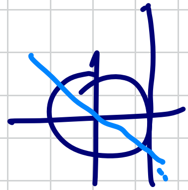
FUNÇÕES TRANSCEDENTAIS

$\boxed{\text{06/08/25 - AULA 14}}$

AULA DE EXERCÍCIOS.

LISTA 04:

$x \in 4^{\circ}q$.



17. Sabendo que $\sec x = \frac{25}{24}$ e $\frac{3\pi}{2} < x < 2\pi$, calcule $\tan 2x$.

$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

(FÓRMULA DA TANGENTE DO ARCO DUPLO)

Como $1 + \tan^2 x = \sec^2 x$, então

$$\tan^2 x = \sec^2 x - 1$$

$$\tan^2 x = \left(\frac{25}{24}\right)^2 - 1$$

$$\tan^2 x = \frac{625}{576} - 1 = \frac{625 - 576}{576}$$

$$\tan^2 x = \frac{49}{576} \Rightarrow \tan x = \pm\sqrt{\frac{49}{576}}$$

$$= -\frac{7}{24}$$

$(x \in 4^{\circ}q)$

Disso:

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x} = \frac{2\cdot\left(-\frac{7}{24}\right)}{1-\frac{49}{576}} =$$

$$= \frac{-\frac{14}{24}}{\frac{576-49}{576}} = \frac{-14}{\cancel{24}} \times \frac{\overset{24}{\cancel{576}}}{527} =$$

$$= -\frac{14\cdot 24}{527} = -\frac{336}{527}$$

24.(

18. Se $\cos x = \frac{3}{5}$ e $\frac{3\pi}{2} < x < 2\pi$, calcule sen $3x$.

$\underbrace{\frac{3\pi}{2} < x < 2\pi}_{x \in 4^\circ q.}$

$$\text{sen}\, 3x = \text{sen}(2x+x) = \text{sen}\, 2x \cdot \cos x + \text{sen}\, x \cdot \cos 2x$$

$\boxed{\text{sen}(a+b) = \text{sen}\, a \cdot \cos b + \text{sen}\, b \cdot \cos a}$

Como $\text{sen}\, 2x = 2\cdot \text{sen}\, x \cdot \cos x$; e

$\cos 2x = \cos^2 x - \text{sen}^2 x$, então:

$$\operatorname{sen} 3x = (2\cdot \operatorname{sen}x \cdot \cos x)\cdot \cos x + \operatorname{sen}x \cdot (\cos^2 x - \operatorname{sen}^2 x)$$

$$\operatorname{sen} 3x = 2\operatorname{sen}x \cdot \cos^2 x + \operatorname{sen}x \cdot \cos^2 x - \operatorname{sen}^3 x$$

$$\boxed{\operatorname{sen} 3x = 3\cdot \operatorname{sen}x \cdot \cos^2 x - \operatorname{sen}^3 x} \quad (*)$$

Como $\cos x = \frac{3}{5}$; $x \in 4.^{\circ}q$; então :

$$\operatorname{sin}^2 x + \cos^2 x = 1$$

$$\operatorname{sen}^2 x + \left(\frac{3}{5}\right)^2 = 1$$

$$\operatorname{sen}^2 x = 1 - \frac{9}{25}$$

$$\operatorname{sen}^2 x = \frac{25-9}{25}$$

$$\operatorname{sen}^2 x = \frac{16}{25}$$

$$\operatorname{sen} x = \pm\sqrt{\frac{16}{25}}$$

$$\operatorname{sen} x = -\frac{4}{5}$$

$x \in 4.^{\circ}q$.

Assim, de $(*)$, vem :

$$\operatorname{sen} 3x = 3\cdot\left(-\frac{4}{5}\right)\left(\frac{3}{5}\right)^2 - \left(-\frac{4}{5}\right)^3$$

$$= -\frac{12}{5} \cdot \frac{4}{25} + \frac{64}{125} =$$

$$= \frac{-48}{125} + \frac{64}{125} = \frac{16}{125}$$

$$\Rightarrow \boxed{\operatorname{sen} 3x = \frac{16}{125}}$$

L4

21. Calcule o valor de $\cos 15°$ de duas formas:

(a) escrevendo $\cos 15° = \cos(45° - 30°)$;

(b) escrevendo $\cos 15° = \cos\left(\frac{30°}{2}\right)$.

