CÁLCULO 3, TURMA T3.

$\boxed{03/06/21 - \text{AULA } 28}$

AULA DE EXERCÍCIOS.

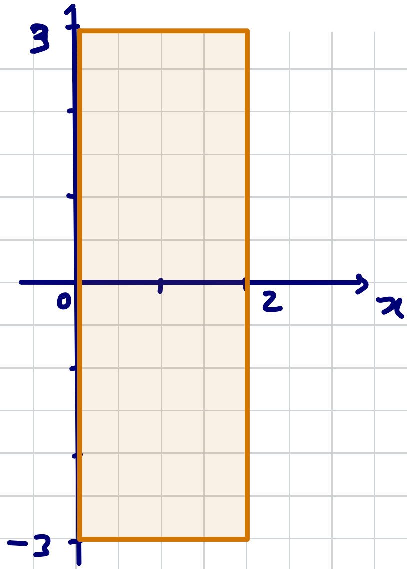
LISTA 08:

1. Calcule cada integral dupla a seguir:

   (a) $\displaystyle\iint_A \operatorname{sen}(x+y)\,dx\,dy$, onde $A = [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}]$.

   (b) $\displaystyle\iint_A \frac{xy^2}{1+x^2}\,dx\,dy$, onde $A = \{(x,y) \in \mathbb{R}^2 : 0 \le x \le 2 \text{ e } -3 \le y \le 3\}$.

   (c) $\displaystyle\iint_A x \operatorname{sen} xy\,dx\,dy$, onde A é o retângulo $0 \le x \le \pi$, $0 \le y \le 1$. (Resp. π)



(b) $$\iint_A \frac{xy^2}{1+x^2}\,dx\,dy =$$

$$= \int_{y=-3}^{y=3} \left(\int_{x=0}^{x=2} \frac{xy^2}{1+x^2}\,dx \right) dy$$

$$= \int_{y=-3}^{y=3} y^2 \cdot \frac{1}{2} \left(\underbrace{\int_{x=0}^{x=2} \frac{2x}{1+x^2}\,dx}_{\int \frac{dr}{r} = \ln|r| + c} \right) dy = \frac{1}{2} \int_{y=-3}^{y=3} y^2 \left(\ln|1+x^2| \right)\Big|_{x=0}^{x=2} dy$$

$r = 1 + x^2$

$\Rightarrow dr = 2x\,dx$

$$= \frac{1}{2}\int_{-3}^{3} y^2 \cdot \left(\ln|1+4| - \underbrace{\ln|1|}_{=0}\right) dy =$$

$$= \frac{1}{2}\ln 5 \int_{-3}^{3} y^2\, dy = \frac{1}{2}\ln 5 \cdot \left(\frac{y^3}{3}\Big|_{-3}^{3} \right) =$$

$$= \frac{1}{2}\ln 5 \cdot \left(\frac{(3)^{\cancel{3}\,2}}{\cancel{3}} - \frac{(-3)^{\cancel{3}}}{\cancel{3}} \right) = \frac{1}{2}\ln 5\,(3^2+3^2)$$

$$= 9\ln 5$$

L8

3. Calcule as integrais iteradas:

(a) $\int_0^1 \int_0^{x^2} (x+2y)\,dy\,dx$ (b) $\int_0^1 \int_x^{2-x} (x^2-y)\,dy\,dx$ (c) $\int_1^2 \int_y^2 xy\,dx\,dy$

(d) $\int_{-1}^1 \int_{x^2}^{1-x^2} 2x^2y^2\,dy\,dx$ (e) $\int_{\frac{\pi}{2}}^{\pi} \int_0^{y^2} \operatorname{sen}\frac{x}{y}\,dx\,dy$

