CÁLCULO 3, TURMA T3

17/04/26 - AULA 11

AULA DE EXERCÍCIOS

LISTA 02

**Lista 02 de Exercícios - Funções de várias variáveis reais**

1. Em cada item a seguir, determine o domínio $\Omega \subset \mathbb{R}^2$ de $f : \Omega \to \mathbb{R}$ e faça um esboço do domínio:

(a) $f(x, y) = \dfrac{1}{x^2 + y^2 - 1}$

(b) $f(x, y) = \sqrt{1 - x^2 - y^2}$

(c) $f(x, y) = \sqrt{x^2 - 4y^2 + 16}$

(d) $f(x, y) = \dfrac{1}{\sqrt{1 - x^2 - y^2}}$

(e) $f(x, y) = \ln(x^2 + y)$

(f) $f(x, y) = \text{arcsen}\,(x + y)$

(g) $f(x, y) = \text{arcsen}\dfrac{x}{x + y}$

(h) $f(x, y) = \arctan \dfrac{x + y}{x - y}$

(d) $f(x,y) = \dfrac{1}{\sqrt{1-x^2-y^2}}$.

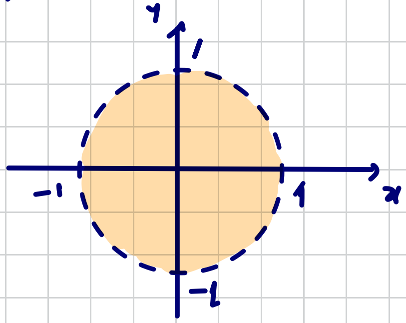
$D(f) = ?$ condição de existência: $1 - x^2 - y^2 > 0$

$-x^2 - y^2 > -1$ $(\times -1)$

$x^2 + y^2 < 1$

$D(f) = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$

GRÁFICO DO DOMÍNIO:



(e) $f(x,y) = \ln(\underbrace{x^2+y}_{a})$.

$D(f) = ?$

condição de existência: $\underline{\underline{x^2+y}} > 0$

[ lembre-se do conceito de logaritmo:

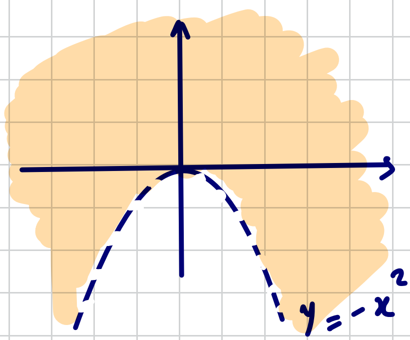
$\log_{\underline{b}} \underline{a} = c \iff b^c = a$.

$\underline{\underline{a > 0}}$ e $b > 0;\ b \neq 1$ ]

Ou seja, devemos impor que $y > -x^2$

$D(f) = \{(x,y) \in \mathbb{R}^2 : y > -x^2\}$.

gráfico do domínio:



(g) $f(x,y) = \arcsin \dfrac{x}{x+y}$

[ **Recordando:** $y = \arcsin x \Leftrightarrow x = \operatorname{sen} y$

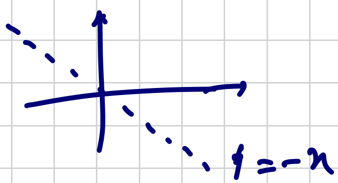
$y \in \mathbb{R}$.

$-1 \le \underset{\operatorname{sen} y}{\underset{\parallel}{x}} \le 1$. ]

condição de existência:

$$-1 \le \frac{x}{x+y} \le 1.$$

Além disso, $x+y \neq 0 \rightarrow \boxed{y \neq -x}$




Assim, temos:

$$D(f) = \left\{ (x,y) \in \mathbb{R}^2 : -1 \le \frac{x}{x+y} \le 1 \text{ e } y \neq -x \right\}.$$

gráfico do domínio:

- $\dfrac{x}{x+y} \le 1$: Como $x+y \neq 0$, temos

dois casos a considerar:

• $x+y>0$. Neste caso:

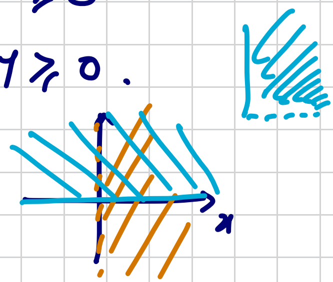
$$\frac{x}{x+y} \leq 1 \iff x \leq 1.(x+y)$$

$$\iff \cancel{x} \leq \cancel{x} + y$$

$$\iff y \geq 0$$

Disso: temos $x+y>0$ e $y \geq 0$.

