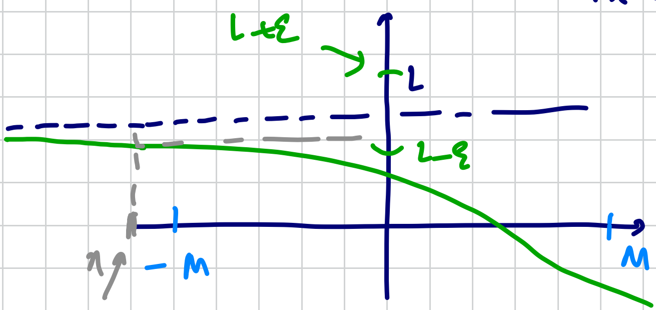
CÁLCULO 1A

28/10/25 - AULA 11

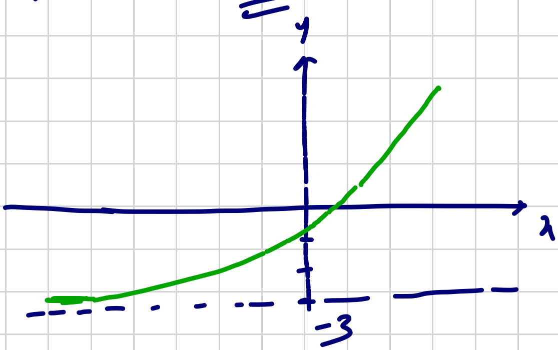
Na aula passada estudamos limites laterais e iniciamos o estudo de limites no infinito.
Vejamos outros casos de limites no infinito.

- $\lim\limits_{x \to -\infty} f(x) = L \overset{\text{def.}}{\iff} \forall \varepsilon > 0, \exists M > 0$ tal que, $\forall x < -M \Rightarrow |f(x) - L| < \varepsilon$.



Ex:

$$\lim_{x \to -\infty} (2)^x - 3 = \underline{\underline{-3}}$$



$y = 2^x - 3$

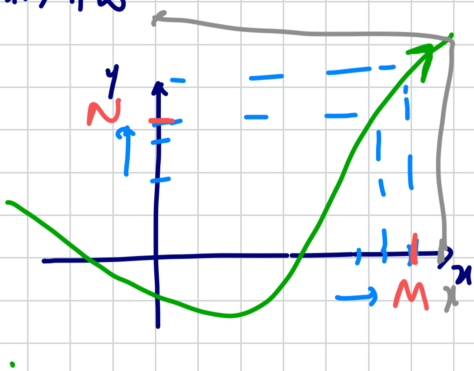
$y + 3 = \underbrace{2^x}_{>0}$

$y + 3 > 0$

$y > -3$

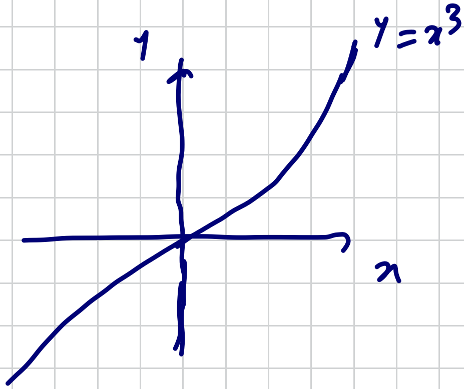
ASSÍNTOTA HORIZ: $y = -3$

• $\lim\limits_{x\to+\infty} f(x) = +\infty \overset{\text{def.:}}{\iff} \forall N>0,\ \exists M>0$ tal que, $\forall x > M \Rightarrow f(x) > N$



Ex.: $f(x) = x^3$

$$\lim_{x\to+\infty} x^3 = +\infty.$$



Na prática, faz-se:

$$\lim_{x\to+\infty} x^3 = (+\infty)^3 = +\infty$$

Outros exemplos:

(a) $\lim_{x\to+\infty} x^4 = (+\infty)^4 = +\infty$

(b) $\lim_{x\to-\infty} x^4 = (-\infty)^4 = (-1)^4 \cdot (+\infty)^4 = +\infty.$

(c) $\lim_{x\to-\infty} x^5 = (-\infty)^5 = -\infty.$

Ou seja, o que precisamos cuidar é a regra de sinais.

E os problemas:

(a) $\lim_{x\to+\infty} x^2 - x = (+\infty)^2 - (+\infty) = +\infty - \infty$

SÍMBOLOS DE INDET.