(b) $$\int_{x=0}^{x=1} \left(\int_{y=x}^{y=2-x} (x^2-y)\,dy \right) dx = \int_{x=0}^{x=1} \left(x^2 y - \frac{y^2}{2} \right)\Big|_{y=x}^{y=2-x} dx =$$

$$= \int_{x=0}^{x=1} \left(x^2(2-x) - \frac{(2-x)^2}{2} - \left[x^2 \cdot x - \frac{x^2}{2} \right] \right) dx$$

$$= \int_0^1 \left(2x^2 - x^3 - \frac{4}{2} + \frac{4x}{2} \cancel{- \frac{x^2}{2}} - x^3 \cancel{+ \frac{x^2}{2}} \right) dx =$$

$$= \int_0^1 \left(-2x^3 + 2x^2 + 2x - 2 \right) dx$$

$$= \left(-\frac{2x^4}{4} + \frac{2x^3}{3} + \frac{2x^2}{2} - 2x \right) \Bigg|_0^1 =$$

$$= \left(-\frac{x^2}{2} + \frac{2x^3}{3} + x^2 - 2x \right) \Bigg|_0^1 =$$

$$= -\frac{1}{2} + \frac{2}{3} + 1 - 2 - 0 = \frac{-3+4+6-12}{6} = -\frac{5}{6}$$

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e) $$\int_{y=\frac{\pi}{2}}^{y=\pi} \left(\int_{x=0}^{x=y^2} \operatorname{sen}\frac{x}{y} \, dx \right) dy = \int_{y=\frac{\pi}{2}}^{y=\pi} \left(y \cdot \int_{x=0}^{x=y^2} \operatorname{sen}\underbrace{\frac{x}{y}}_{w} \cdot \underbrace{\frac{1}{y} dx}_{dw} \right) dy$$

$$\int \operatorname{sen} w \, dw = -\cos w + C$$

$$w = \frac{x}{y} \rightarrow dw = \frac{1}{y} \cdot dx$$

$$= \int_{y=\frac{\pi}{2}}^{y=\pi} y \cdot \left(-\cos \frac{x}{y}\right)\Bigg|_{x=0}^{x=y^2} dy = \int_{y=\frac{\pi}{2}}^{y=\pi} y \cdot \left(-\cos \frac{y^{\cancel{2}}}{\cancel{y}} - (-\cos 0)\right) dy$$

$$= \int_{\frac{\pi}{2}}^{\pi} y \cdot \left(-\cos y + 1\right) dy = \underbrace{-\int_{\frac{\pi}{2}}^{\pi} y \cdot \cos y \, dy}_{\text{INTEGRAR POR PARTES } (*)} + \int_{\frac{\pi}{2}}^{\pi} dy =$$

$$(*) \int y \cos y \, dy = \int u \, dv = u \cdot v - \int v \, du$$

$$\begin{cases} u = y \Rightarrow du = dy \\ dv = \cos y \, dy \Rightarrow v = \int \cos y \, dy = \operatorname{sen} y \end{cases}$$

$$\Rightarrow \int y \cos y \, dy = y \cdot \operatorname{sen} y - \int \operatorname{sen} y \cdot dy$$

$$= y \operatorname{sen} y - (-\cos y) + C$$

$$= y \cdot \operatorname{sen} y + \cos y + C.$$

$$= -\left(y \cdot \operatorname{sen} y + \cos y\right)\Bigg|_{\frac{\pi}{2}}^{\pi} + y\Bigg|_{\frac{\pi}{2}}^{\pi} =$$

$$= -\Big[\pi \underbrace{\operatorname{sen}\pi}_{0} + \underbrace{\cos\pi}_{2} - \Big(\frac{\pi}{2}\underbrace{\operatorname{sen}\frac{\pi}{2}}_{1} + \underbrace{\cos\frac{\pi}{2}}_{0}\Big)\Big] + \underbrace{\pi - \frac{\pi}{2}}_{=\frac{\pi}{2}}$$

$$\underbrace{= -\Big[1 - \frac{\pi}{2}\Big] + \frac{\pi}{2}}_{} = -1 + \frac{\pi}{2} + \frac{\pi}{2} = \pi - 1$$

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4. Em cada item a seguir, esboce o domínio Ω e calcule a integral indicada.

(a) $\Omega = \{(x, y) \in \mathbb{R}^2 \ x \geq 0, y \geq 0, x + y \leq 1\}$ e $f(x, y) = x^2 y$.

(b) Ω é o quadrado de vértices em $(1, 0)$, $(-1, 0)$, $(0, 1)$ e $(0, -1)$ e $f(x, y) = xe^y$.

