FUNÇÕES TRANSCENDENTAIS

03/12/25 - AULA 09

Na aula passada vimos redução a 1º quadrante e as fórmulas da adição e da subtração do seno e do cosseno.

A saber:

$$\text{sen}(a+b) = \text{sen}\,a \cdot \cos b + \text{sen}\,b \cdot \cos a.$$

$$\text{sen}(a-b) = \text{sen}\,a \cdot \cos b - \text{sen}\,b \cdot \cos a$$

$$\cos(a+b) = \cos a \cdot \cos b - \text{sen}\,a \cdot \text{sen}\,b.$$

$$\cos(a-b) = \cos a \cdot \cos b + \text{sen}\,a \cdot \text{sen}\,b.$$

No que segue, vamos desenvolver as fórmulas da tangente da soma e da diferença de dois arcos.

- $$\tan(a+b) = \frac{\text{sen}(a+b)}{\cos(a+b)} =$$

$$= \frac{\text{sen}\,a \cdot \cos b + \text{sen}\,b \cdot \cos a}{\cos a \cdot \cos b - \text{sen}\,a \cdot \text{sen}\,b} =$$

$$= \frac{\dfrac{\operatorname{sen} a \cdot \cancel{\cos b}}{\cos a \cdot \cancel{\cos b}} + \dfrac{\operatorname{sen} b \cdot \cancel{\cos a}}{\cancel{\cos a} \cdot \cos b}}{\underbrace{\dfrac{\cancel{\cos a} \cdot \cancel{\cos b}}{\cancel{\cos a} \cdot \cancel{\cos b}}}_{1} - \dfrac{\operatorname{sen} a \cdot \operatorname{sen} b}{\cos a \cdot \cos b}} =$$

DIVIDA O NUM. E O DENOMINADOR POR $\cos a . \cos b$

para obtermos tangentes

$$= \frac{\dfrac{\operatorname{sen} a}{\cos a} + \dfrac{\operatorname{sen} b}{\cos b}}{1 - \dfrac{\operatorname{sen} a}{\cos a} \cdot \dfrac{\operatorname{sen} b}{\cos b}} = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}.$$

conclusão:

$$\boxed{\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \cdot \tan b}}$$

FÓRMULA DA TANGENTE DA SOMA DE DOIS ARCOS.

Ex.:01) Calcule $\tan 75°$.

solução:

$$\tan 75° = \tan(30° + 45°) = \frac{\tan 30° + \tan 45°}{1 - \tan 30° \cdot \tan 45°}$$

$$= \frac{\frac{\sqrt{3}}{3} + 1}{1 - \frac{\sqrt{3}}{3} \cdot 1} = \frac{\frac{\sqrt{3}+3}{\cancel{3}}}{\frac{3-\sqrt{3}}{\cancel{3}}} =$$

$$= \frac{3+\sqrt{3}}{3-\sqrt{3}} \cdot \frac{3+\sqrt{3}}{3+\sqrt{3}} = \frac{(3+\sqrt{3})^2}{9 - (\cancel{\sqrt{3}})^{\cancel{2}}}$$

$$= \frac{(3)^2 + 6\sqrt{3} + (\cancel{\sqrt{3}})^{\cancel{2}}}{9-3} =$$

$$= \frac{9 + 6\sqrt{3} + 3}{6} = \frac{12 + 6\sqrt{3}}{6}$$

$$= \frac{\cancel{6}(2+\sqrt{3})}{\cancel{6}} = 2 + \sqrt{3}$$

$$\rightarrow \boxed{\tan 75° = 2 + \sqrt{3}}$$

EX. 02)

$$\tan 375^\circ = ?$$

SOL: $\begin{array}{r|l} 375^\circ & 360 \\ \hline 15^\circ & 1 \end{array}$ $\rightarrow$ 1ºQ

$$\tan 375^\circ = \tan 15^\circ = \tan(45^\circ - 30^\circ)$$

Neste caso, precisamos desenvolver a fórmula da tangente da diferença:

$$\tan(a-b) = ?$$

PELA FÓRMULA DA ADIÇÃO

$$\tan(a-b) = \tan(a+(-b)) =$$

$$= \frac{\tan a + \tan(-b)}{1 - \tan a \cdot \tan(-b)} =$$

- $\tan(-b) = -\tan b$

$$= \frac{\tan a - \tan b}{1 - \tan a \cdot (-\tan b)}$$

$$= \frac{\tan a - \tan b}{1 + \tan a . \tan b}$$

conclusão :

$$\boxed{\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a . \tan b}}$$

FÓRMULA DA TANGENTE DA DIFERENÇA ENTRE DOIS ARCOS.

De posse dessa nova fórmula podemos seguir no exemplo:

$$\tan 375^\circ = \tan 15^\circ = \tan(45^\circ - 30^\circ) =$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ . \tan 30^\circ} =$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1 . \frac{\sqrt{3}}{3}} = \frac{\frac{3-\sqrt{3}}{\cancel{3}}}{\frac{3+\sqrt{3}}{\cancel{3}}} =$$

$$= \frac{3-\sqrt{3}}{3+\sqrt{3}} \times \frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{(3-\sqrt{3})^2}{9-3} =$$

$$= \frac{9 - 6\sqrt{3} + 3}{6} = \frac{12 - 6\sqrt{3}}{6} = \frac{\cancel{6}(2-\sqrt{3})}{\cancel{6}} = 2-\sqrt{3}.$$

$$\Rightarrow \boxed{\tan 375^\circ = 2-\sqrt{3}}$$

LISTA 04

7. Sendo $\tan y = 2$ e $x + y = 135^\circ$, calcule o valor de $\tan x$.

Solução: $x + y = 135^\circ \Rightarrow x = 135^\circ - y$

Demo:

$$\tan x = \tan(135^\circ - y) = \frac{\tan 135^\circ - \overbrace{\tan y}^{2}}{1 + \tan 135^\circ \cdot \underbrace{\tan y}_{2}}$$

Note que:

- $\underbrace{\tan 135^\circ}_{2^{\circ}Q} = -\tan(180^\circ - 135^\circ) = -\tan 45^\circ = -1.$

Demo, segue que:

$$\tan x = \frac{\tan 135^\circ - \tan y}{1 + \tan 135^\circ \cdot \tan y} = \frac{-1-2}{1+(-1)\cdot 2} =$$

$$= \frac{-3}{1-2} = \frac{-3}{-1} = 3$$

FÓRMULAS DO ARCO DUPLO: Vamos determinar fórmulas para $\text{sen}\,2a$, $\cos 2a$ e $\tan 2a$. Suas deduções são simples, bastando escrever

$$2a = a + a.$$

Assim, tem-se:

- $$\text{sen}\,2a = \text{sen}(a+a) = \text{sen}\,a \cdot \cos a + \text{sen}\,a \cdot \cos a$$
$$= 2 \cdot \text{sen}\,a \cdot \cos a$$

$$\Rightarrow \boxed{\text{sen}\,2a = 2 \cdot \text{sen}\,a \cdot \cos a}$$

- $$\cos 2a = \cos(a+a) = \cos a \cdot \cos a - \text{sen}\,a \cdot \text{sen}\,a$$
$$= \cos^2 a - \text{sen}^2 a$$

$$\Rightarrow \boxed{\cos 2a = \cos^2 a - \text{sen}^2 a}$$

$$\bullet\ \tan 2a = \tan(a+a) = \frac{\tan a + \tan a}{1 - \tan a \cdot \tan a}$$

$$= \frac{2 \cdot \tan a}{1 - \tan^2 a}$$

$$\Rightarrow \boxed{\tan 2a = \frac{2\tan a}{1 - \tan^2 a}}$$

obs.: $\nexists \tan 90°$. Uma forma "robusta" de concluir isto é escrever: $90° = 2 \cdot 45°$.
e disso:

$$\tan 90° = \tan(2 \cdot 45°) = \frac{2 \cdot \tan 45°}{1 - \tan^2 45°} =$$

$$= \frac{2 \cdot 1}{1 - (1)^2} = \frac{2}{1-1} = \frac{2}{0} = \nexists$$

Ex.!

