}} - \cancel{x} + \frac{x^2}{2} + \cancel{\frac{x^2}{4}} \right) dx$$

$$= \int_0^1 (1 - 2x + x^2) dx = \left(x - x^2 + \frac{x^3}{3} \right) \Big|_0^1 =$$

$$= 1 - 1 + \frac{1}{3} - (0 - 0 + 0) = \frac{1}{3} \text{ u.r.}$$

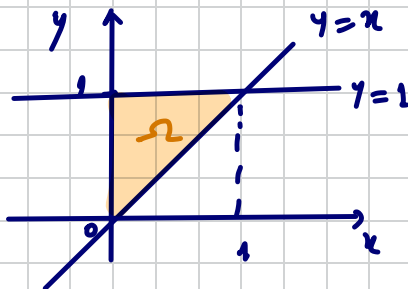
(unidades de volume)

04) Calcule $\int_{x=0}^{x=1} \int_{y=x}^{y=1} \sin y^2 dy dx$.

Solução: Note que a integral mais interna ($\sin y$) temos $\int \sin y^2 dy$; neste caso teríamos

$\int \sin u du$, com $u = y^2 \Rightarrow du = 2y dy$; mas não temos a variável y no diferencial!

Fica faltando variável. Isso indica que devemos trocar a ordem de integração.



$$\int_{x=0}^{x=1} \int_{y=x}^{y=1} \sin y^2 dy dx = \int_{y=0}^{y=1} \int_{x=0}^{x=y} \sin y^2 dx dy =$$

$$= \int_{y=0}^{y=1} \sin y^2 \left(\int_{x=0}^{x=y} dx \right) dy = \int_{y=0}^{y=1} \sin y^2 \cdot x \Big|_{x=0}^{x=y} dy =$$

$$= \int_{y=0}^{y=1} \sin y^2 \cdot (y - 0) dy = \frac{1}{2} \int_0^1 \sin y^2 \cdot 2y dy =$$

$u = y^2 \Rightarrow du = 2y dy$

$$= \frac{1}{2} \cdot (-\cos y^2) \Big|_0^1 = \frac{1}{2} \cdot (-\cos 1 + \underbrace{\cos 0}_{=1})$$

$$= \frac{1 - \cos 1}{2}$$

05) Calcule $\int_0^1 \int_{3y}^3 e^{x^2} dx dy$. (tema)

06) Esboce a região de integração e faça a mudança de ordem de integração para

$$\int_0^4 \int_0^{\sqrt{x}} f(y, x) dy dx. \quad (\text{tema})$$