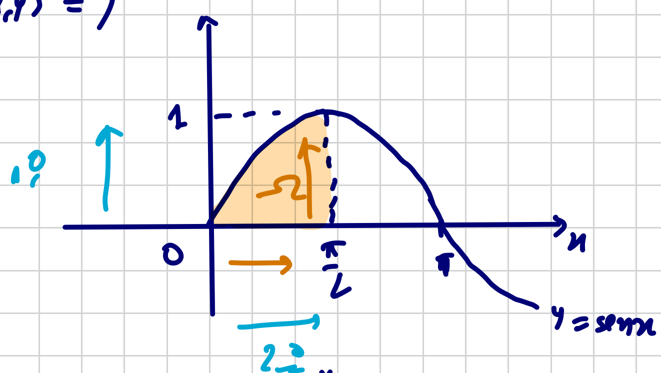
(c) $\Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq \sin x,\ 0 \leq x \leq \frac{\pi}{2}\}$ e $f(x, y) = y$.

(d) Ω é o domínio delimitado pela parábola $y = x^2$, o eixo horizontal e a reta $x = 1$ e $f(x, y) = xe^y$.

(e) Ω é o semicírculo $x^2 + y^2 \leq 1$, $x \geq 0$ e $f(x, y) = y$.

(c) $\Omega = \{(x,y) \in \mathbb{R}^2 : \quad 0 \leq y \leq \operatorname{sen} x \ ; \ 0 \leq x \leq \frac{\pi}{2}\}$

e $f(x,y) = y$



$$\iint_\Omega f(x,y)\, dA = \int_{x=0}^{x=\frac{\pi}{2}} \int_{y=0}^{y=\operatorname{sen} x} y \,.\, dy \,.\, dx =$$

$$= \int_{x=0}^{x=\frac{\pi}{2}} \left.\frac{y^2}{2}\right|_{y=0}^{y=\operatorname{sen} x} dx = \int_0^{\frac{\pi}{2}} \left(\frac{\operatorname{sen}^2 x}{2} - 0\right) dx =$$

$$= \frac{1}{2} \int_0^{\frac{\pi}{2}} \operatorname{sen}^2 x \, dx = \frac{1}{2} \int_0^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} \, dx =$$

$$\operatorname{sen}^2 x = \frac{1-\cos 2x}{2}$$

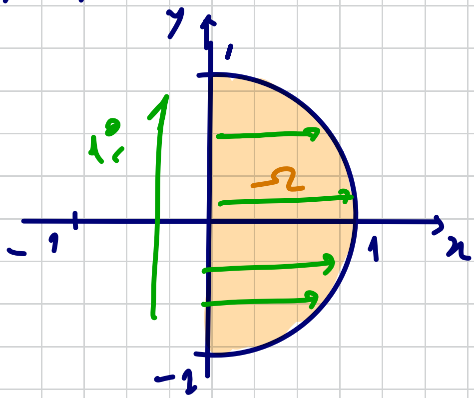
$$= \frac{1}{4} \int_0^{\frac{\pi}{2}} dx - \frac{1}{4} \cdot \frac{1}{2} \int_0^{\frac{\pi}{2}} \cos 2x \, 2\, dx =$$

$$u = 2x \Rightarrow du = 2dx$$

$$= \frac{1}{4} \left. x \right|_0^{\frac{\pi}{2}} - \frac{1}{8} \left. \operatorname{sen} 2x \right|_0^{\frac{\pi}{2}} = \frac{1}{4}\left(\frac{\pi}{2} - 0\right) - \frac{1}{8}\cancel{(\underbrace{\operatorname{sen}\pi}_{0} - \underbrace{\operatorname{sen} 0}_{0})}$$

$$= \frac{\pi}{8}$$

(e) Ω é o semicírculo $x^2+y^2 \le 1$, $x \geq 0$ e $f(x,y) = y$.



$$\iint_\Omega f(x,y)\,dA = \int_{y=-1}^{y=1} \int_{x=0}^{x=\sqrt{1-y^2}} y \,dx\,dy =$$

$$= \int_{y=-1}^{y=1} y \left(\int_{x=0}^{x=\sqrt{1-y^2}} dx \right) dy = \int_{y=-1}^{y=1} y \cdot \left. x \right|_{x=0}^{x=\sqrt{1-y^2}} dy =$$

$$= \int_{y=-1}^{y=1} y \cdot \sqrt{1-y^2} \cdot dy = -\frac{1}{2} \int_{-1}^{1} (1-y^2)^{1/2} (-2y\,dy) =$$

