CÁLCULO 3, TURMA T_2

$\boxed{17/07/26 \text{ – AULA 45}}$

AULA DE EXERCÍCIOS

LISTA 12

8. Use o teorema de Green para calcular cada integral de linha a seguir, ao longo da curva dada com orientação positiva.

   (a) $\int_\gamma e^y dx + 2xe^y dy$, onde γ é o quadrado de lados $x = 0$, $x = 1$, $y = 0$ e $y = 1$.

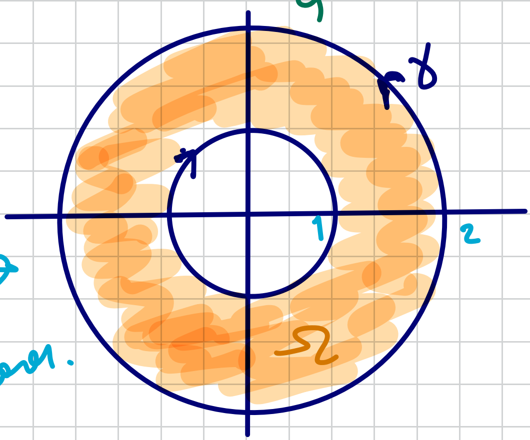
   (b) $\int_\gamma (ye^{\sqrt{x}})dx + (2x + \cos y^2)dy$, onde γ é a fronteira da região delimitada pelas parábolas $y = x^2$ e $x = y^2$

   (c) $\int_\gamma xe^{-2x}dx + (x^4 + 2x^2y^2)dy$, onde γ é a região entre as circunferências $x^2 + y^2 = 1$ e $x^2 + y^2 = 4$.

FRONTEIRA DA

FIZ PARTE DELA NA T3, MAS ABANDONAMOS PELO VOLUME DE CÁLCULOS.

(c) $$\oint_\gamma \underbrace{x e^{-2x}}_{P} dx + (\underbrace{x^4 + 2x^2y^2}_{Q}) dy = ?$$



$0 \leq \theta \leq 2\pi$

$1 \leq \rho \leq 2$

como Ω é circular, usaremos coord. polares.

$$\oint_\gamma P dx + Q dy = \iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA \text{ , onde:}$$

$$P = x \cdot e^{-2x} \Rightarrow \frac{\partial P}{\partial y} = 0$$

$$Q = x^4 + 2x^2y^2 \Rightarrow \frac{\partial Q}{\partial x} = 4x^3 + 4xy^2$$

$$\Rightarrow \oint_{\gamma} P\,dx + Q\,dy = \iint_{\Omega} \left(4x^3 + 4xy^2 - 0\right) dA =$$

$$= \iint_{\Omega} 4x\underbrace{\left(x^2+y^2\right)}_{\rho^2} \underline{\underline{dA}}$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{\rho=1}^{\rho=2} 4 \cdot \rho\cos\theta \cdot \rho^2 \cdot \underline{\underline{\rho\,d\rho\,d\theta}} =$$

$$= \int_{\theta=0}^{\theta=2\pi} \int_{\rho=1}^{\rho=2} 4\rho^4 \cdot \cos\theta\,d\rho\,d\theta =$$

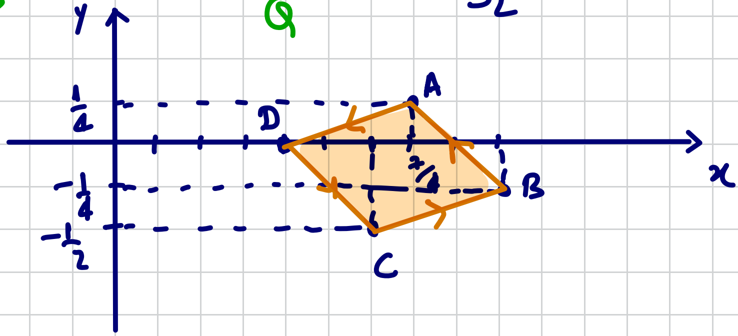
$$= \int_{\theta=0}^{\theta=2\pi} \cos\theta\,d\theta \int_{\rho=1}^{\rho=2} 4\rho^4\,d\rho = \operatorname{sen}\theta\Big|_0^{2\pi} \cdot \left. 4\frac{\rho^5}{5} \right|_1^2 =$$

$$= \underbrace{\left(\operatorname{sen}2\pi - \operatorname{sen}0\right)}_{=0} \cdot \left(\frac{4}{5}\cdot(2)^5 - \frac{4}{5}\cdot(1)^5\right) = 0$$

