01) $x, y \in 1^{\underline{o}}\,q.$

$$x = 105^\circ - y$$

$$\Rightarrow \operatorname{sen} x = \operatorname{sen}(105^\circ - y) =$$

$$= \operatorname{sen} 105^\circ . \cos y - \operatorname{sen} y . \cos 105^\circ$$

- $\operatorname{sen} 105^\circ = \operatorname{sen}(60^\circ + 45^\circ) =$

$$= \operatorname{sen} 60^\circ . \cos 45^\circ + \operatorname{sen} 45^\circ . \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

- $\cos 105^\circ = \cos(60^\circ + 45^\circ) =$

$$= \cos 60^\circ . \cos 45^\circ - \operatorname{sen} 60^\circ . \operatorname{sen} 45^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} = \frac{\sqrt{2} - \sqrt{6}}{4}$$

Temos $\cot y = 3$.

$$1 + \cot^2 y = \csc^2 y$$

$$\csc^2 y = 1 + (3)^2 \Rightarrow \csc y = \sqrt{10}$$

$$\boxed{\operatorname{sen} y = \frac{1}{\sqrt{10}}}$$

$$\operatorname{sen}^2 y + \cos^2 y = 1.$$

$$\cos y = \sqrt{1 - \operatorname{sen}^2 y} = \sqrt{1 - \frac{1}{10}}$$

$$\Rightarrow \boxed{\cos y = \frac{3}{\sqrt{10}}}$$

Assim, teremos:

$$\operatorname{sen} x = \operatorname{sen} 105^\circ . \cos y - \operatorname{sen} y . \cos 105^\circ$$

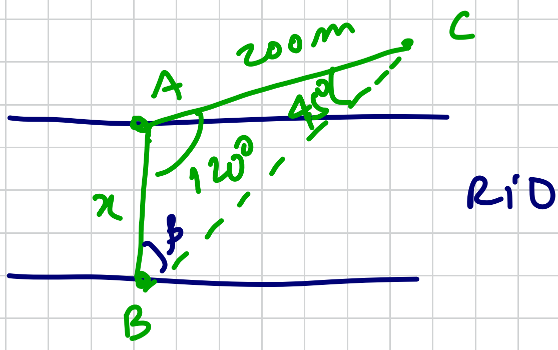
$$= \frac{\sqrt{6}+\sqrt{2}}{4} \cdot \frac{3}{\sqrt{10}} - \frac{1}{\sqrt{10}} \cdot \frac{\sqrt{2}-\sqrt{6}}{4}$$

$$= \frac{1}{4\sqrt{10}} \left(3\sqrt{6} + 3\sqrt{2} - \sqrt{2} + \sqrt{6} \right)$$

$$= \frac{1}{4\sqrt{10}} \cdot \left(4\sqrt{6} + 2\sqrt{2} \right) = \frac{\cancel{2}\cancel{\sqrt{2}}(2\sqrt{3}+1)}{\cancel{4}_{2}\sqrt{5}\cdot\cancel{\sqrt{2}}}$$

$$= \frac{2\sqrt{3}+1}{2\sqrt{5}}$$

02)



$x = ?$

Pela lei dos senos:

$$\frac{x}{\operatorname{sen} 45^\circ} = \frac{200m}{\operatorname{sen} \beta} \quad ; \text{ onde}$$

$$\beta = 180^\circ - 120^\circ - 45^\circ$$

$$\beta = 15^\circ$$

$$\operatorname{sen} 15^\circ = \operatorname{sen}(60^\circ - 45^\circ) =$$

$$= \operatorname{sen} 60^\circ \cdot \cos 45^\circ - \operatorname{sen} 45^\circ \cdot \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} - \sqrt{2}}{4}$$