L4:

10. (UFCE) Se $\sin x + \cos x = \dfrac{1}{\sqrt{3}}$, então o valor de $\operatorname{sen} 2x$ é

(a) $-\dfrac{1}{2}$. (b) $-\dfrac{1}{3}$. (c) $\dfrac{1}{3}$. (d) $\dfrac{2}{3}$. (e) $-\dfrac{2}{3}$.

$$\operatorname{sen} x + \cos x = \frac{1}{\sqrt{3}}, \qquad \operatorname{sen} 2x = ?$$

Elevando ao quadrado, vem:

$$\boxed{\operatorname{sen} 2x = 2 \cdot \operatorname{sen} x \cdot \cos x}$$

Fórmula.

$$(\operatorname{sen} x + \cos x)^2 = \left(\frac{1}{\sqrt{3}}\right)^2$$

$$\operatorname{sen}^2 x + \underbrace{2 \cdot \operatorname{sen} x \cdot \cos x}_{\operatorname{sen} 2x} + \cos^2 x = \frac{1}{3}$$

$$(\underbrace{\operatorname{sen}^2 x + \cos^2 x}_{=1}) + \operatorname{sen} 2x = \frac{1}{3}$$

$$\Rightarrow \operatorname{sen} 2x = \frac{1}{3} - 1 = \frac{1-3}{3} = -\frac{2}{3}$$

L4

18. Se $\cos x = \frac{3}{5}$ e $\frac{3\pi}{2} < x < 2\pi$, calcule sen $3x$.

$x \in 4^{\underline{o}}q$.

$\text{sen}\,3x = ?$

$$\text{sen}\,3x = \text{sen}(2x + x) = \underbrace{\text{sen}\,2x}_{2\,\text{sen}\,x\cos x} \cdot \cos x + \text{sen}\,x \cdot \underbrace{\cos 2x}_{\cos^2 x - \text{sen}^2 x}$$

$$= (2\,\text{sen}\,x \cdot \cos x) \cdot \cos x + \text{sen}\,x \cdot (\cos^2 x - \text{sen}^2 x)$$

$$= 2 \cdot \text{sen}\,x \cdot \cos^2 x + \text{sen}\,x \cdot \cos^2 x - \text{sen}^3 x$$

$$= 3 \cdot \text{sen}\,x \cdot \cos^2 x - \text{sen}^3 x$$

Como $\cos x = \frac{3}{5}$; $x \in 4^{\underline{o}}q$, então, pela razão trigonométrica fundamental:

$\text{sen}^2 x + \cos^2 x = 1$; vem:

$$\operatorname{sen}^2 x + \left(\frac{3}{5}\right)^2 = 1$$

$$\operatorname{sen}^2 x = 1 - \frac{9}{25}$$

$$\operatorname{sen}^2 x = \frac{16}{25} \Rightarrow \operatorname{sen} x = \sqrt{\frac{16}{25}}$$

$$\Rightarrow \operatorname{sen} x = \underbrace{-}\frac{4}{5}$$

$x \in 4^{\circ}q.$

(e neste quadrante o seno é negativo)

Assim, vamos, finalmente, encontrar:

$$\operatorname{sen} 3x = 3 \operatorname{sen} x \cdot \cos^2 x - \operatorname{sen}^3 x$$

$$= 3 \cdot \left(-\frac{4}{5}\right) \cdot \left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^3$$

$$= -\frac{108}{5^3} + \frac{64}{5^3} = -\frac{44}{125}$$

FÓRMULAS DO ARCO METADE:

Da fórmula $\cos 2a = \cos^2 a - \operatorname{sen}^2 a$;

como $\operatorname{sen}^2 a + \cos^2 a = 1$, então:

$$\operatorname{sen}^2 a = 1 - \cos^2 a.$$

Levando para a fórmula acima, do arco duplo, vamos obter:

$$\cos 2a = \cos^2 a - (1 - \cos^2 a)$$

$$\cos 2a = \cos^2 a - 1 + \cos^2 a$$

$$\cos 2a = 2\cos^2 a - 1$$

$$1 + \cos 2a = 2 \cdot \cos^2 a$$

$$\Rightarrow \cos^2 a = \frac{1 + \cos 2a}{2}$$

$$\Rightarrow \cos a = \pm \sqrt{\frac{1 + \cos 2a}{2}}$$

Escreva $a = \frac{x}{2}$. Assim:

$$\boxed{\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}}$$

Do mesmo modo, se isolarmos $\operatorname{sen}^2 x$, e procedendo analogamente, vamos obter:

$$\boxed{\operatorname{sen}\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$$

E, $\tan\frac{x}{2} = \dfrac{\operatorname{sen}\frac{x}{2}}{\cos\frac{x}{2}} = \dots = \pm\sqrt{\dfrac{1-\cos x}{1+\cos x}}$

$$\Rightarrow \boxed{\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}}$$

Ex.: L4:

12. Determine o valor de $\cos 37,5°$.

solução: Note que

$$37,5° = \frac{75°}{2}$$

Então:

$$\cos 37,5° = +\sqrt{\frac{1+\cos 75°}{2}} \quad ; \text{ onde:}$$

$$\cos 75° = \cos(30°+45°) = \cos 30° \cdot \cos 45° - \operatorname{sen} 30° \cdot \cos 45°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$$

Disso, segue que:

$$\cos 37,5° = \sqrt{\frac{1+\frac{\sqrt{6}-\sqrt{2}}{4}}{2}} = \sqrt{\frac{\frac{4+\sqrt{6}-\sqrt{2}}{4}}{2}}$$

$$= \sqrt{\frac{4+\sqrt{6}-\sqrt{2}}{4} \cdot \frac{1}{2}} = \sqrt{\frac{4+\sqrt{6}-\sqrt{2}}{8}}$$

L4

21. Calcule o valor de $\cos 15°$ de duas formas:

(a) escrevendo $\cos 15° = \cos(45° - 30°)$;

(b) escrevendo $\cos 15° = \cos\left(\frac{30°}{2}\right)$.

Aparentemente, os resultados obtidos em (a) e em (b) parecem ser diferentes. No entanto, representam o mesmo valor, ou seja, são iguais. Como você mostraria isso?

(a) $\cos 15° = \cos(45° - 30°) =$

$$= \cos 30° \cdot \cos 45° + \operatorname{sen} 30° \cdot \operatorname{sen} 45°$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{1}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

$$\Rightarrow \boxed{\cos 15° = \frac{\sqrt{6} + \sqrt{2}}{4}}$$

(b) $\cos 15° = \cos\left(\frac{30°}{2}\right) = \sqrt{\frac{1 + \cos 30°}{2}} =$

$$= \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} = \sqrt{\frac{\frac{2 + \sqrt{3}}{2}}{2}} =$$

$$= \sqrt{\frac{2 + \sqrt{3}}{2} \times \frac{1}{2}} = \frac{\sqrt{2 + \sqrt{3}}}{2}$$

$$\Rightarrow \boxed{\cos 15^\circ = \frac{\sqrt{2+\sqrt{3}}}{2}}$$

Escreva $x = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ e $y = \dfrac{\sqrt{2+\sqrt{3}}}{2}$;

ambos são positivos. Elevando ao quadrado, vamos obter:

$$x^2 = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \frac{6+2.\sqrt{12}+2}{16}$$

$$= \frac{8+2\sqrt{3.4}}{16} =$$

$$= \frac{8+2.2.\sqrt{3}}{16}$$

$$= \frac{4(2+\sqrt{3})}{16} = \frac{2+\sqrt{3}}{4}$$

$$y^2 = \left(\frac{\sqrt{2+\sqrt{3}}}{2}\right)^2 = \frac{2+\sqrt{3}}{2^2} = \frac{2+\sqrt{3}}{4}$$

Como $x^2 = y^2$ e $x, y > 0$, segue que $x = y$; ou seja,

$$\frac{\sqrt{6}+\sqrt{2}}{4} = \frac{\sqrt{2+\sqrt{3}}}{2}$$

FÓRMULAS DA PROSTAFÉRESE OU DE TRANSFORMAÇÃO DE SOMA EM PRODUTO.