Então, $x>0$.




• $x+y<0$. Neste caso:

?

$$\frac{x}{\underbrace{x+y}_{<0}} \leq 1 \iff x \geq 1.(x+y)$$

$$\iff \cancel{x} \geq \cancel{x} + y$$

$$y \leq 0$$

Disso, temos:

$x+y<0$ ; $y \leq 0$.

Então:

(f) $f(x,y) = \arcsin(x+y)$.

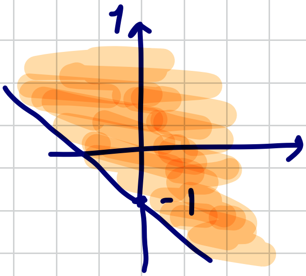
$D(f) = ?$ condição de existência:

[lembre: $y = \arcsin x \iff x = \sin y$, $-1 \leq x \leq 1$]

No nosso caso: $\underbrace{-1 \leq x+y}\ \underbrace{\leq 1}$.

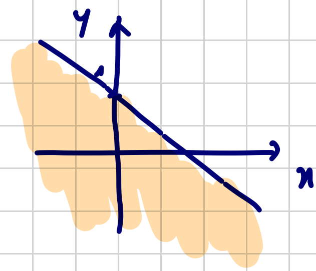
$x + y \geq -1$.

$y \geq -1 - x$



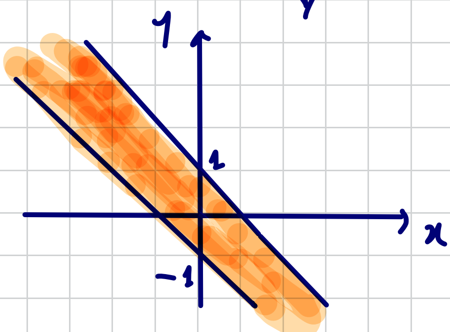
$x + y \leq 1$.

$y \leq 1 - x$



$\Downarrow$ "$\cap$" pois deve satisfazer ambos.

gráfico do domínio. $\longrightarrow$



$D(f) = \{(x,y) \in \mathbb{R}^2 : -1 \leq x+y \leq 1\}$.

2. Obtenha o domínio de cada função a seguir e faça um esboço gráfico de $f$.

(a) $f(x,y) = \sqrt{16 - x^2 - y^2}$

(b) $f(x,y) = 4x^2 + 9y^2$

(c) $f(x,y) = x^2 - y^2$

(d) $f(x,y) = \sqrt{x+y}$

(b) $z = x^2 - y^2$ $\qquad D(f) = \mathbb{R}^2$.

gráfico do domínio:

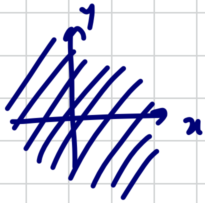


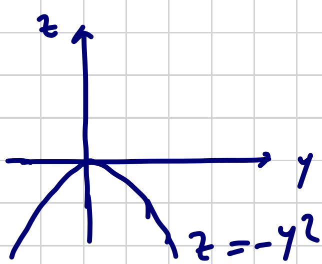

gráfico de $f$:

$$z = x^2 - y^2$$

traços nos planos coordenados:

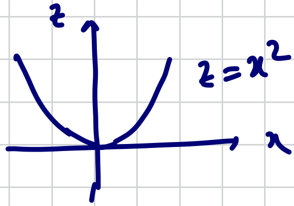
- $x = 0$: (plano $yz$).

$$z = -y^2 \quad \text{(parábola)}$$



• $y=0$ (plano $xz$):

