CÁLCULO 3, TURMA T2

03/06/24 — AULA 28

AULA DE EXERCÍCIOS.

LISTA 08.

4. Em cada item a seguir, esboce o domínio Ω e calcule a integral indicada.

(a) $\Omega = \{(x, y) \in \mathbb{R}^2 \; x \geq 0, y \geq 0, x + y \leq 1\}$ e $f(x, y) = x^2 y$.

(b) Ω é o quadrado de vértices em $(1, 0)$, $(-1, 0)$, $(0, 1)$ e $(0, -1)$ e $f(x, y) = xe^y$.

(c) $\Omega = \{(x, y) \in \mathbb{R}^2 : 0 \leq y \leq \sin x,\ 0 \leq x \leq \frac{\pi}{2}\}$ e $f(x, y) = y$.

(d) Ω é o domínio delimitado pela parábola $y = x^2$, o eixo horizontal e a reta $x = 1$ e $f(x, y) = xe^y$.

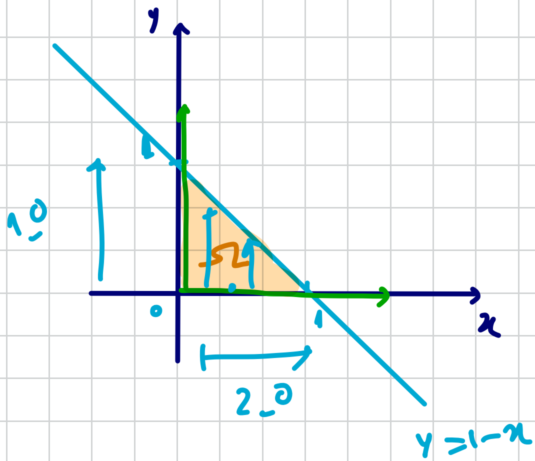
(e) Ω é o semicírculo $x^2 + y^2 \leq 1$, $x \geq 0$ e $f(x, y) = y$.

(a) $\Omega = \{(x,y) \in \mathbb{R}^2 : \quad x \geq 0, \quad y \geq 0, \quad x + y \leq 1\}$.

$f(x,y) = x^2 y$.

$x + y = 1$.

$y = 1 - x$



$$\iint_{\Omega} f(x,y)\, dA = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x} x^2 y \, dy \, dx =$$

$$= \int_{x=0}^{x=1} x^2 \left(\int_{y=0}^{y=1-x} y \, dy \right) dx = \int_{x=0}^{x=1} x^2 \cdot \left. \frac{y^2}{2} \right|_{y=0}^{y=1-x} dx =$$

$$= \int_{x=0}^{x=1} x^2 \cdot \left[\frac{(1-x)^2}{2} - \frac{(0)^2}{2} \right] dx =$$

$$= \int_0^1 x^2 \cdot \left(\frac{1-2x+x^2}{2} \right) dx =$$

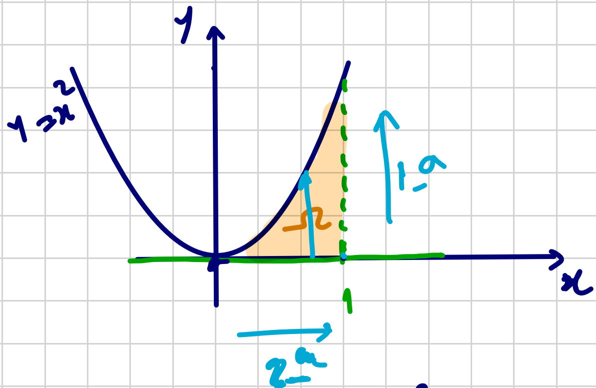
$$= \frac{1}{2} \int_0^1 \left(x^2 - 2x^3 + x^4 \right) dx = \frac{1}{2} \cdot \left. \left(\frac{x^3}{3} - \frac{2x^4}{4} + \frac{x^5}{5} \right) \right|_0^1 =$$

$$= \frac{1}{2} \left. \left(\frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} \right) \right|_0^1 = \frac{1}{2} \cdot \left(\frac{1}{3} - \frac{1}{2} + \frac{1}{5} - 0 \right)$$

$$= \frac{1}{2} \cdot \frac{10 - 15 + 6}{30} = \frac{1}{60}$$

(d) Ω é delimitado pela parábola $y = x^2$, o eixo horizontal e a reta $x = 1$.