Assim:

$$\frac{x}{\frac{\sqrt{2}}{2}} = \frac{200\,m}{\frac{\sqrt{6}-\sqrt{2}}{4}}$$

$$\frac{\sqrt{6}-\sqrt{2}}{4} \cdot x = 100\sqrt{2}$$

$$x = \frac{4 \cdot 100\sqrt{2}}{\sqrt{6}-\sqrt{2}} = \frac{400\cancel{\sqrt{2}}}{\cancel{\sqrt{2}}(\sqrt{3}-1)}$$

$$= \frac{400}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} =$$

$$\frac{400\,(1+\sqrt{3})}{3-1} = \underline{200\,(1+\sqrt{3})\,m}$$

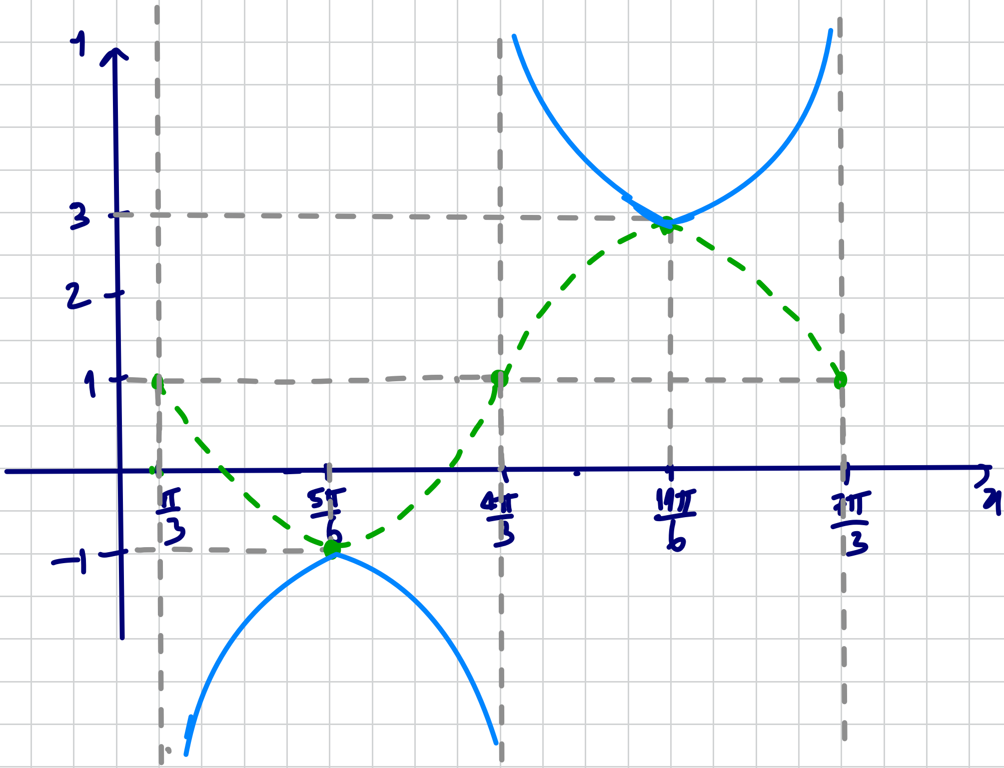
03)

(a) $y = 1 - 2.\csc\left(\frac{\pi}{3} - x\right)$.

$\underline{\underline{1^o}}$: $y = 1 - 2.\sin\left(\underbrace{\frac{\pi}{3} - x}_{=t}\right)$

$t = \frac{\pi}{3} - x \Rightarrow x = t + \frac{\pi}{3}$

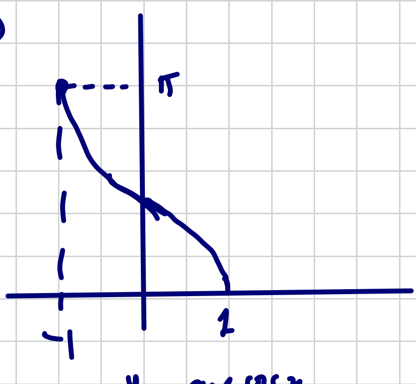
t	$y = 1 - 2\sin t$	$x = t + \frac{\pi}{3}$
0	1	$\frac{\pi}{3}$
$\frac{\pi}{2}$	$1 - 2.1 = -1$	$\frac{\pi}{2} + \frac{\pi}{3} = \frac{3\pi + 2\pi}{6} = \frac{5\pi}{6}$
π	$1 - 0 = 1$	$\pi + \frac{\pi}{3} = \frac{4\pi}{3}$
$\frac{3\pi}{2}$	$1 - 2.(-1) = 3$	$\frac{3\pi}{2} + \frac{\pi}{3} = \frac{11\pi}{6}$
2π	$1 - 2.0 = 1$	$2\pi + \frac{\pi}{3} = \frac{7\pi}{3}$