01) $\operatorname{sen} p + \operatorname{sen} q = 2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$

02) $\operatorname{sen} p - \operatorname{sen} q = 2 \cdot \operatorname{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right)$

03) $\cos p + \cos q = 2 \cdot \cos\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$

04) $\cos p - \cos q = -2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \operatorname{sen}\left(\frac{p-q}{2}\right)$

Vamos mostrar / deduzir a primeira:

Temos que

$$+\begin{cases} \text{sen}(a+b) = \text{sen}\,a \cdot \cos b + \text{sen}\,b \cdot \cancel{\cos a} \\ \text{sen}(a-b) = \text{sen}\,a \cdot \cos b - \text{sen}\,b \cdot \cancel{\cos a}. \end{cases}$$

$$\text{sen}\underbrace{(a+b)}_{p} + \text{sen}\underbrace{(a-b)}_{q} = 2 \cdot \text{sen}\,a \cdot \cos b. \quad (*)$$

Escreva

$$+\begin{cases} a+b = p \\ a-b = q \end{cases} \longrightarrow b = p - a$$

$$2a = p+q \qquad b = p - \frac{p+q}{2}$$

$$\Rightarrow a = \frac{p+q}{2} \qquad b = \frac{2p - p - q}{2}$$

$$b = \frac{p-q}{2}$$

Assim, teremos em $(*)$:

$$\text{sen}\,p + \text{sen}\,q = 2 \cdot \text{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

Para deduzir a segunda fórmula, subtraímos as duas fórmulas do seno:

$$
- \begin{cases} \operatorname{sen}(a+b) = \cancel{\operatorname{sen} a \cdot \cos b} + \operatorname{sen} b \cdot \cos a \\ \operatorname{sen}(a-b) = \cancel{\operatorname{sen} a \cdot \cos b} - \operatorname{sen} b \cdot \cos a \end{cases}
$$

$$+$$

$$
\operatorname{sen}\underbrace{(a+b)}_{p} - \operatorname{sen}\underbrace{(a-b)}_{q} = 2 \cdot \operatorname{sen} \underbrace{b}_{\frac{p-q}{2}} \cdot \cos \underbrace{a}_{\frac{p+q}{2}}
$$

$$
\Rightarrow \operatorname{sen} p - \operatorname{sen} q = 2 \cdot \operatorname{sen}\left(\frac{p-q}{2}\right) \cdot \cos\left(\frac{p+q}{2}\right)
$$

Analogamente demonstram-se as fórmulas 03 e 04, usando as fórmulas do cosseno da soma e da diferença de dois arcos.

DE UMA PROVA DO SEMESTRE PASSADO:

Questão 06. Prove a igualdade: (1,5 pt)

$$\frac{\cos\frac{\alpha}{2} - \operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \operatorname{sen}\frac{\alpha}{2}} = \frac{1 - \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}}$$

lembre que $\cos(\theta) = \operatorname{sen}\left(\frac{\pi}{2} - \theta\right)$

SOLUÇÃO:

$$\frac{\cos\frac{\alpha}{2} - \operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \operatorname{sen}\frac{\alpha}{2}} = \frac{\overbrace{\operatorname{sen}\left(\frac{\pi}{2} - \frac{\alpha}{2}\right)}^{p} - \overbrace{\operatorname{sen}\frac{\alpha}{2}}^{q}}{\operatorname{sen}\left(\frac{\pi}{2} - \frac{\alpha}{2}\right) + \operatorname{sen}\frac{\alpha}{2}}$$