A função é $f(x, y) = x \cdot e^y$.



$$\iint_\Omega f(x,y) \cdot dA = \int_{x=0}^{x=1} \int_{y=0}^{y=x^2} x \cdot e^y \, dy \, dx =$$

$$= \int_{x=0}^{x=1} x \cdot \left(\int_{y=0}^{y=x^2} e^y \, dy \right) dx = \int_{x=0}^{x=1} x \cdot \left. e^y \right|_{y=0}^{y=x^2} dx =$$

$$= \int_{x=0}^{x=1} x \cdot \left(e^{x^2} - \underbrace{e^0}_{=1} \right) dx = \int_0^1 \left(x \cdot e^{x^2} - x \right) dx =$$

$$= \frac{1}{2}\int_0^1 2x \cdot e^{x^2}\, dx - \int_0^1 x \cdot dx =$$

$$\int e^u\, du$$

$$u = x^2 \Rightarrow du = 2x\, dx$$

$$= \frac{1}{2}\, e^{x^2}\Big|_0^1 - \frac{x^2}{2}\Big|_0^1 = \frac{1}{2}\cdot\left(e^1 - \underbrace{e^0}_{1}\right) - \frac{1}{2} + 0$$

$$= \frac{1}{2}e - \frac{1}{2} - \frac{1}{2} = \frac{e-2}{2} \quad //$$

5. Calcule as integrais duplas abaixo (será preciso inverter a ordem de integração)

(a) $\int_0^\pi \int_x^\pi \frac{\operatorname{sen} y}{y}\, dy dx$

(b) $\int_0^2 \int_x^2 2y^2 \operatorname{sen} xy\, dy dx$

(c) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\, dy dx$

(d) $\int_0^1 \int_y^1 \sqrt{1+x^2}\, dx dy$

(e) $\int_0^1 \int_y^1 e^{x^2}\, dx dy$

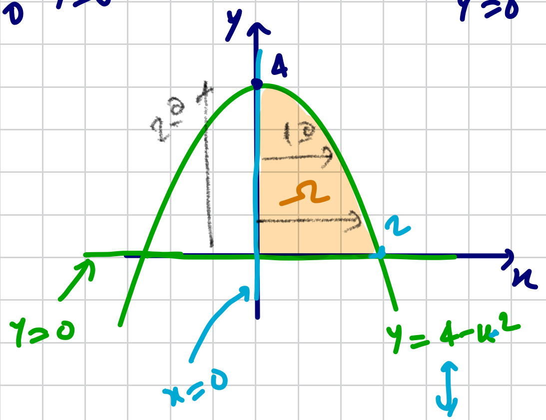
(f) $\int_1^2 \int_1^2 y e^{xy}\, dx dy$

(c) Note que $\int \frac{e^{2y}}{4-y}\, dy$ (mais interna)

não é calculável. Então, precisamos trocar a ordem de integração. Com efeito, os limites de integração serão alterados, razão pela qual precisamos fazer o desenho da região de integração.

$$\int_{x=0}^{x=2}\int_{y=0}^{y=4-x^2} \frac{x\,e^{2y}}{4-y}\,dy\,dx = \int_{y=0}^{y=4}\int_{x=0}^{x=\sqrt{4-y}} \frac{x\,e^{2y}}{4-y}\,dx\,dy =$$

CONSTANTE PARA x



$$x^2 = 4-y$$
$$x = \sqrt{4-y}$$

$$= \int_{y=0}^{y=4} \frac{e^{2y}}{4-y}\cdot\left(\int_{x=0}^{x=\sqrt{4-y}} x\,dx\right)dy$$

$$= \int_{y=0}^{y=4} \frac{e^{2y}}{4-y}\cdot \left.\frac{x^2}{2}\right|_{x=0}^{x=\sqrt{4-y}} dy$$

$$= \int_{y=0}^{y=4} \frac{e^{2y}}{4-y}\left[\frac{(\cancel{\sqrt{4-y}})^{\cancel{2}}}{2} - \frac{0}{2}\right]dy =$$