Aparentemente, os resultados obtidos em (a) e em (b) parecem ser diferentes. No entanto, representam o mesmo valor, ou seja, são iguais. Como você mostraria isso?

(a) $\cos 15° = \cos(45° - 30°) = \cos 45° \cdot \cos 30° + \operatorname{sen} 45° \cdot \operatorname{sen} 30°$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6} + \sqrt{2}}{4}$$

(b) $\cos 15° = \cos \frac{30°}{2} = +\sqrt{\frac{1 + \cos 30°}{2}} = \sqrt{\frac{1 + \frac{\sqrt{3}}{2}}{2}} =$

1º Q

$$\cos \left(\frac{x}{2}\right) = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$= \sqrt{\frac{\frac{2+\sqrt{3}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{3}}{2} \times \frac{1}{2}} = \underbrace{\sqrt{\frac{2+\sqrt{3}}{4}}}$$

Então $x = \dfrac{\sqrt{6}+\sqrt{2}}{4}$ e $y = \sqrt{\dfrac{2+\sqrt{3}}{4}}$.

Note que $x, y > 0$.

Eleve ambos ao quadrado:

$$y^2 = \left(\sqrt{\frac{2+\sqrt{3}}{4}}\right)^{\cancel{2}} = \frac{2+\sqrt{3}}{4}$$

$$x^2 = \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2 = \frac{6+2\sqrt{6}.\sqrt{2}+2}{16}$$

$$= \frac{8+2\sqrt{12}}{16} = \frac{4+\sqrt{12}}{8}$$

$$= \frac{4+\sqrt{3.2^{\cancel{2}}}}{8} = \frac{4+2\sqrt{3}}{8} = \frac{2+\sqrt{3}}{4}$$

Então, $x^2 = y^2 \Leftrightarrow x = \pm y$; e como $x, y > 0$, $x = y$, ou seja, obtemos que

$$\underbrace{\cos 15^\circ = \frac{\sqrt{6}+\sqrt{2}}{4} = \sqrt{\frac{2+\sqrt{3}}{4}}}.$$

L4]

22. Mostre que

(a) $\operatorname{sen} 40° + \operatorname{sen} 20° = \cos 10°$.

(b) $\operatorname{sen} 105° + \operatorname{sen} 15° = \frac{\sqrt{6}}{2}$.

(c) $\cos 130° + \cos 110° + \cos 10° = 0$.

(d) $\cos 220^0 + \cos 100° + \cos 20° = 0$.

(a) lembrar da fórmula:

$$\operatorname{sen} p + \operatorname{sen} q = 2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

Assim :

$$\operatorname{sen} 40° + \operatorname{sen} 20° = 2 \cdot \operatorname{sen}\left(\frac{40°+20°}{2}\right) \cdot \cos\left(\frac{40°-20°}{2}\right)$$

$$= 2 \operatorname{sen} 30° \cdot \cos 10° = \cancel{2} \cdot \frac{1}{\cancel{2}} \cdot \cos 10° = \cos 10°$$

L4]

23. Prove que $\dfrac{\operatorname{sen}(x+y)}{\cos(x-y)} = \dfrac{\tan x + \tan y}{1 + \tan x \cdot \tan y}$.

$$\frac{\operatorname{sen}(x+y)}{\cos(x-y)} = \frac{\operatorname{sen} x \cdot \cos y + \operatorname{sen} y \cdot \cos x}{\cos x \cdot \cos y + \operatorname{sen} x \cdot \operatorname{sen} y} =$$

DIVIDIR NUM. E DENOM. POR $\cos x \cdot \cos y$

$$= \frac{\dfrac{\operatorname{sen} x \cdot \cancel{\cos y}}{\cos x \cdot \cancel{\cos y}} + \dfrac{\operatorname{sen} y \cdot \cancel{\cos x}}{\cancel{\cos x} \cdot \cos y}}{\dfrac{\cancel{\cos x} \cdot \cancel{\cos y}}{\cancel{\cos x} \cdot \cancel{\cos y}} + \dfrac{\operatorname{sen} x}{\cos x} \cdot \dfrac{\operatorname{sen} y}{\cos y}} = \frac{\tan x + \tan y}{1 + \tan x \cdot \tan y}.$$