$\int u^k du \quad u = 1-y^2$

$\Rightarrow du = -2y\,dy$

$$= -\frac{1}{2} \frac{(1-y^2)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Bigg|_{-1}^{1} = -\frac{1}{\cancel{2}}\frac{\cancel{2}}{3}(1-y^2)^{\frac{3}{2}}\Bigg|_{-1}^{1}$$

$$= -\frac{1}{3}\cdot(1-y^2)^{\frac{3}{2}}\Bigg|_{-1}^{1} = -\frac{1}{3}\cdot\Big[\underbrace{(1-1)^{\frac{3}{2}}}_{0} - \underbrace{(1-(-1)^2)^{\frac{3}{2}}}_{0}\Big]$$

$$= -\frac{1}{3}\cdot 0 = 0$$

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5. Calcule as integrais duplas abaixo (será preciso inverter a ordem de integração)

(a) $\int_0^{\pi}\int_x^{\pi}\frac{\operatorname{sen} y}{y}dydx$

(b) $\int_0^{2}\int_x^{2}2y^2\operatorname{sen} xydydx$

(c) $\int_0^{2}\int_0^{4-x^2}\frac{xe^{2y}}{4-y}dydx$

(d) $\int_0^{1}\int_y^{1}\sqrt{1+x^2}dxdy$

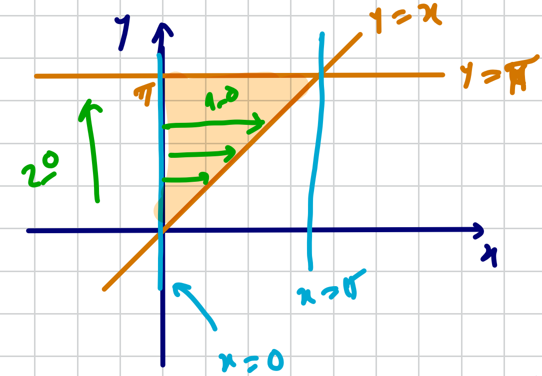
(e) $\int_0^{1}\int_y^{1}e^{x^2}dxdy$

(f) $\int_1^{2}\int_1^{2}ye^{xy}dxdy$

(a) $\int_{x=0}^{x=\pi}\int_{y=x}^{y=\pi}\frac{\operatorname{sen} y}{y}dydx$

Como não é possível calcular $\int \frac{\operatorname{sen} y}{y}dy$, precisamos trocar a ordem de integração.

por essa razão devemos desenhar a região para ajustar os limites de integração.



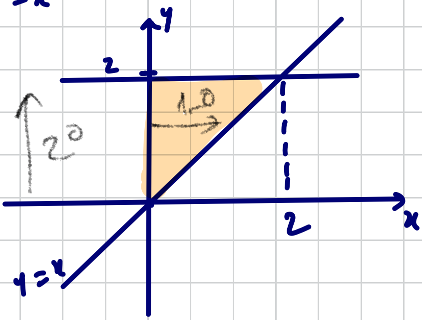
$$\int_{x=0}^{y=\pi}\int_{y=x}^{y=\pi}\frac{\operatorname{sen} y}{y}\,dy\,dx = \int_{y=0}^{y=\pi}\left(\int_{x=0}^{x=y}\underbrace{\frac{\operatorname{sen} y}{y}}_{\text{CONSTANTE PARA } x.}dx\right)dy =$$

$$= \int_{y=0}^{x=\pi}\frac{\operatorname{sen} y}{y}\cdot\left(\int_{x=0}^{x=y}dx\right)dy = \int_{y=0}^{y=\pi}\frac{\operatorname{sen} y}{y}\cdot x\Big|_{x=0}^{x=y}dy =$$

$$= \int_{0}^{\pi}\frac{\operatorname{sen} y}{\cancel{y}}\cdot\cancel{y}\cdot dy = \int_{0}^{\pi}\operatorname{sen} y\,dy = -\cos y\Big|_{0}^{\pi} =$$

$$= -\left(\underbrace{\cos\pi}_{0} - \underbrace{\cos 0}_{1}\right) = -(-1) = 1 \,//$$