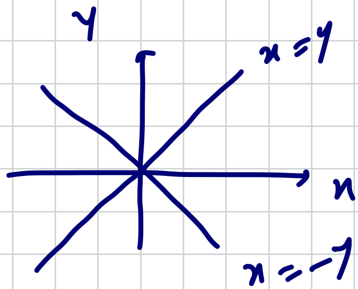
$$z = x^2 \quad \text{(parábola)}$$

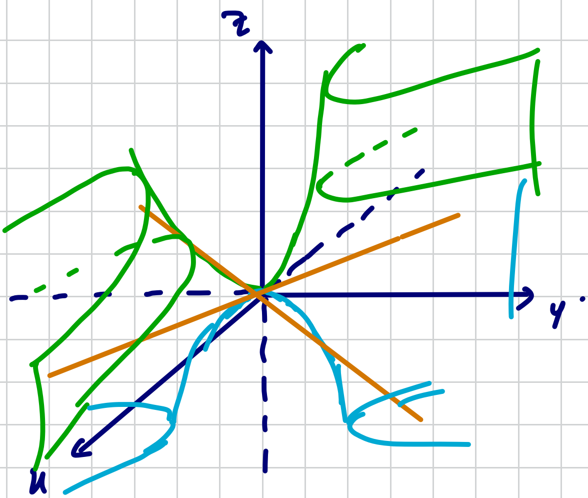



• $z=0$: $\quad x^2 - y^2 = 0 \iff (x+y)(x-y)=0$

$$\begin{cases} x+y=0 \Rightarrow x=-y \\ x-y=0 \Rightarrow x=y \end{cases}$$

(retas concorrentes)



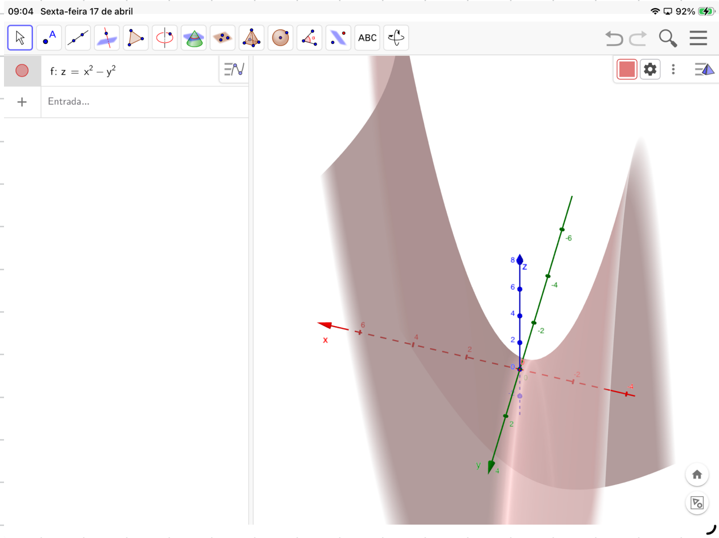

$z = k$ tem-se:

$$x^2 - y^2 = k$$

$$\frac{x^2}{k} - \frac{y^2}{k} = 1$$

(hipérboles)


HIPERBOLÓIDE PARABÓLICO.




(c) $f(x,y) = 4x^2 + 9y^2 \geq 0$

$D(f) = \mathbb{R}^2$

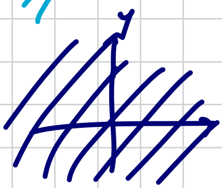



gráfico de f:

$z = 4x^2 + 9y^2$

$z = 0: \; 4x^2 + 9y^2 = 0$

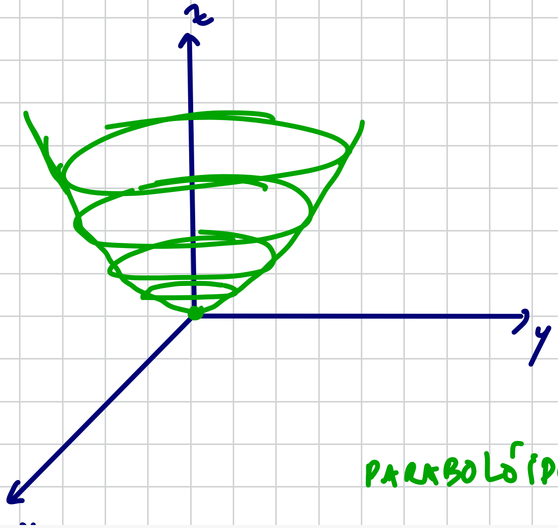
$\iff$

$(x,y) = (0,0)$

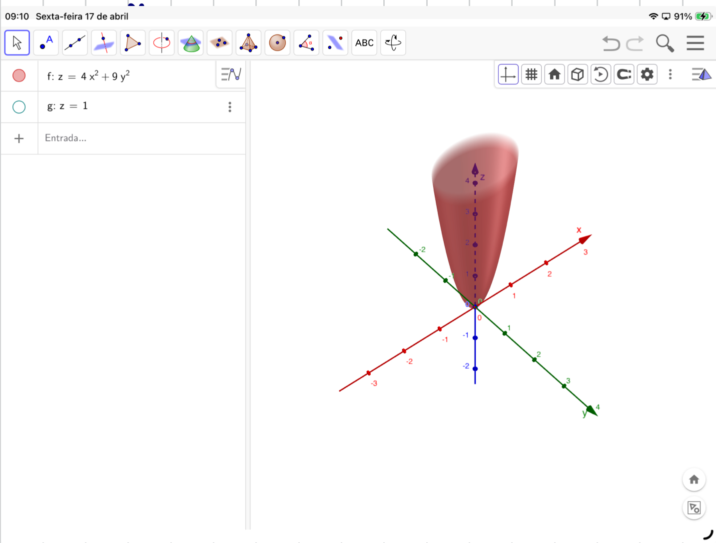
origem.

$z = k: \quad 4x^2 + 9y^2 = k \quad \div k.$

$$\frac{4x^2}{k} + \frac{9y^2}{k} = 1. \quad \text{(elipses)}$$




PARABOLÓIDE ELÍPTICO.



3. Esboce a curva de cada função abaixo.

(a) $f(t) = (t, t^2, t^3)$, $0 \le t \le 1$.

(b) $f(t) = (4-4t)\vec{i} + (4-4t)\vec{j}$, $t \in [0,2]$.

(c) $f(t) = (\ln t, t, 1)$, $t > 0$

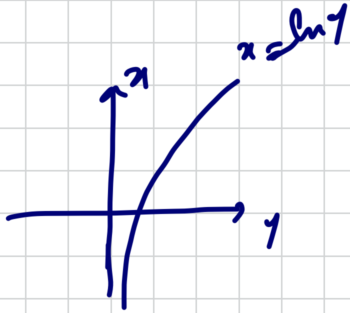
(d) $f(t) = (6 \operatorname{sen} t, 4, 25 \cos t)$, $0 \le t \le 2\pi$.

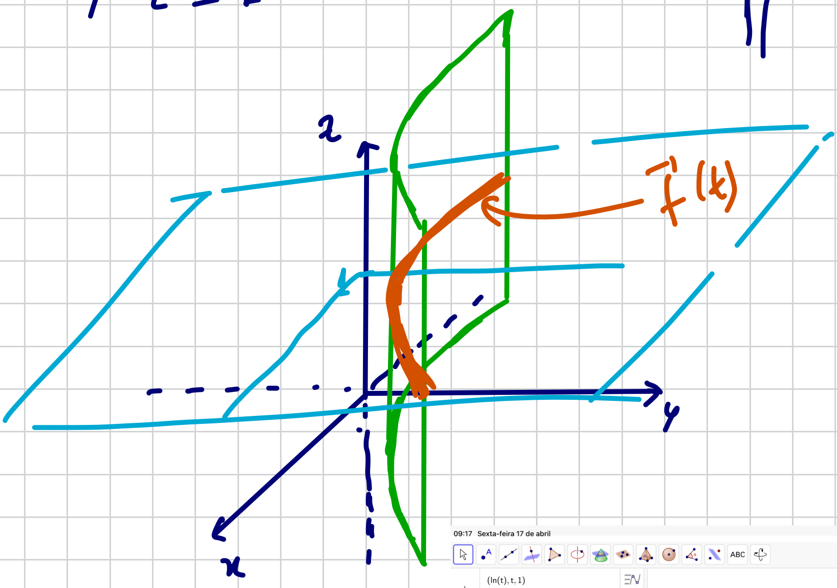
(e) $f(t) = (8 - 4 \operatorname{sen} t, 2\cos t, 4 \operatorname{sen} t)$

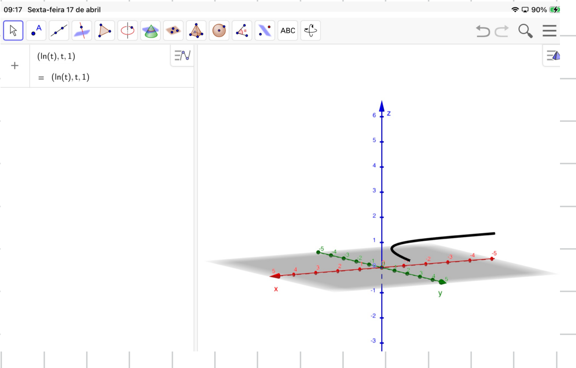
(f) $f(t) = (\operatorname{sen} t, t, \cos t)$

