CÁLCULO II.

06/08/25 — AULA 25

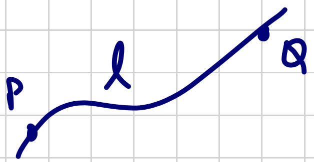
AULA DE EXERCÍCIOS:

LISTA 10

2. Ache o comprimento da curva $x^2 = (2y+3)^3$ de $P(1,-1)$ a $Q(7\sqrt{7},2)$.

$$x = \sqrt{(2y+3)^3} = (2y+3)^{\frac{3}{2}}$$

$$x = f(y) \qquad \overset{\|}{f(y)}$$



$$\ell = \int_{a=-1}^{b=2} \sqrt{1+[f'(y)]^2}\, dy$$

$$f(y) = (2y+3)^{\frac{3}{2}}$$

$$\Rightarrow f'(y) = \frac{3}{\cancel{2}}(2y+3)^{\frac{1}{2}} \cdot \cancel{2}$$

$$f'(y) = 3\sqrt{2y+3}$$

$$\Rightarrow \ell = \int_{-1}^{2} \sqrt{1+\left[3\sqrt{2y+3}\right]^2}\, dy$$

$$= \int_{-1}^{2} \sqrt{1 + 9\cdot(2y+3)}\, dy$$

$$= \int_{-1}^{2} \sqrt{1 + 18y + 27}\, dy = \int_{-1}^{2} \sqrt{18y+28}\, dy$$

$\underline{\underline{1^o}}$: $\int \sqrt{18y+28}\, dy = \frac{1}{18}\int (18y+28)^{1/2}\, 18\, dy =$

$$\begin{cases} u = 18y+28 \\ du = 18\, dy \end{cases}$$

$$= \frac{1}{18} \cdot \frac{(18y+28)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c = \frac{1}{\cancel{18}_{9}} \cdot \frac{\cancel{2}}{3} \cdot (18y+28)^{\frac{3}{2}} + c$$

$$= \frac{1}{27}\sqrt{(18y+28)^3} + c = \frac{18y+28}{27} \cdot \sqrt{18y+28} + c$$

$$\Rightarrow l = \int_{-1}^{2} \sqrt{18y+28}\, dy = \left(\frac{18y+28}{27} \cdot \sqrt{18y+28} \right) \Bigg|_{-1}^{2} =$$

$$= \frac{36+28}{27} \cdot \sqrt{36+28} - \frac{-18+28}{27} \cdot \sqrt{-18+28}$$

$$= \frac{64}{27}\sqrt{64} - \frac{10}{27}\sqrt{10}$$

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L10)

5. Determine o comprimento do arco da curva $y = \ln(1 - x^2)$ de $x = 0$ a $x = \frac{1}{2}$.

$$l = \int_0^{\frac{1}{2}} \sqrt{1 + [f'(x)]^2}\, dx.$$

$$f(x) = \ln(1 - x^2)$$

$$\Rightarrow f'(x) = \frac{-2x}{1 - x^2}$$

Assim, teremos:

$$l = \int_0^{\frac{1}{2}} \sqrt{1 + \left(\frac{-2x}{1-x^2}\right)^2}\, dx$$

$$= \int_0^{\frac{1}{2}} \sqrt{\frac{(1-x^2)^2 + 4x^2}{(1-x^2)^2}}\, dx =$$

$$= \int_0^{1/2} \frac{\sqrt{1-2x^2+x^4+4x^2}}{1-x^2}\,dx =$$

$$= \int_0^{1/2} \frac{\sqrt{x^4+2x^2+1}}{1-x^2}\,dx = \int_0^{1/2} \frac{\sqrt{\cancel{(x^2+1)^2}}}{1-x^2}\,dx$$

$$= \int_0^{1/2} \frac{1+x^2}{1-x^2}\,dx = \int_0^{1/2} \frac{-1(1-x^2)+2}{1-x^2}\,dx =$$

$$\begin{array}{r|l} \cancel{x^2}+1 & -x^2+1 \\ \hline -\cancel{x^2}+1 & -1 \\ \hline 2 & \end{array}$$

$$x^2+1 = -1.(-x^2+1)+2$$

$$\int_0^{1/2} -dx + 2\int_0^{1/2} \frac{dx}{1-x^2} \longrightarrow \int \frac{dv}{a^2-v^2} = \frac{1}{2a}\ln\left|\frac{v-a}{v+a}\right| + c$$

$$= -x\Big|_0^{1/2} + \cancel{2}.\frac{1}{\cancel{2}.1}.\ln\left|\frac{x-1}{x+1}\right|\Bigg|_0^{1/2} =$$

$$= -\frac{1}{2} - (0) + \ln\left|\frac{\frac{1}{2}-1}{\frac{1}{2}+1}\right| - \underbrace{\ln\left|\frac{-1}{1}\right|}_{= \ln 1 = 0}$$

$$= -\frac{1}{2} + \ln\left|\frac{-\frac{1}{2}}{\frac{3}{2}}\right| = -\frac{1}{2} + \ln\frac{1}{3}$$