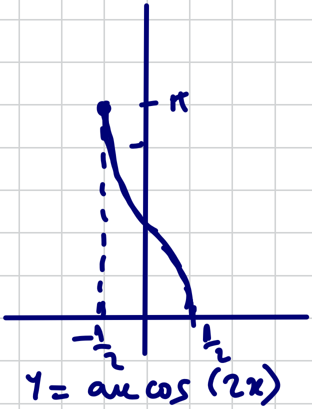
$D(f) = \mathbb{R} \setminus \{ k\pi + \frac{\pi}{3} \}$

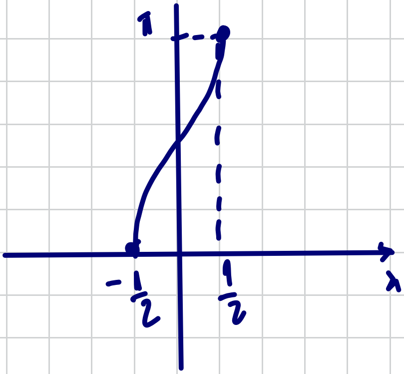
$Im(f) = (-\infty, -1] \cup [3, +\infty)$.

(b)



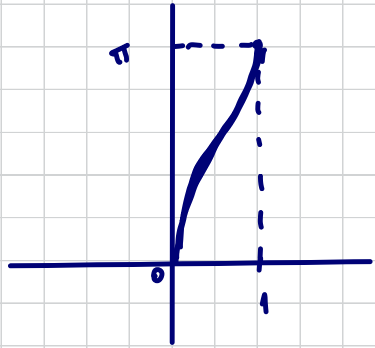
$\rightsquigarrow$





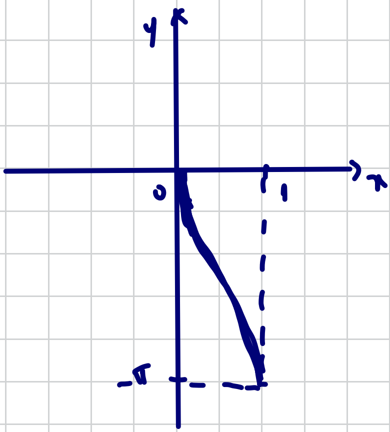
$y = \arccos(\underbrace{-2x}_{\Downarrow})$