$$= \int_0^4 \frac{e^{2y}}{\cancel{4-y}}\cdot\frac{(\cancel{4-y})}{2}\,dy = \frac{1}{2}\cdot\frac{1}{2}\int_0^4 e^{\overset{u}{2y}}\,(\overset{du}{2dy})$$

$$\int e^u\,du = e^u + c$$

$u = 2y$

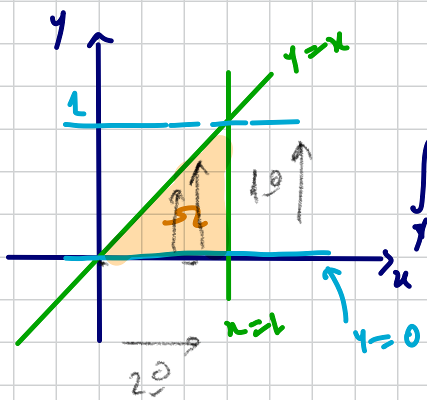
$du = 2dy$

$$= \frac{1}{4} \cdot e^{2y}\Big|_{0}^{4} = \frac{1}{4}\cdot\left(e^{8} - e^{0}\right) = \frac{e^{8}-1}{4}$$ //

e) $$\int_{y=0}^{y=1}\int_{x=y}^{x=1} e^{x^2}\,dx\,dy$$

obs.: $\int e^{x^2}dx$ não tem como calcular.

Então, precisamos trocar a ordem de integração.



Disso, temos:

$$\int_{y=0}^{y=1}\int_{x=y}^{x=1} e^{x^2}\,dx\,dy = \int_{x=0}^{x=1}\int_{y=0}^{y=x} e^{x^2}\,dy\,dx =$$

$$= \int_{x=0}^{x=1} e^{x^2}\left(\int_{y=0}^{y=x} dy\right)dx = \int_{x=0}^{x=1} e^{x^2}\cdot y\Big|_{y=0}^{y=x}\,dx =$$

$$= \int_{x=0}^{x=1} e^{\overbrace{x^2}^{r}} \cdot (x-0)\,dx = \frac{1}{2}\int_0^1 e^{x^2}\,\underbrace{2x\,dx}_{dr} =$$

$\int e^r\,dr = e^r + C$

$r = x^2 \Rightarrow dr = 2x\,dx$

$$= \frac{1}{2}\cdot\left(e^{x^2}\right)\Big|_0^1 = \frac{1}{2}\cdot\left(e^1 - \underbrace{e^0}_{=1}\right) = \frac{e-1}{2}$$

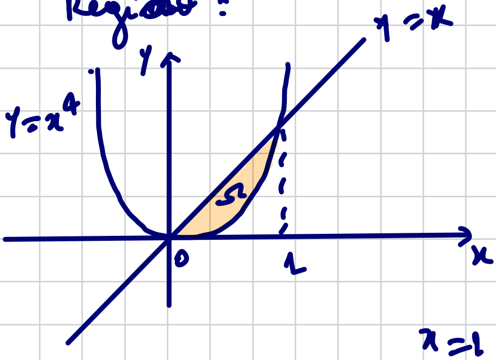
28:

6. Calcule o volume do sólido abaixo do plano $x + 2y - z = 0$ e acima da região limitada por $y = x$ e $y = x^4$.

$\downarrow$

$z = f(x,y) = x + 2y$

Região:



$x = x^4 \Leftrightarrow x = 1$

$$V = \iint_\Omega f(x,y)\,dA = \int_{x=0}^{x=1}\int_{y=x^4}^{y=x} (x+2y)\,dy\,dx =$$

$$= \int_{x=0}^{x=1} \left(xy + y^2\right)\Big|_{y=x^4}^{y=x} dx =$$

$$= \int_{x=0}^{x=1} \left[x \cdot x + x^2 - \left(x \cdot x^4 + (x^4)^2\right)\right] dx =$$

$$= \int_0^1 \left(x^2 + x^2 - x^5 - x^8\right) dx = \int_0^1 \left(2x^2 - x^5 - x^8\right) dx =$$