(c) $f(t) = (\ln t, t, 1)$ ; $t > 0$.

$$\begin{cases} x = \ln t \\ y = t \\ z = 1 \end{cases} \;>\; x = \ln y$$



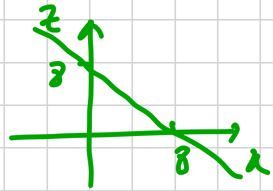



(e) $\vec{f}(t) = (8 - 4\,\text{sen}\,t,\ 2\cos t,\ 4\,\text{sen}\,t)$

$$\begin{cases} x = 8 - 4\,\text{sen}\,t \\ y = 2\cos t \\ z = 4\,\text{sen}\,t \end{cases}$$

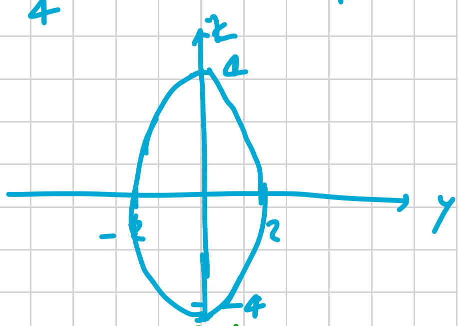
$x = 8 - z$

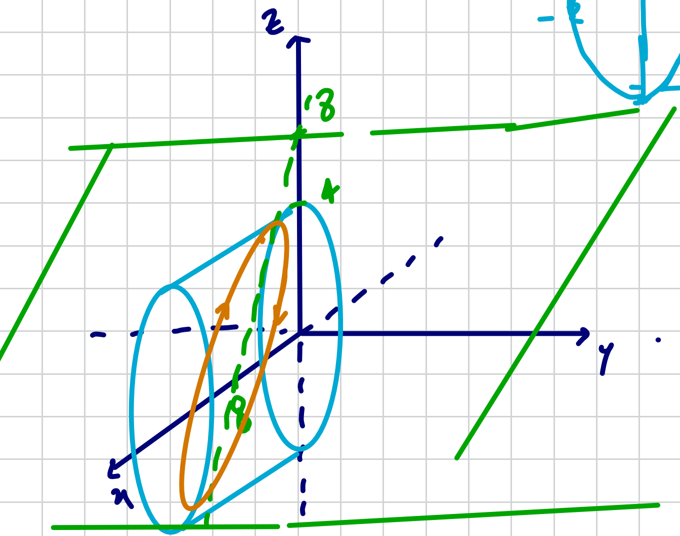


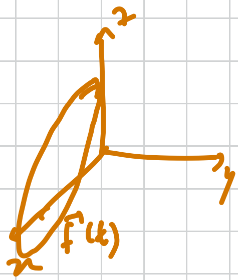

$\frac{y}{2} = \cos t$ ; $\frac{z}{4} = \text{sen}\,t$.

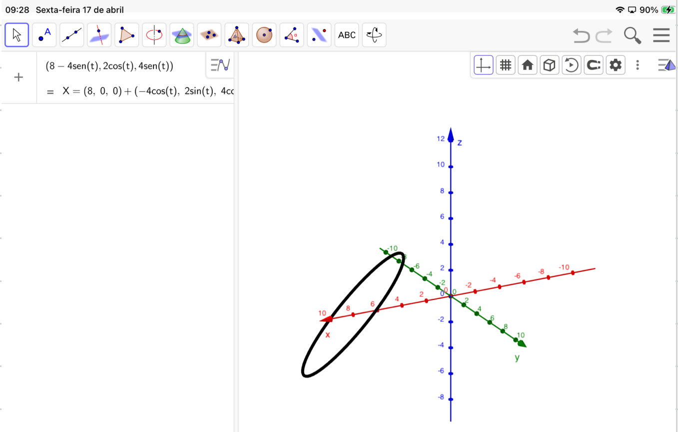
$\text{sen}^2 t + \cos^2 t = 1$.

$\frac{z^2}{16} + \frac{y^2}{4} = 1$. (elipse no plano yz)





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ABC
(8 − 4sen(t), 2cos(t), 4sen(t))
= X = (8, 0, 0) + (−4cos(t), 2sin(t), 4co
z
x
y