$1 \iff x = \frac{1}{2}$

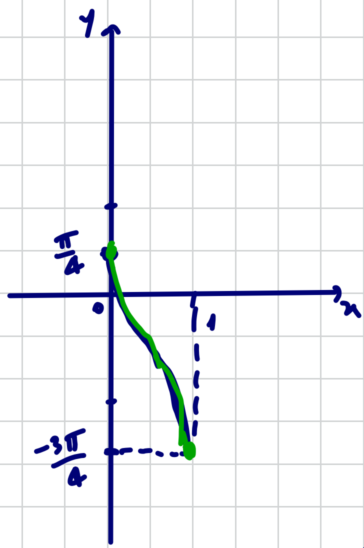


$y = \arccos(\underbrace{1-2x}_{\Downarrow})$

$1 \iff x = 0$



$y = -\arccos(1-2x)$



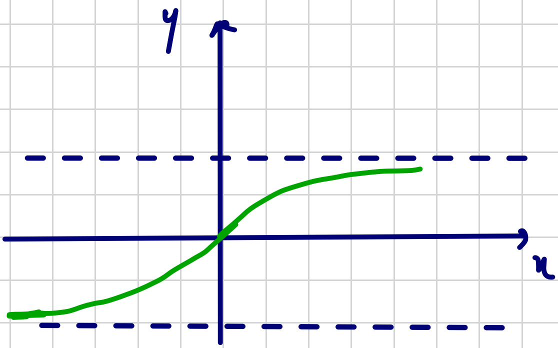
$y = \frac{\pi}{4} - \arccos(1-2x)$

$0 + \frac{\pi}{4} = \frac{\pi}{4}$

$-\pi + \frac{\pi}{4} = -\frac{3\pi}{4}$

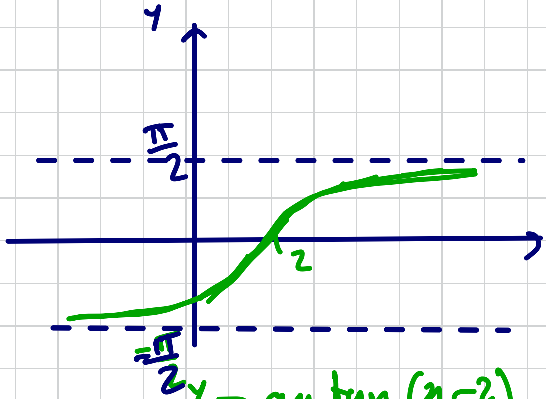
$D(f) = [0, 1]$; $\mathrm{Im}(f) = \left[-\frac{3\pi}{4}, \frac{\pi}{4}\right]$.

(c) $y = \frac{\pi}{3} + \arctan(x-2)$



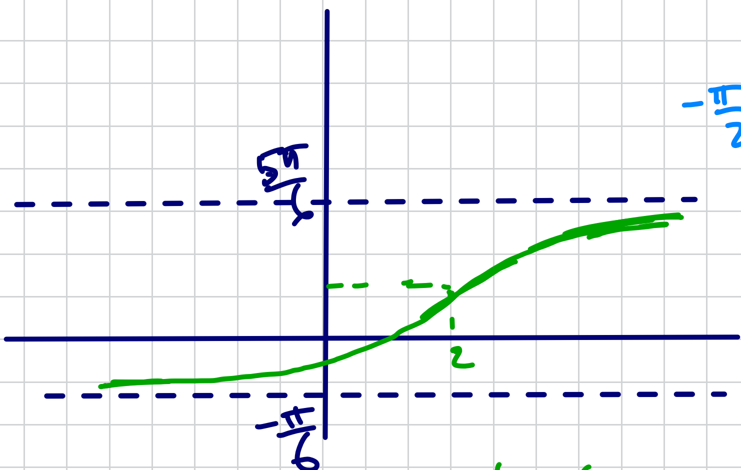
$y = \arctan x$

$\rightsquigarrow$



$y = \arctan(x-2)$

$\rightsquigarrow$



$y = \frac{\pi}{3} + \arctan(x-2)$

$-\frac{\pi}{2} + \frac{\pi}{3} = \frac{-\pi}{6}$

$+\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

$D(f) = \mathbb{R}$; $\quad Im(f) = \left(-\frac{\pi}{6}, \frac{5\pi}{6}\right)$

o4) mostrar que:

$$\arcsin\frac{4}{5} - \arccos\frac{2}{\sqrt{5}} = \arctan\frac{1}{2}.$$

Sejam $\alpha = \arcsin\frac{4}{5} \Leftrightarrow \operatorname{sen}\alpha = \frac{4}{5}$

$\beta = \arccos\frac{2}{\sqrt{5}} \Leftrightarrow \cos\beta = \frac{2}{\sqrt{5}}$

Então:

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha.\tan\beta}.$$

Como $\operatorname{sen}\alpha = \frac{4}{5}$, então

$$\underbrace{\cos\alpha} = \sqrt{1-\left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}} = \underline{\frac{3}{5}}$$

e como $\cos\beta = \frac{2}{\sqrt{5}}$, então:

$$\underbrace{\operatorname{sen}\beta} = \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \underline{\frac{1}{\sqrt{5}}}.$$

Assim:

- $$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{\frac{4}{\cancel{5}}}{\frac{3}{\cancel{5}}} = \frac{4}{3}$$

- $$\tan\beta = \frac{\operatorname{sen}\beta}{\cos\beta} = \frac{\frac{1}{\cancel{\sqrt{5}}}}{\frac{2}{\cancel{\sqrt{5}}}} = \frac{1}{2}.$$