$$= \frac{\cancel{2}\cdot \operatorname{sen}\left(\frac{\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\alpha}{2}}{2}\right)\cdot \cos\left(\frac{\frac{\pi}{2} - \cancel{\frac{\alpha}{2}} + \cancel{\frac{\alpha}{2}}}{2}\right)}{\cancel{2}\cdot \operatorname{sen}\left(\frac{\frac{\pi}{2} - \cancel{\frac{\alpha}{2}} + \cancel{\frac{\alpha}{2}}}{2}\right)\cdot \cos\left(\frac{\frac{\pi}{2} - \frac{\alpha}{2} - \frac{\alpha}{2}}{2}\right)}$$

$$\operatorname{sen}p - \operatorname{sen}q = 2\cdot \operatorname{sen}\left(\frac{p-q}{2}\right)\cdot \cos\left(\frac{p+q}{2}\right)$$

$$\operatorname{sen}p + \operatorname{sen}q = 2\cdot \operatorname{sen}\left(\frac{p+q}{2}\right)\cdot \cos\left(\frac{p-q}{2}\right)$$

$$= \frac{\operatorname{sen}\left(\frac{\frac{\pi}{2}-\alpha}{2}\right)\cdot \overbrace{\cancel{\cos\left(\frac{\pi}{4}\right)}}^{\frac{\sqrt{2}}{2}}}{\underbrace{\cancel{\operatorname{sen}\left(\frac{\pi}{4}\right)}}_{\frac{\sqrt{2}}{2}}\cdot \cos\left(\frac{\frac{\pi}{2}-\alpha}{2}\right)} =$$

$$= \frac{\operatorname{sen}\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}{\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)} =$$

$$= \frac{\operatorname{sen}\frac{\pi}{4}\cdot\cos\frac{\alpha}{2}-\operatorname{sen}\frac{\alpha}{2}\cdot\cos\frac{\pi}{4}}{\cos\frac{\pi}{4}\cdot\cos\frac{\alpha}{2}+\operatorname{sen}\frac{\pi}{4}\cdot\operatorname{sen}\frac{\alpha}{2}} =$$

$$= \frac{\frac{\cancel{\sqrt{2}}}{2}\cdot\cos\frac{\alpha}{2}-\operatorname{sen}\frac{\alpha}{2}\ \frac{\cancel{\sqrt{2}}}{2}}{\frac{\sqrt{2}}{\cancel{2}}\cdot\cos\frac{\alpha}{2}+\frac{\sqrt{2}}{\cancel{2}}\ \operatorname{sen}\frac{\alpha}{2}} =$$

$$\frac{\cos\frac{\alpha}{2} - \operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2} + \operatorname{sen}\frac{\alpha}{2}} = \frac{\frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} - \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}{\frac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \frac{\operatorname{sen}\frac{\alpha}{2}}{\cos\frac{\alpha}{2}}}$$

$1 =$

$1 =$

$$= \frac{1 - \tan\frac{\alpha}{2}}{1 + \tan\frac{\alpha}{2}}$$

Lista 04:

22. Mostre que

(a) $\operatorname{sen} 40^\circ + \operatorname{sen} 20^\circ = \cos 10^\circ$.

(b) $\operatorname{sen} 105^\circ + \operatorname{sen} 15^\circ = \frac{\sqrt{6}}{2}$.

(c) $\cos 130^\circ + \cos 110^\circ + \cos 10^\circ = 0$.

(d) $\cos 220^0 + \cos 100^\circ + \cos 20^\circ = 0$.

(a) $\operatorname{sen} 40^\circ + \operatorname{sen} 20^\circ = \cos 10^\circ$.

De fato:

$$\underbrace{\operatorname{sen} 40^\circ}_{p} + \underbrace{\operatorname{sen} 20^\circ}_{q} = 2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right) =$$

$$= 2 \cdot \operatorname{sen}\left(\frac{40^\circ + 20^\circ}{2}\right) \cdot \cos\left(\frac{40^\circ - 20^\circ}{2}\right) =$$

$$= 2 \cdot \operatorname{sen} 30^\circ \cdot \cos 10^\circ = \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \cos 10^\circ =$$

$$= \cos 10^\circ$$