L5 ; 02 (k)

(k) $\int \frac{(x^2+x)dx}{(x-1)(x^2+1)} = \ln|x-1| + \arctan(x) + c$

$$\frac{x^2+x}{(x-1)\cdot(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}.$$

$$= \frac{A\cdot(x^2+1) + (Bx+C)(x-1)}{(x-1)(x^2+1)}$$

$$= \frac{Ax^2+A+Bx^2-Bx+Cx-C}{(x-1)(x^2+1)}$$

$$x^2+x\,0 = Ax^2 + A + Bx^2 - Bx + Cx - C.$$

$$\begin{cases} A + B = 1 \\ -B + C = 1 \\ A - C = 0 \end{cases} \quad +$$

$$2A = 2 \Rightarrow A = 1$$

$$\Rightarrow B = 1 - A$$

$$B = 1 - 1 \Rightarrow B = 0$$

$$-\underbrace{B}_{0} + C = 1$$

$$C = 1$$

$$\Rightarrow \frac{x^2+x}{(x-1)(x^2+1)} = \frac{1}{x-1} + \frac{0x+1}{x^2+1}$$

$$\int \frac{du}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$$

$$\Rightarrow \int \frac{(x^2+x)\,dx}{(x-1)(x^2+1)} = \int \frac{dx}{x-1} + \int \frac{dx}{x^2+1}$$

$$= \ln|x-1| + \frac{1}{1}.\arctan\left(\frac{x}{1}\right)+C$$

$$= \ln|x-1| + \arctan x + C.$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$u = x-1$$
$$du = dx$$

L5 – 01(d)

$$\text{(d)} \int \operatorname{sen}^2 2x \cos^5 2x\,dx = \int \operatorname{sen}^2 2x \cdot \Big(\underbrace{\cos^2 2x}_{1-\operatorname{sen}^2 2x}\Big)^2 \underbrace{\cos 2x\,dx}_{\text{vai compor o diferencial}}$$

$$= \int \operatorname{sen}^2 2x \cdot (1-\operatorname{sen}^2 2x)^2 \cdot \cos 2x\,dx$$

$$= \int \operatorname{sen}^2 2x \cdot (1 - 2\operatorname{sen}^2 2x + \operatorname{sen}^4 2x) \cdot \cos 2x\,dx$$

$$= \frac{1}{2}\int \operatorname{sen}^2 2x\,(2\cos 2x\,dx) - \frac{\cancel{2}}{\cancel{2}}\int \operatorname{sen}^4 2x\,(2\cos 2x\,dx) + \frac{1}{2}\int \operatorname{sen}^6 2x\,(2\cos 2x\,dx)$$

$$\int v^k\,dv$$

$$\underline{v = \operatorname{sen} 2x} \Rightarrow dv = 2\cos 2x\,dx$$

$$= \frac{1}{2}\int v^2\,dv - \int v^4\,dv + \frac{1}{2}\int v^6\,dv =$$

$$= \frac{1}{2}\frac{v^3}{3} - \frac{v^5}{5} + \frac{1}{2}\frac{v^7}{7} + c =$$

$$= \frac{1}{6}\cdot(\operatorname{sen} 2x)^3 - \frac{1}{5}(\operatorname{sen} 2x)^5 + \frac{1}{14}\cdot(\operatorname{sen} 2x)^7 + c\,.$$

$$= \frac{1}{6}\,\text{sen}^3 2x - \frac{1}{5}\cdot\text{sen}^5 2x + \frac{1}{14}\cdot\text{sen}^7 2x + c.$$

25.02-(g)

(g) $\displaystyle\int \frac{(2x-1)dx}{x^3-2x^2-5x+6} = -\frac{1}{6}\ln|x-1| + \frac{1}{2}\ln|x-3| - \frac{1}{3}\ln|x+2| + c$

Como o denominador não está fatorado, inicialmente precisamos efetuar uma fatoração, a partir dos zeros do denominador.

$$x^3-2x^2-5x+6=0.$$

Termo independente: 6

seus divisores: $\pm 1; \pm 2; \pm 3; \pm 6$.

algum talvez possa ser raiz. Tem que verificar.

quando $x=1$, tem-se:

$1-2-5+6=0$ (OK!)