Portanto:

$$\tan(\alpha-\beta) = \frac{\frac{4}{3} - \frac{1}{2}}{1 + \frac{\cancel{4}^{2}}{3}\cdot\frac{1}{\cancel{2}}} = \frac{\frac{8-3}{6}}{\frac{5}{3}} =$$

$$= \frac{\cancel{5}}{6} \times \frac{3}{\cancel{5}} = \frac{1}{2}$$

$$\Rightarrow \quad \alpha - \beta = \operatorname{arc\,tan}\left(\frac{1}{2}\right)$$

$$\Rightarrow \quad \operatorname{arc\,sen}\frac{4}{5} - \operatorname{arc\,cos}\frac{2}{\sqrt{5}} = \operatorname{arc\,tan}\frac{1}{2}.$$

$\square$

05)

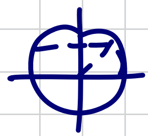
(a) $2 \cdot \sec x = \tan x + \cot x$

$$2 \cdot \frac{1}{\cos x} = \frac{\operatorname{sen} x}{\cos x} + \frac{\cos x}{\operatorname{sen} x}$$

$$\frac{2}{\cos x} = \frac{\overbrace{\operatorname{sen}^2 x + \cos^2 x}^{=1}}{\operatorname{sen} x \cdot \cos x}$$

$$\frac{2}{\cancel{\cos x}} = \frac{1}{\operatorname{sen} x \cdot \cancel{\cos x}} \Rightarrow \underbrace{\operatorname{sen} x = \frac{1}{2}}_{x \in 1^{\circ}q \text{ ou } 2^{\circ}q}$$

- $1^{\circ}q$: $x = \frac{\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$



- $2^{\circ}q$: $x = \pi - \frac{\pi}{6} + 2k\pi$, $k \in \mathbb{Z}$

$$x = \frac{5\pi}{6} + 2k\pi \text{ , } k \in \mathbb{Z}.$$

$$S = \left\{ \frac{\pi}{6} + 2k\pi ; \frac{5\pi}{6} + 2k\pi , \forall k \in \mathbb{Z} \right\}.$$

(b) $\cos 2x + \cos x + 1 = 0$

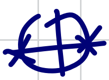
$$\underbrace{\cos 2x} = \cos^2 x - \text{sen}^2 x$$
$$= \cos^2 x - (1 - \cos^2 x)$$
$$= \underbrace{2\cos^2 x - 1}.$$

$$2\cos^2 x \cancel{-1} + \cos x \cancel{+1} = 0$$

$$\Leftrightarrow \quad 2\cos^2 x + \cos x = 0$$

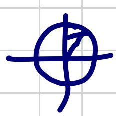
$$\Leftrightarrow \quad \cos x\,(2\cos x + 1) = 0$$

$$\Leftrightarrow \begin{cases} \cos x = 0 \\ 2\cos x = -1 \end{cases}$$



- $\cos x = 0 \Leftrightarrow x = k\pi, \ \forall k \in \mathbb{Z}.$

- $\cos x = -\frac{1}{2}$ ($x \in 2^{\circ}q$ ou $x \in 4^{\circ}q$)



- $2^{\circ}q$: $x = \pi - \frac{\pi}{3} + 2k\pi = \frac{2\pi}{3} + 2k\pi$

- $4^{\circ}q$: $x = \pi + \frac{\pi}{3} + 2k\pi = \frac{4\pi}{3} + 2k\pi.$

$S = \{ k\pi \ ; \ \frac{2\pi}{3} + 2k\pi \ \text{ou} \ \frac{4\pi}{3} + 2k\pi, \ \forall k \in \mathbb{Z} \}.$

06) A forma mais simples de resolver:

$$\frac{\cos\frac{a}{2} - \operatorname{sen}\frac{a}{2}}{\cos\frac{a}{2} + \operatorname{sen}\frac{a}{2}} \underset{\div \cos\frac{a}{2}}{=} \frac{\frac{\cos\frac{a}{2}}{\cos\frac{a}{2}} - \frac{\operatorname{sen}\frac{a}{2}}{\cos\frac{a}{2}}}{\frac{\cos\frac{a}{2}}{\cos\frac{a}{2}} + \frac{\operatorname{sen}\frac{a}{2}}{\cos\frac{a}{2}}} = \frac{1 - \tan\frac{a}{2}}{1 + \tan\frac{a}{2}}$$