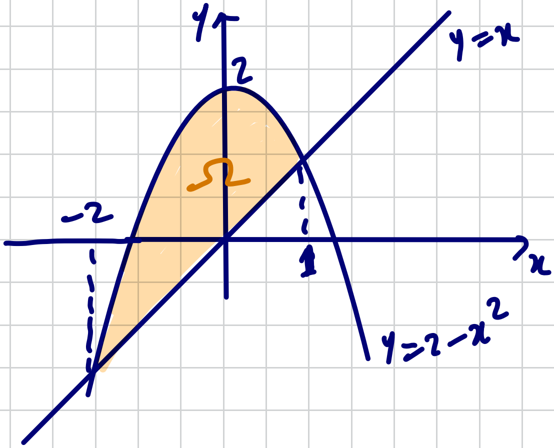
$$\left(\frac{2x^3}{3} - \frac{x^6}{6} - \frac{x^9}{9}\right)\Big|_0^1 = \frac{2}{3} - \frac{1}{6} - \frac{1}{9} - 0 =$$

$$= \frac{12 - 3 - 2}{18} = \frac{7}{18} \text{ u.v.}$$

L8

$f(x,y) = x^2$

9. Encontre o volume do sólido que é limitado superiormente pelo cilindro $z = x^2$ e inferiormente pela região delimitada pela parábola $y = 2 - x^2$ e pela reta $y = x$ no plano xy.



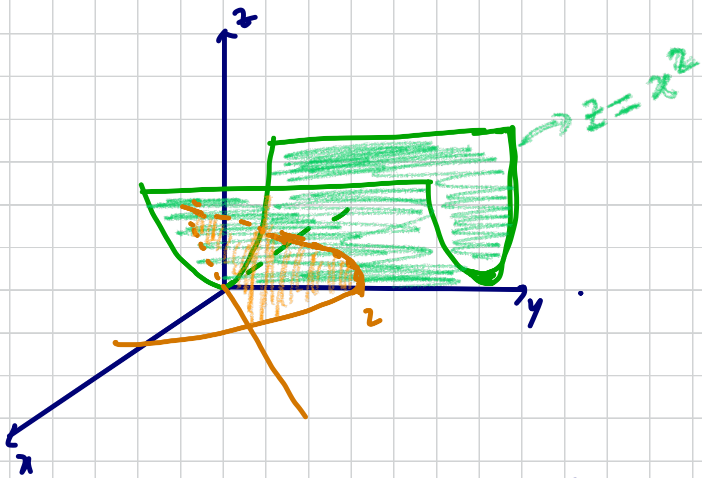
interceptos:

$$2 - x^2 = x$$

$$x^2 + x - 2 = 0$$

$$x = \frac{-1 \pm \sqrt{1+8}}{2}$$

$$x = \frac{-1 \pm 3}{2} \to x = 1,\quad x = -2$$



$$V = \iint_{\Omega} f(x,y)\, dA = \int_{x=-2}^{x=1} \int_{y=x}^{y=2-x^2} x^2 \, dy \, dx =$$

$$= \int_{x=-2}^{x=1} x^2 \left(\int_{y=x}^{y=2-x^2} dy \right) dx = \int_{x=-2}^{x=1} x^2 \cdot y \Big|_{y=x}^{y=2-x^2} dx =$$

$$= \int_{x=-2}^{x=1} x^2 \cdot (2 - x^2 - x)\, dx = \int_{-2}^{1} (2x^2 - x^4 - x^3)\, dx =$$

$$\begin{array}{r} 200 \\ -137 \end{array}$$

$$= \left(\frac{2x^3}{3} - \frac{x^5}{5} - \frac{x^4}{4}\right)\Bigg|_{-2}^{1} =$$

$$= \frac{2}{3} - \frac{1}{5} - \frac{1}{4} - \left(\frac{2 \cdot (-2)^3}{3} - \frac{(-2)^5}{5} - \frac{(-2)^4}{4}\right) =$$

$$= \frac{2}{3} - \frac{1}{5} - \frac{1}{4} + \frac{16}{3} - \frac{32}{5} + 4 = \frac{\overset{6}{\frown}\!\!\!\!18}{3} - \frac{33}{5} - \frac{1}{4} + 4$$

$$= 10 - \frac{33}{5} - \frac{1}{4} = \frac{200 - 132 - 5}{20} = \frac{63}{20} \text{ u. a.}$$