(b) $\lim_{x\to+\infty} x - x^2 = \infty - (\infty)^2 = \infty - \infty$

(c) $\lim_{x\to+\infty} \frac{x}{x^2} = \frac{(\infty)}{\infty^2}$ $= \frac{\infty}{\infty}$

SÍMBOLOS DE

(d) $\lim_{x\to+\infty} \dfrac{x^2}{x} \to \dfrac{(\infty)^2}{\infty} = \dfrac{\infty}{\infty}$ INDETERMINAÇÃO

Ou seja, $\infty - \infty$ e $\dfrac{\infty}{\infty}$ são símbolos de indeterminação. Nestes casos, o resultado do cálculo de limite dependerá das funções envolvidas.

Voltando aos exemplos acima:

(a) $\lim_{x\to+\infty} x^2 - x \underset{\substack{\uparrow \\ \infty-\infty \\ (\text{INDET.})}}{=} \lim_{x\to+\infty} x(x-1) = (+\infty)\cdot(\underbrace{+\infty-1}_{+\infty})$

$= +\infty\cdot(+\infty) = +\infty$

(b) $\lim_{x\to+\infty} x - x^2 \underset{\substack{\uparrow \\ \infty-\infty \\ (\text{INDEF.})}}{=} \lim_{x\to+\infty} x(1-x) = +\infty\cdot(1-(+\infty))$

$= +\infty\cdot(-\infty) = \underline{\underline{-\infty}}$

(c) $\lim_{x\to+\infty} \dfrac{x}{x^2} \underset{\substack{\frac{\infty}{\infty} \\ (\text{INDE})}}{=} \lim_{x\to+\infty} \dfrac{1}{x} = \dfrac{1}{+\infty} = 0$

(d) $\lim\limits_{x\to+\infty} \frac{x^{\cancel{2}}}{\cancel{x}} \underset{\uparrow}{=} \lim\limits_{x\to+\infty} x = +\infty$ //

$\frac{\infty}{\infty}$ (IND.)

Mesmos raciocínios se tivermos $x \to -\infty$

Ex.:

- $\lim\limits_{x\to-\infty} \frac{x^3}{x^4} = \lim\limits_{x\to-\infty} \frac{\cancel{x^3}}{\cancel{x^4}} = \lim\limits_{x\to-\infty} \frac{1}{x} = \frac{1}{-\infty} = 0$

- $\lim\limits_{x\to-\infty} \frac{x^3}{x} = \lim\limits_{x\to-\infty} \frac{x^{\cancel{3}}}{\cancel{x}} = \lim\limits_{x\to-\infty} x^2 = (-\infty)^2 = +\infty$

E para calcular, por exemplo:

$$\lim_{x\to+\infty} \frac{2x^2-3x+2}{4x-3x^2} = ?$$

Nestes casos, vamos pôr o termo de mais

alto grau do numerador e do denominador em evidência. Assim, escrevemos:

$$\lim_{x\to+\infty} \frac{2x^2-3x+1}{4x-3x^2} = \lim_{x\to+\infty} \frac{2\cancel{x^2}\left(1-\frac{3\cancel{x}}{2x^{\cancel{2}}}+\frac{1}{2x^2}\right)}{-3\cancel{x^2}\left(\frac{4\cancel{x}}{-3x^{\cancel{2}}}+1\right)} =$$

$$= \lim_{x\to+\infty} \frac{2\left(\underset{=}{1}-\underbrace{\frac{3}{2x}}_{\to 0}+\underbrace{\frac{1}{2x^2}}_{\to 0}\right)}{-3.\left(\underbrace{\frac{4}{-3x}}_{\to 0}+\underset{=}{1}\right)} = \frac{2.1}{-3.1} = -\frac{2}{3}$$

Em resumo, $\lim_{x\to+\infty} \frac{f(x)}{g(x)}$, onde $f(x)$ e $g(x)$ são polinômios, basta tomar o grau mais alto do numerador e do denominador. "Limpando" o cálculo acima, pode-se fazer:

$$\lim_{x\to+\infty} \frac{2x^2-3x+1}{4x-3x^2} = \lim_{x\to+\infty} \frac{2\cancel{x^2}}{-3\cancel{x^2}} = -\frac{2}{3}$$

Vejamos outros exemplos:

Ex.: Calcule os limites:

(a) $\lim\limits_{x\to+\infty} \dfrac{5x^3-3x^2+2}{7x-x^2} = \lim\limits_{x\to+\infty} \dfrac{5x^{\cancel{3}}}{-\cancel{x^2}} = \lim\limits_{x\to+\infty} -5x$