1

LA

26. Obtenha todos os pares (x, y), com $x, y \in [0, 2\pi]$, tais que

$$\begin{cases} \operatorname{sen}(x+y) + \operatorname{sen}(x-y) = \frac{1}{2} \\ \operatorname{sen} x + \cos y = 1 \end{cases}.$$

$$\operatorname{sen} x \cdot \cos y + \cancel{\operatorname{sen} y \cdot \cos x} + \operatorname{sen} x \cdot \cos y - \cancel{\operatorname{sen} y \cos x} = \frac{1}{2}$$

$$\Rightarrow \boxed{2 \cdot \operatorname{sen} x \cos y = \frac{1}{2}}$$

Pela 2ª equação:

$\operatorname{sen} x + \cos y = 1$; elevando ao quadrado, vem:

$$(\operatorname{sen} x + \cos y)^2 = 1^2 = 1$$

$$\operatorname{sen}^2 x + \underbrace{2 \cdot \operatorname{sen} x \cdot \cos y}_{1/2} + \cos^2 y = 1$$

$$\operatorname{sen}^2 x + \cos^2 y = 1 - \frac{1}{2} \Rightarrow \boxed{\operatorname{sen}^2 x + \cos^2 y = \frac{1}{2}}$$

$$\begin{cases} \operatorname{sen} x \cdot \cos y = \frac{1}{4} \\ \operatorname{sen} x + \cos y = 1 \end{cases} \Longrightarrow \underline{\operatorname{sen} x = 1 - \cos y}$$

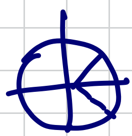
$$(1 - \cos y)\cos y = \frac{1}{4}$$

$$\cos y - \cos^2 y - \frac{1}{4} = 0 \qquad (\times -4)$$

$$4\cos^2 y - 4\cos y + 1 = 0$$

$$\cos y = \frac{4 \pm \sqrt{16 - 16}}{2 \cdot 4} = \cos y = \frac{1}{2}.$$

$y \in$ 1º q ou 4º q



$$\begin{cases} 1^{\text{o}}\text{q}: \ y = \frac{\pi}{3} \text{ rad.} \\ 4^{\text{o}}\text{q}: \ y = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \text{ rad.} \end{cases}$$

Por fim:

$$\operatorname{sen} x = 1 - \underbrace{\cos y}_{\frac{1}{2}} = 1 - \frac{1}{2} = \frac{1}{2}.$$

$x \in$ 1º q ou 2º q.

$$\begin{cases} 1^{\text{o}}\text{q}: \ x = \frac{\pi}{6} \text{ rad.} \\ 2^{\text{o}}\text{q}: \ x = \pi - \frac{\pi}{6} = \frac{5\pi}{6} \text{ rad.} \end{cases}$$

SOLUÇÃO: $x = \frac{\pi}{6}$ ou $\frac{5\pi}{6}$ e $y = \frac{\pi}{3}$ ou $\frac{5\pi}{3}$.

LISTA 05:

5. Esboçar o gráfico de cada função abaixo, indicando domínio e imagem:

(a) $f(x) = \arccos(3+2x)$

(b) $f(x) = \dfrac{2\pi}{5} - 3\,\mathrm{arcsen}\,(1-x)$

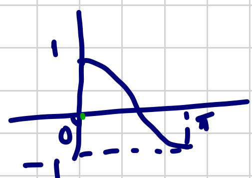
(c) $f(x) = \dfrac{\pi}{3} + 2\arctan(3x-1)$

(d) $f(x) = -\mathrm{arccsc}(3-2x)$

(e) $f(x) = -\dfrac{\pi}{4} + \mathrm{arccot}\,(2x-1)$

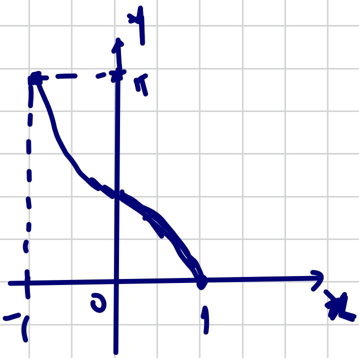
(f) $f(x) = \pi - 2\,\mathrm{arcsec}(x+1)$