L12

9. Utilize o Teorema de Green para calcular $\oint_\gamma \cos(x-3y)dx + \ln(x+y)dy$, onde γ é o quadrilátero $ABCD$ de vértices $A(\frac{7}{4},\frac{1}{4})$, $B(\frac{9}{4},-\frac{1}{4})$, $C(\frac{3}{2},-\frac{1}{2})$ e $D(1,0)$.

$$\oint_\gamma \underbrace{\cos(x-3y)}_{P}dx + \underbrace{\ln(x+y)}_{Q}dy = \iint_\Omega \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)dA =$$



$$P = \cos(x-3y) \Rightarrow \frac{\partial P}{\partial y} = -\operatorname{sen}(x-3y).(-3) = +3.\operatorname{sen}(x-3y)$$

$$Q = \ln(x+y) \Rightarrow \frac{\partial Q}{\partial x} = \frac{1}{x+y}$$

$$= \iint_\Omega \left(\frac{1}{x+y} - 3.\operatorname{sen}(x-3y)\right).dA =$$

DEVEMOS EFETUAR UMA MUDANÇA DE VARIÁVEL

$$-\begin{cases} u = x+y \\ v = x-3y \end{cases}$$

SUBTRAINDO, VEM:

$$u - v = x + y - (x-3y)$$

$$u - v = \cancel{x} + y - \cancel{x} + 3y$$

$$4y = u - v$$

$$\boxed{y = \frac{1}{4}u - \frac{1}{4}v}$$

$$u = x + y$$

$$\Rightarrow x = u - y$$

$$x = u - \frac{1}{4}u + \frac{1}{4}v \Rightarrow \boxed{x = \frac{3}{4}u + \frac{1}{4}v}$$

$$= \iint_{\Omega'} \left(\frac{1}{u} - 3\,\mathrm{sen}\, v\right) \left|\det J(T)(u,v)\right| . \, du\, dv \stackrel{\triangle}{=}$$

onde:

$$J(T)(u,v) = \begin{bmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

$$\Rightarrow \det J(T)(u,v) = \begin{vmatrix} \frac{3}{4} & \frac{1}{4} \\ \frac{1}{4} & -\frac{1}{4} \end{vmatrix} = -\frac{3}{16} - \frac{1}{16}$$

$-$ $+$

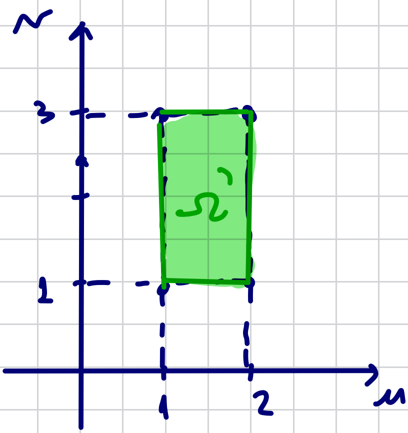
$$= -\frac{4}{16} = -\frac{1}{4}.$$

Disso, segue que:

$$\triangleq \iint_{\Omega'} \left(\frac{1}{u} - 3.\operatorname{sen} v\right) . \left|-\frac{1}{4}\right| . du\, dv =$$

$$= \frac{1}{4} . \iint_{\Omega'} \left(\frac{1}{u} - 3 \operatorname{sen} v\right) du\, dv \ \boxed{=}$$