(b) $\displaystyle\int_{x=0}^{x=2}\int_{y=x}^{y=2} 2y^2 \operatorname{sen} xy \; dy\, dx = 2\int_{y=0}^{y=2}\int_{x=0}^{x=y} y^2 \operatorname{sen} xy \, dx\; dy =$



$$= 2\cdot\int_{y=0}^{y=2} \frac{y^{\cancel{2}}}{\cancel{y}}\left(\int_{x=0}^{x=y} \underbrace{\operatorname{sen} \overbrace{xy}^{r}}\,(\overbrace{y\,dx}^{dr})\right) dy =$$

$\int \operatorname{sen} r \, dr = -\cos r + c$

$r = xy \Rightarrow dr = y\,dx$

$$= 2\cdot\int_{y=0}^{y=2} y\left(-\cos xy\right)\Big|_{x=0}^{x=y} dy =$$

$$= 2\int_0^2 y\cdot\left(-\cos y^2 - (-\underbrace{\cos 0}_{1})\right)\cdot dy =$$

$$= -2\int_0^2 \underbrace{y\cdot\cos y^2}\, dy + 2\int_0^2 y\, dy =$$

$\hookrightarrow u = y^2 \Rightarrow du = 2y\,dy$

$$= -\int_0^2 \cos y^2 \,(2y\,dy) + \left. \cancel{2}\frac{y^2}{\cancel{2}} \right|_0^2 =$$

$$= -\operatorname{sen} y^2 \Big|_0^2 + 4 = -\operatorname{sen} 4 + \underbrace{\operatorname{sen} 0}_{0} + 4$$

$$= 4 - \operatorname{sen} 4 \;//$$

$$V = \iiint_\Omega dV$$

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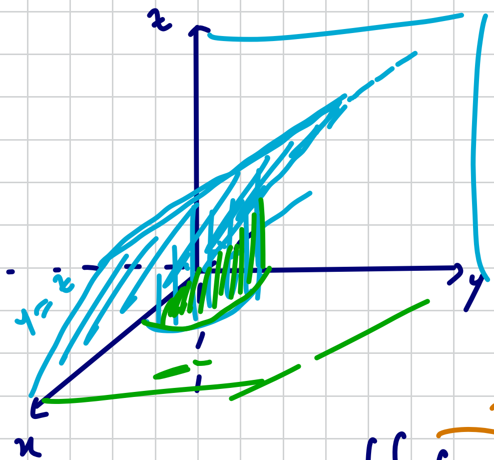
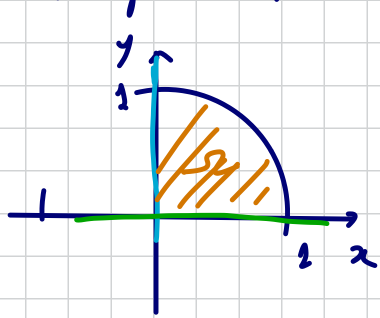
8. Calcule o volume do sólido limitado pelo cilindro $x^2 + y^2 = 1$ e pelos planos $y = z$, $x = 0$, $z = 0$ no primeiro octante.



no plano xy:



$$V = \iint_\Omega \overbrace{f(x,y)}^{= z = y} dA = \iint_\Omega y \, dA =$$

$$= \int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} y \, dy \, dx = \int_{x=0}^{x=1} \frac{y^2}{2} \Big|_{y=0}^{y=\sqrt{1-x^2}} dx =$$

$$= \frac{1}{2} \int_{x=0}^{x=1} \left(\left(\cancel{\sqrt{1-x^2}} \right)^{\cancel{2}} - 0^2 \right) dx =$$

$$= \frac{1}{2} \int_0^1 (1-x^2) dx = \frac{1}{2} \cdot \left(x - \frac{x^3}{3} \right) \Big|_0^1 =$$

$$= \frac{1}{2} \cdot \left(1 - \frac{1}{3} - 0 \right) = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = \underline{\underline{\frac{1}{3} \text{ u.r.}}}$$