(a) $y = \arccos(3+2x) \iff 3+2x = \cos y$

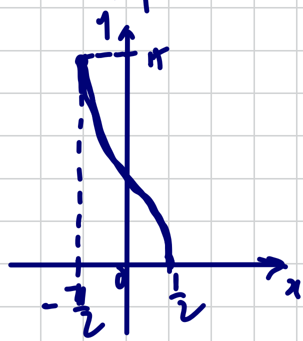


$\cos : [0,\pi] \to [-1,1]$ bijet.

$\arccos : [-1,1] \to [0,\pi]$



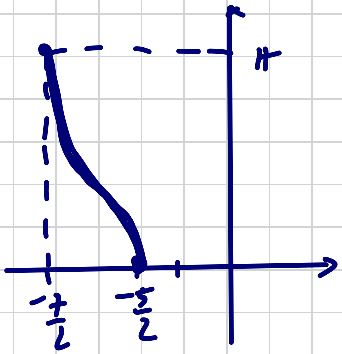
$y = \arccos x$



$y = \arccos 2x$

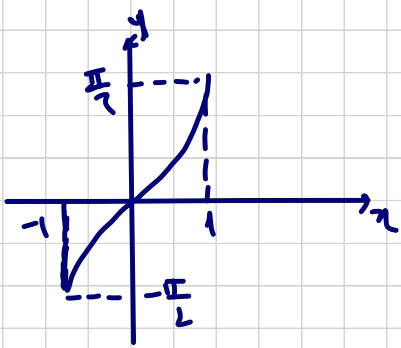
$\frac{1}{2} - 3 = \frac{1}{2} - \frac{6}{2} = -\frac{5}{2}$

$-\frac{1}{2} - 3 = -\frac{1}{2} - \frac{6}{2} = -\frac{7}{2}$

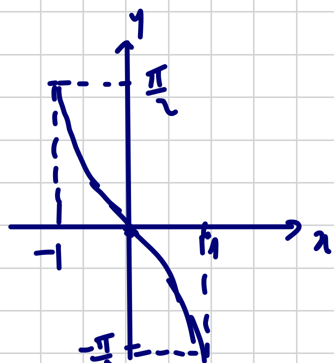


$y = \arccos(3+2x)$

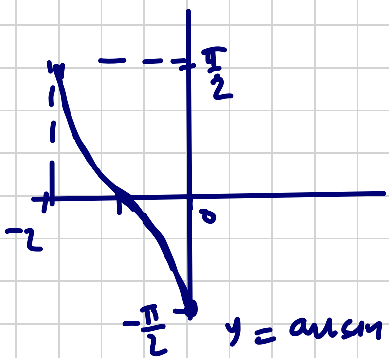
(b) $y = \frac{2\pi}{5} - 3 \cdot \text{arcsen}(1-x)$