(x,y)	$(u,v) = (x+y,\ x-3y)$
$A\left(\frac{7}{4}, \frac{1}{4}\right)$	$T(A) = \left(\frac{7}{4} + \frac{1}{4},\ \frac{7}{4} - \frac{3}{4}\right) = (2,1)$
$B\left(\frac{9}{4}, -\frac{1}{4}\right)$	$T(B) = \left(\frac{9}{4} - \frac{1}{4},\ \frac{9}{4} + \frac{3}{4}\right) = (2,3)$
$C\left(\frac{3}{2}, -\frac{1}{2}\right)$	$T(C) = \left(\frac{3}{2} - \frac{1}{2},\ \frac{3}{2} + \frac{3}{2}\right) = (1,3)$
$D(1,0)$	$T(D) = (1+0,\ 1+3.0) = (1,1)$



$$\boxed{=} \frac{1}{4}\int_{v=1}^{v=3}\int_{u=1}^{u=2}\left(\frac{1}{u} - 3\cdot \operatorname{sen} v\right) du\, dv =$$

$$= \frac{1}{4}\int_{v=1}^{v=3}\int_{u=1}^{u=2}\frac{1}{u}\, du\, dv - \frac{3}{4}\int_{v=1}^{v=3}\int_{u=1}^{u=2} \operatorname{sen} v\, du\, dv$$

$$= \frac{1}{4}\int_{v=1}^{v=3} dv \cdot \int_{u=1}^{u=2}\frac{du}{u} - \frac{3}{4}\cdot\int_{v=1}^{v=3} \operatorname{sen} v\, dv \cdot \int_{u=1}^{u=2} du$$

$$= \frac{1}{4}\, v\Big|_1^3 \cdot \ln|u|\Big|_1^2 - \frac{3}{4}(-\cos v)\Big|_1^3 \cdot u\Big|_1^2 =$$

$$= \frac{1}{4}\cdot(3-1)\cdot(\ln 2 - \ln 1) + \frac{3}{4}(\cos 3 - \cos 1)\cdot(2-1)$$

$$= \frac{2}{4}\cdot(\ln 2 - 0) + \frac{3}{4}(\cos 3 - \cos 1)\cdot 1$$

$$= \frac{1}{2}\ln 2 + \frac{3}{4}(\cos 3 - \cos 1)$$

LISTA 11

4. Encontre a divergência e o rotacional do campo vetorial dado:

(a) $\vec{F}(x,y,z) = (e^x \cos y, e^x \operatorname{sen} y, z)$

(b) $\vec{F}(x,y,z) = (x^2, y^2, z^2)$

(a) $\vec{F}(x,y,z) = (\underbrace{e^x \cdot \cos y}_{F_1}, \underbrace{e^x \cdot \operatorname{sen} y}_{F_2}, \underbrace{z}_{F_3})$

- $\operatorname{div} \vec{F} = \dfrac{\partial F_1}{\partial x} + \dfrac{\partial F_2}{\partial y} + \dfrac{\partial F_3}{\partial z}$

$$= e^x \cdot \cos y + e^x \cdot \cos y + 1 = 2 \cdot e^x \cos y + 1$$

- $\operatorname{rot} \vec{F} = \left| \begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ e^x \cos y & e^x \operatorname{sen} y & z \end{matrix} \right| \begin{matrix} \vec{i} & \vec{j} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} \\ e^x \cos y & e^x \operatorname{sen} y \end{matrix}$

$$= \Big(\underbrace{\frac{\partial}{\partial y}(z)}_{0} - \underbrace{\frac{\partial}{\partial z}(e^x \operatorname{sen} y)}_{0}\Big)\vec{i} + \Big(\underbrace{\frac{\partial}{\partial z}(e^x \cos y)}_{=0} - \underbrace{\frac{\partial}{\partial x}(z)}_{=0}\Big)\vec{j} + \Big(\frac{\partial}{\partial x}(e^x \operatorname{sen} y) - \frac{\partial}{\partial y}(e^x \cos y)\Big)\vec{k}$$

$$= 0 \cdot \vec{i} + 0\vec{j} + (e^x \cdot \operatorname{sen} y - (-e^x \cdot \operatorname{sen} y))\vec{k}$$

$$= 0\vec{i} + 0\vec{j} + 2 \operatorname{sen} y \cdot e^x \cdot \vec{k} = (0, 0, 2 \operatorname{sen} y \cdot e^x)$$