Então, divide o polinômio por $x-1$:

$$\begin{array}{l|l} x^3-2x^2-5x+6 & x-1 \\ \cline{2-2} -x^3+x^2 & x^2-x-6 \\ \cline{1-1} \quad -x^2-5x+6 & \\ \quad +x^2-x & \\ \cline{1-1} \qquad -6x+6 & \\ \qquad +6x-6 & \\ \cline{1-1} \qquad\quad 0 & \end{array}$$

$$\Rightarrow x^3-2x^2-5x+6 = (x-1)\cdot(x^2-x-6)$$

$$x^2-x-6=0$$

$$x = \frac{1 \pm \sqrt{1+24}}{2}$$

$$x = \frac{1+5}{2} = 3$$

ou

$$x = \frac{1-5}{2} = -2$$

$$\Rightarrow \quad x^3 - 2x^2 - 5x + 6 = (x-1)(x^2 - x - 6)$$

$$= (x-1)(x-3)(x-(-2))$$

$$= (x-1)(x-3)(x+2)$$

$$\int \frac{(2x-1)\,dx}{(x-1)(x-3)(x+2)}$$

$$\frac{2x-1}{(x-1)(x-3)(x+2)} = \frac{A}{x-1} + \frac{B}{x-3} + \frac{C}{x+2}$$

$$= \frac{A(x-3)(x+2) + B(x-1)(x+2) + C(x-1)(x-3)}{(x-1)(x+3)(x+2)}$$

$$2x-1 = A(x-3)(x+2) + B(x-1)\cdot(x+2) + C(x-1)(x-3)$$

(válido p/ caso I):

- quando $x = 1$:

$$2 \cdot 1 - 1 = A \cdot (1-3) \cdot (1+2) + 0 + 0$$

$$1 = -6A \rightarrow \boxed{A = -\frac{1}{6}}$$

• quando $x = 3$:

$$2.(3) - 1 = 0 + B(3-1)(3+2) + 0$$

$$5 = 10B \Rightarrow \boxed{B = \frac{1}{2}}$$

• quando $x = -2$:

$$2.(-2) - 1 = C.(-2-1).(-2-3)$$

$$-5 = +15C \Rightarrow \boxed{C = -\frac{1}{3}}$$

$$\Rightarrow \int \frac{(2x-1)dx}{(x-1)(x-3)(x+2)} = -\frac{1}{6}\int \frac{dx}{x-1} + \frac{1}{2}\int \frac{dx}{x-3} - \frac{1}{3}\int \frac{dx}{x+2}$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$= -\frac{1}{6}\ln|x-1| + \frac{1}{2}\ln|x-3| - \frac{1}{3}.\ln|x+2| + C.$$

L6 01 (c)

(c) $\int \frac{dx}{4 - 5\sin x}$

$$\begin{cases} \tan\frac{x}{2} = z \\ \operatorname{sen} x = \frac{2z}{z^2+1} \\ dx = \frac{dz}{z^2+1} \end{cases}$$

$$\int \frac{dx}{4-5\operatorname{sen} x} = \int \frac{\frac{dz}{z^2+1}}{4-5\cdot\left(\frac{2z}{z^2+1}\right)} = \int \frac{\frac{dz}{\cancel{z^2+1}}}{\frac{4z^2+4-10z}{\cancel{z^2+1}}}$$

$$= \int \frac{dz}{4z^2-10z+4}$$

$$4z^2-10z+4 = 4\cdot\left[(z+k)^2+l\right]$$

$$= 4z^2+8kz+4k^2+4l$$

$$\begin{cases} 8k=-10 \\ 4k^2+4l=4 \end{cases} \Rightarrow \left\{ k=-\frac{5}{4} \right.$$

$$k^2+l=1$$

$$\left(-\frac{5}{4}\right)^2+l=1$$

$$l = 1-\frac{25}{16}$$

$$l = -\frac{9}{16}$$

$$4z^2-10z+4 = 4\cdot\left[\left(z-\frac{5}{4}\right)^2-\frac{9}{16}\right]$$

Dimo:

$$\int \frac{dx}{4-5\sin x} = \int \frac{dz}{4\left[\left(z-\frac{5}{4}\right)^2-\frac{9}{16}\right]} =$$

$$\frac{1}{4}\int \frac{dz}{\underbrace{\left(z-\frac{5}{4}\right)}_{u}^2-\left(\frac{3}{4}\right)^2} = \frac{1}{\cancel{4}}\cdot\frac{1}{2\cdot\frac{3}{\cancel{4}}}\cdot \ln\left|\frac{z-\frac{5}{4}-\frac{3}{4}}{z-\frac{5}{4}+\frac{3}{4}}\right| + c$$

$$\int \frac{du}{u^2-a^2} = \frac{1}{2a}\ln\left|\frac{u-a}{u+a}\right| + c$$

$$= \frac{1}{6}\cdot \ln\left|\frac{z-2}{z-\frac{1}{2}}\right| + c = \frac{1}{6}\cdot \ln\left|\frac{\tan\frac{x}{2}-2}{\tan\frac{x}{2}-\frac{1}{2}}\right| + c$$

$$z = \tan\frac{x}{2}$$