$= -5.(+\infty) = -\infty.$

(b) $\lim\limits_{x\to-\infty} \dfrac{4x^5-3x^7+1}{2x^3-x^2+9} = \lim\limits_{x\to-\infty} \dfrac{-3x^{\cancel{7}}}{2x^{\cancel{3}}} = \lim\limits_{x\to-\infty} \dfrac{-3x^4}{2}$

$= -\frac{3}{2}.(-\infty)^4 = -\frac{3}{2}.(+\infty) = -\infty$

(c) $\lim\limits_{x\to-\infty} \dfrac{x-x^2+x^3}{x-x^5+2} = \lim\limits_{x\to-\infty} \dfrac{x^{\cancel{3}}}{-x^{\cancel{5}}} = \lim\limits_{x\to-\infty} -\dfrac{1}{x^2}$

$= -\dfrac{1}{(-\infty)^2} = -\dfrac{1}{+\infty} = 0$

EXERCÍCIO: Com auxílio de limites laterais e limites no infinito, esboce o gráfico da função

$$f(x) = \frac{2x}{x-1}.$$

SOLUÇÃO: $D(f) = ?$ $D(f) = \mathbb{R} \setminus \{1\}$, pois

$$x - 1 \neq 0 \iff x \neq 1$$

- $$\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} \frac{2x}{x-1} = \frac{2.(1)}{\underbrace{\underbrace{1}_{0,99} - 1}_{0^-}} = \frac{2}{0^-} = -\infty$$

- $$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} \frac{2x}{x-1} = \frac{2.1}{\underbrace{1}_{1,01} - 1} = \frac{2}{0^+} = +\infty$$

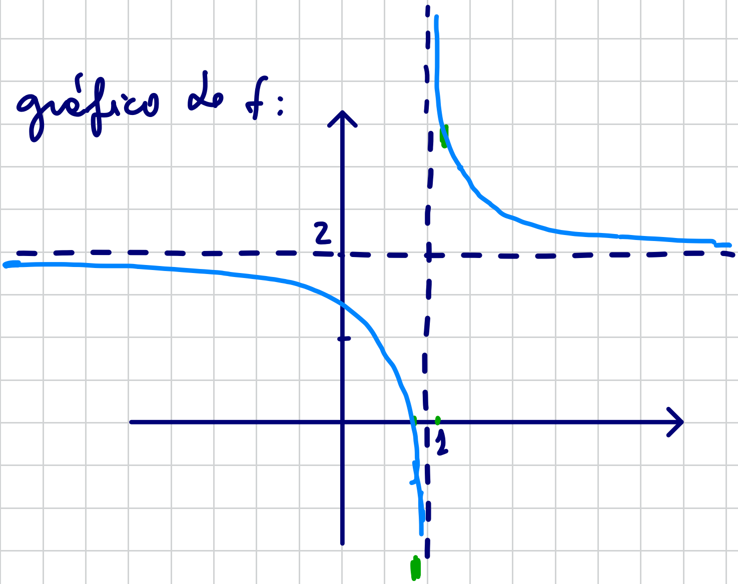
1

1,01.

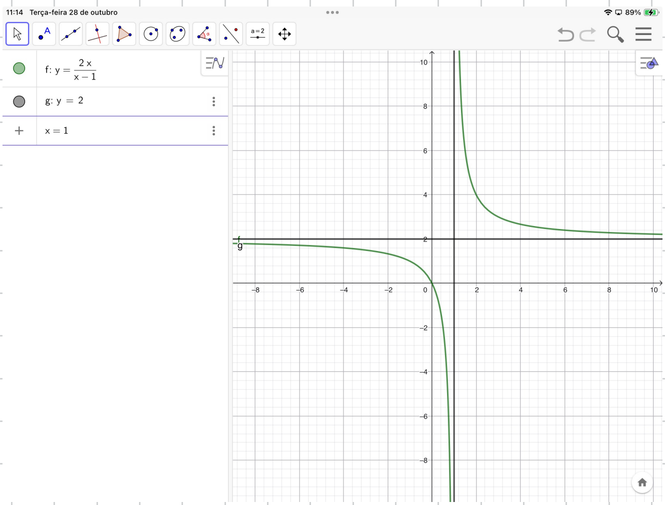
- $$\lim_{x \to +\infty} f(x) = \lim_{x \to +\infty} \frac{2x}{x-1} = \lim_{x \to +\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$$

- $$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{2x}{x-1} = \lim_{x \to -\infty} \frac{2\cancel{x}}{\cancel{x}} = 2$$

Esboço gráfico de f:



PELO GEOGEBRA:



02) Idem para $f(x) = \dfrac{2x-4}{x^2-1}$.