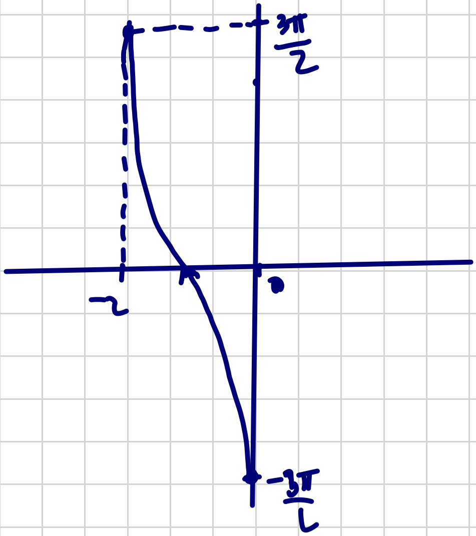
$y = \text{arcsen}\, x$



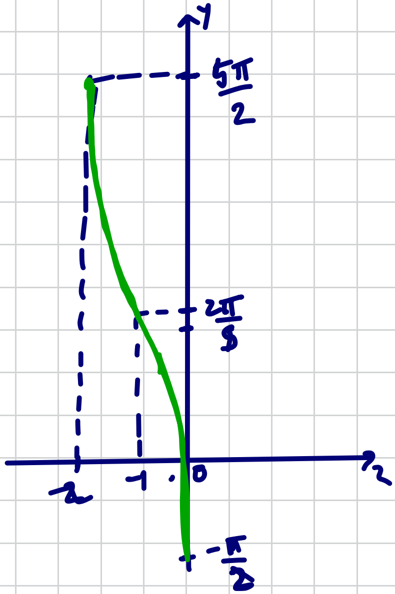
$y = \arcsin(-x)$



$y = \arcsin(1-x)$



$y = 3 \cdot \operatorname{arcsen}(1-x)$



$y = \frac{2\pi}{5} + 3 \cdot \operatorname{arcsen}(1-x)$

$-\frac{3\pi}{2} + \frac{2\pi}{5} = \frac{-15\pi + 10\pi}{10}$

$= -\frac{\pi}{2}$

$\frac{3\pi}{2} + \frac{2\pi}{5} = \frac{15\pi + 10\pi}{10}$

$= \frac{5\pi}{2}$

L5

8. Mostre que $\operatorname{arcsen}\frac{1}{\sqrt{5}} + \operatorname{arcsen}\frac{2}{\sqrt{5}} = \frac{\pi}{2}$.

Escreva $\alpha = \arcsin\frac{1}{\sqrt{5}} \iff \operatorname{sen}\alpha = \frac{1}{\sqrt{5}}$

$\beta = \operatorname{arcsen}\frac{2}{\sqrt{5}} \iff \operatorname{sen}\beta = \frac{2}{\sqrt{5}}$

A mostrar: $\alpha + \beta = \frac{\pi}{2}$.

$$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \operatorname{sen}\beta \cdot \cos\alpha$$

$$= \frac{1}{\sqrt{5}} \cdot \cos\beta + \frac{2}{\sqrt{5}} \cdot \cos\alpha$$

Pela relação trigonom. fundamental;

$$\operatorname{sen}^2\alpha + \cos^2\alpha = 1$$

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$\cos\alpha = +\sqrt{1 - \left(\frac{1}{\sqrt{5}}\right)^2} = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}}$$

Do mesmo modo,

$$\operatorname{sen}^2\beta + \cos^2\beta = 1$$

$$\Rightarrow \cos^2\beta = 1 - \sin^2\beta$$

$$\Rightarrow \cos\beta = +\sqrt{1 - \operatorname{sen}^2\beta}$$

$$\Rightarrow \cos\beta = \sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = \sqrt{1-\frac{4}{5}} = \frac{1}{\sqrt{5}}$$

Portanto,

$$\operatorname{sen}(\alpha+\beta) = \frac{1}{\sqrt{5}}\cdot\cos\beta + \frac{2}{\sqrt{5}}\cdot\operatorname{sen}\beta$$

$$= \frac{1}{\sqrt{5}}\cdot\frac{1}{\sqrt{5}} + \frac{2}{\sqrt{5}}\cdot\frac{2}{\sqrt{5}} = \frac{1}{5}+\frac{4}{5} = 1.$$

$$\Rightarrow \operatorname{sen}(\alpha+\beta) = 1 \iff \alpha+\beta = \operatorname{arcsen}(1) = \frac{\pi}{2}$$

$$\Rightarrow \alpha+\beta = \frac{\pi}{2}.$$

L5|

10. Demonstre que $\operatorname{arcsen}\frac{77}{85} - \operatorname{arcsen}\frac{3}{5} = \operatorname{arccos}\frac{15}{17}$.

Escreva $\alpha = \operatorname{arcsen}\frac{77}{85} \iff \operatorname{sen}\alpha = \frac{77}{85}$

$\beta = \operatorname{arcsen}\frac{3}{5} \iff \operatorname{sen}\beta = \frac{3}{5}$

Mostrar:

$$\alpha-\beta = \operatorname{arccos}\frac{15}{17}$$

Note que:

$$\cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + \operatorname{sen}\alpha \cdot \operatorname{sen}\beta. = \frac{15}{17}$$

(...) se segue o mesmo procedimento de antes.