Solução: $D(f) = ?$ $\quad x^2-1 \neq 0 \iff x \neq \pm 1$.

$D(f) = \mathbb{R} \setminus \{-1, 1\}$. $\quad x=-1$ e $x=1$ são assíntotas verticais.

• $\lim\limits_{x\to 1^+} f(x) = \lim\limits_{x\to 1^+} \dfrac{2x-4}{x^2-1} = \lim\limits_{x\to 1^+} \dfrac{2(x-2)}{(x+1)(x-1)}$

1 $\quad$ 1,01

$$= \frac{2.(\overbrace{1-2}^{-1})}{(1+1).(\underbrace{1-1}_{\substack{1,01 \\ 0^+}})} = \frac{-\cancel{2}}{\cancel{2}.(0)^+} = -\frac{1}{0^+} = -\infty$$

• $\lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^-} \dfrac{2x-4}{x^2-1} = \lim\limits_{x\to 1^-} \dfrac{2(x-2)}{(x+1)(x-1)} =$

0,99 $\quad$ 1

$$= \frac{2.(1-2)}{(1+1)(\underbrace{1-1}_{0,99})} = \frac{-\cancel{2}}{\cancel{2}.(0)^-} = -\frac{1}{0^-} = +\infty$$

- $\lim\limits_{x\to -1^+} f(x) = \lim\limits_{x\to -1^+} \dfrac{2x-4}{x^2-1} = \lim\limits_{x\to -1^+} \dfrac{2(x-2)}{(x+1)(x-1)}$

-1 $\quad -0{,}99$

$$= \frac{2.(-1-2)}{\underbrace{(-1+1)}_{\underbrace{-0{,}99+1}_{0}}.(-1-1)} = \frac{\cancel{-6}^{\,3}}{0^+.(\cancel{-2})} = \frac{3}{0^+} = +\infty$$

- $\lim\limits_{x\to -1^-} f(x) = \lim\limits_{x\to -1^-} \dfrac{2(x-2)}{(x+1)(x-1)} = \dfrac{2.(-1-2)}{\underbrace{(-1+1)}_{\underbrace{-1{,}01+1}_{\underbrace{-0{,}01}}}.(-1-1)}$

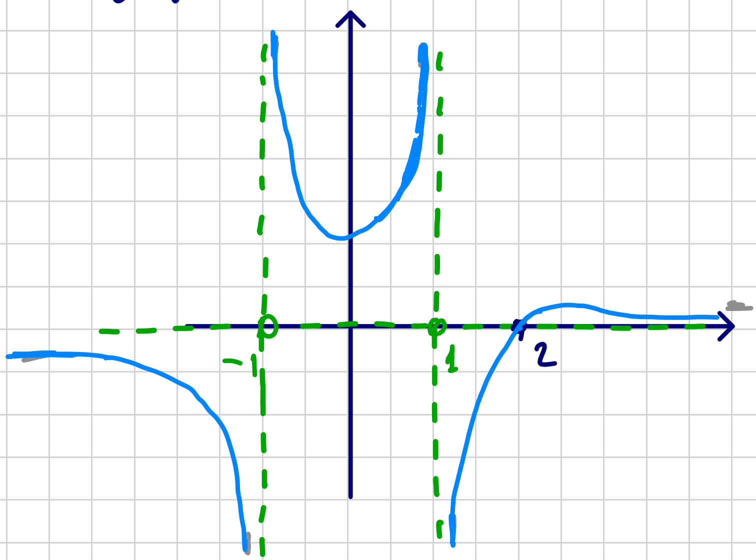
-1 $\quad -1{,}01$

$$= -\frac{6}{0^-.(-2)} = +\frac{3}{0^-} = -\infty$$

- $\lim\limits_{x\to \pm\infty} f(x) = \lim\limits_{x\to \pm\infty} \dfrac{2x-4}{x^2-1} = \lim\limits_{x\to \pm\infty} \dfrac{2\cancel{x}}{x^{\cancel{2}}} =$

$$= \lim_{x\to \pm\infty} \frac{2}{x} = \frac{2}{(\pm\infty)} = 0$$

Esboço gráfico de f:



$$f(x) = \frac{2x-4}{x^2-1}$$ tem zeros? $\iff 2x-4=0$

$\iff \boxed{x=2}$

ESBOÇO GRÁFICO PELO GEOGEBRA: