FUNÇÕES TRANSCEDENTAIS

30/07/25 – AULA 13

EQUAÇÕES TRIGONOMÉTRICAS

São equações às quais a variável está ligada a algum valor trigonométrico.

Ex.: Resolver cada equação:

(a) $\operatorname{sen} x + \cos^2 x = 1.$

$\operatorname{sen}^2 x + \cos^2 x = 1.$

$\cos^2 x = 1 - \operatorname{sen}^2 x$

Solução:

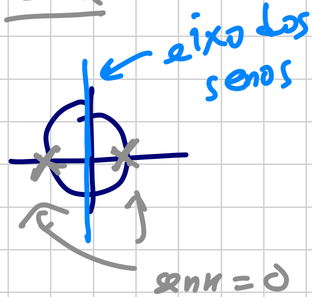
$$\operatorname{sen} x + \cancel{1} - \operatorname{sen}^2 x = \cancel{1}$$

é interessante fazer esta mudança pois a eq. ficará apenas na variável $\operatorname{sen} x$

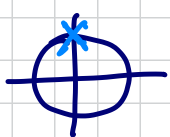
$$\operatorname{sen} x - \operatorname{sen}^2 x = 0$$

$$\operatorname{sen} x \cdot (1 - \operatorname{sen} x) = 0$$

$$\begin{cases} \operatorname{sen} x = 0 \\ 1 - \operatorname{sen} x = 0 \end{cases}$$



- $\operatorname{sen} x = 0 \iff x = k\pi, \quad k \in \mathbb{Z}.$
- $1 - \operatorname{sen} x = 0 \iff \operatorname{sen} x = 1$



$$\Leftrightarrow x = \frac{\pi}{2} + 2k\pi, \ k \in \mathbb{Z}.$$

<u>Solução</u>: $S = \{ k\pi \text{ ou } \frac{\pi}{2} + 2k\pi, \ \forall k \in \mathbb{Z} \}$.

Uma verificação: $x = \frac{\pi}{2}$ é uma solução. De facto; no problema original, tem-se:

$$\operatorname{sen} x + \cos^2 x = 1. \quad \left(x = \frac{\pi}{2}\right)$$

$$\underbrace{\operatorname{sen}\frac{\pi}{2}}_{=1} + \underbrace{\cos^2\frac{\pi}{2}}_{(0)^2} = 1 + 0^2 = 1 + 0 = 1.$$

ok!

(b) $2\tan x - \cos x = 0$.

<u>Solução</u>: $2\tan x - \cos x = 0 \Leftrightarrow 2\tan x = \cos x$

$$\Leftrightarrow 2 \cdot \frac{\operatorname{sen} x}{\cos x} = \cos x \qquad (\times \cos x)$$

$$\Leftrightarrow 2\operatorname{sen} x = \cos^2 x$$

$$\Leftrightarrow 2\operatorname{sen} x = 1 - \operatorname{sen}^2 x$$

$\operatorname{sen}^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \operatorname{sen}^2 x$

$$\Leftrightarrow \operatorname{sen}^2 x + 2\operatorname{sen} x - 1 = 0$$

$$\operatorname{sen} x = \frac{-2 \pm \sqrt{4+4}}{2}$$

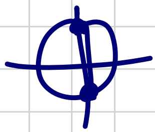
$$\operatorname{sen} x = \frac{-2 \pm \sqrt{8}}{2} = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$\operatorname{sen} x = -1 \pm \sqrt{2}$$

$\longrightarrow \operatorname{sen} x = \underline{-1+\sqrt{2}}$

$\longrightarrow \operatorname{sen} x = -1-\sqrt{2} < -1$ ($\nexists$)

DEIXAREMOS ASSIM, POIS NÃO CONHECEMOS ARCO x TAL QUE $\operatorname{sen} x = -1+\sqrt{2}$



EXERCÍCIOS:

1. Resolva as equações abaixo:

(a) $2 - 2\cos x = \operatorname{sen} x \cdot \tan x$

(b) $2\operatorname{sen}^2 x + 6\cos x = 5 + \cos 2x$

(c) $(1 - \tan x)(1 + \operatorname{sen} 2x) = 1 + \tan x$

(d) $\cos 2x - 3\operatorname{sen} x + 1 = 0$

(e) $\operatorname{sen} x + \cos x = -1$

(f) $\sqrt{3}\ \operatorname{sen} x - \cos x = -\sqrt{3}$

(g) $\cos x + \sqrt{3}\operatorname{sen} x = 1$

(h) $\operatorname{sen} 7x + \operatorname{sen} 5x = 0$

(i) $\cos 3x + \ \operatorname{sen} 2x = 0$

(j) $\operatorname{sen} 7x + \cos 3x = \cos 5x - \operatorname{sen} x$

SOLUÇÕES DOS INDICADOS:

(a) $2 - 2\cos x = \operatorname{sen} x \cdot \tan x$

$$2 - 2\cos x = \operatorname{sen} x \cdot \frac{\operatorname{sen} x}{\cos x}$$

$$2(1 - \cos x) = \frac{\operatorname{sen}^2 x}{\cos x} \qquad \times \cos x$$

$$2(1-\cos x)\cdot\cos x = \operatorname{sen}^2 x$$

$$2\cos x - 2\cdot\cos^2 x = \underline{\operatorname{sen}^2 x}$$

$$\boxed{\begin{array}{l}\operatorname{sen}^2 x + \cos^2 x = 1.\\ \operatorname{sen}^2 x = 1-\cos^2 x\end{array}}$$

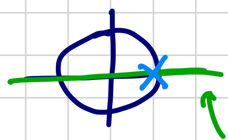
$$2\cos x - 2\cos^2 x = 1-\cos^2 x$$

$$2\cos x \underbrace{-\,2\cos^2 x + \cos^2 x}\, - 1 = 0$$

$$-\cos^2 x + 2\cos x - 1 = 0 \qquad (\times -1)$$

$$\cos^2 x - 2\cos x + 1 = 0$$

$$\cos x = \frac{+2 \pm \sqrt{4-4}}{2\cdot 1}$$



eixo dos cossenos

$$\cos x = \frac{2 \pm 0}{2} \Rightarrow \cos x = 1$$

$$\Leftrightarrow x = 2k\pi \ , \ \forall k \in \mathbb{Z}.$$

$$S = \{2k\pi : k \in \mathbb{Z}\}.$$

(b) $2\operatorname{sen}^2 x + 6\cos x = 5 + \underline{\cos 2x}$

$\hookrightarrow$ $\boxed{\cos 2x = \cos^2 x - \operatorname{sen}^2 x}$

FÓRMULA DO ARCO DUPLO.

$$2\operatorname{sen}^2 x + 6\cos x = 5 + \cos^2 x - \operatorname{sen}^2 x$$

$$2\,\text{sen}^2x + 6\cos x - 5 - \cos^2 x + \text{sen}^2 x = 0$$

$$3\,\text{sen}^2x + 6\cos x - 5 - \cos^2 x = 0$$

$$\text{sen}^2x + \cos^2 x = 1$$
$$\text{sen}^2x = 1 - \cos^2 x$$

$$3.(1-\cos^2 x) + 6\cos x - 5 - \cos^2 x = 0$$

$$3 - 3\cos^2 x + 6\cos x - 5 - \cos^2 x = 0$$

$$-4\cos^2 x + 6\cos x - 2 = 0 \qquad [\div(-2)]$$

$$2\cos^2 x - 3\cos x + 1 = 0$$

[EQUAÇÃO DE 2º GRAU NA VARIÁVEL $\cos x$]

$$\cos x = \frac{+3 \pm \sqrt{9-8}}{2.2} = \frac{3 \pm 1}{4}$$

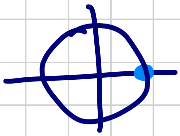
$$\cos x = \frac{3+1}{4} \Rightarrow \boxed{\cos x = 1}$$

ou

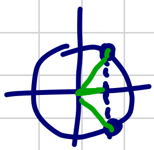
ou

$$\cos x = \frac{3-1}{4} \Rightarrow \boxed{\cos x = \frac{1}{2}}$$

• $\cos x = 1 \iff x = 2k\pi \quad , \forall k \in \mathbb{Z}.$



• $\cos x = \frac{1}{2} \iff x \in 1.^{\circ}q$ ou $x \in 4.^{\circ}q.$



$\underline{\underline{1.^{\circ}q}}$: $x = \frac{\pi}{3} + 2k\pi \quad , \forall k \in \mathbb{Z}.$

$\underline{\underline{4.^{\circ}q}}$: $x = 2\pi - \frac{\pi}{3} + 2k\pi$

$x = \frac{5\pi}{3} + 2k\pi. \quad \forall k \in \mathbb{Z}$

<u>SOLUÇÃO:</u>

$S = \{ 2k\pi$ ou $\frac{\pi}{3} + 2k\pi$ ou $\frac{5\pi}{3} + 2k\pi, \forall k \in \mathbb{Z} \}.$

(c) $(1 - \tan x).(1 + \underline{\operatorname{sen} 2x}) = 1 + \tan x$

↳ FÓRMULA DE ARCO DUPLO:

$\operatorname{sen} 2x = 2.\operatorname{sen} x . \cos x$

$(1 - \tan x)(1 + 2.\operatorname{sen} x.\cos x) = 1 + \tan x$

$$\cancel{1} + 2\,\text{sen}x\cdot\cos x - \tan x - 2\cdot\text{sen}x\cdot\cos x\cdot\tan x = \cancel{1} + \tan x$$

$$2\,\text{sen}x\cdot\cos x - \tan x - 2\cdot\text{sen}x\cdot\cancel{\cos x}\cdot\frac{\text{sen}x}{\cancel{\cos x}} - \tan x = 0$$

$$2\,\text{sen}x\cdot\cos x - 2\tan x - 2\,\text{sen}^2x = 0 \quad (\div 2)$$

$$\text{sen}x\cdot\cos x - \tan x - \text{sen}^2x = 0$$

$$\text{sen}x\cdot\cos x - \frac{\text{sen}x}{\cos x} - \text{sen}^2x = 0$$

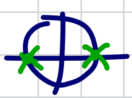
$$\text{sen}x\left(\cos x - \frac{1}{\cos x} - \text{sen}x\right) = 0 \iff$$

- $\text{sen}x = 0$

ou

- $\cos x - \dfrac{1}{\cos x} - \text{sen}x = 0$

- $\text{sen}x = 0 \iff \boxed{x = k\pi \ ; \ \forall k \in \mathbb{Z}}$



- $\cos x - \dfrac{1}{\cos x} - \text{sen}x = 0 \qquad \times(\cos x)$

$$\iff \cos^2x - 1 - \text{sen}x\cdot\cos x = 0$$

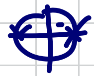
$$sen^2x + cos^2x = 1.$$
$$cos^2x = 1 - sen^2x$$

$$\Leftrightarrow \cancel{1} - sen^2x - \cancel{1} - senx \cdot cosx = 0$$

$$\Leftrightarrow -sen^2x - senx \cdot cosx = 0 \quad \times(-1)$$

$$sen^2x + senx \cdot cosx = 0$$

$$senx(senx + cosx) = 0$$



$$\Leftrightarrow \begin{cases} senx = 0 \Leftrightarrow \boxed{x = k\pi, \ k \in \mathbb{Z}} \\ senx + cosx = 0 \end{cases}$$

$$senx + cosx = 0$$

$$senx + sen\left(\frac{\pi}{2} - x\right) = 0 \Leftrightarrow$$

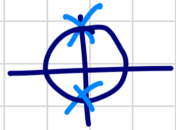
$$senp + senq = 2 \cdot sen\frac{p+q}{2} \cdot cos\frac{p-q}{2}$$

$$\Leftrightarrow 2 \cdot sen\left(\frac{\cancel{x} + \frac{\pi}{2} - \cancel{x}}{2}\right) \cdot cos\frac{x - \left(\frac{\pi}{2} - x\right)}{2} = 0$$

$$\Leftrightarrow 2 \cdot sen\frac{\pi}{4} \cdot cos\left(\frac{2x - \frac{\pi}{2}}{2}\right) = 0$$

$$\cancel{2} \cdot \frac{\sqrt{2}}{\cancel{2}} \cdot cos\left(x - \frac{\pi}{4}\right) = 0$$

$$\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)=0 \Leftrightarrow$$



$$\cos\left(x-\frac{\pi}{4}\right)=0$$

$$\Leftrightarrow x-\frac{\pi}{4}=k\pi+\frac{\pi}{2}, \quad \forall k\in\mathbb{Z}$$

$$\Leftrightarrow x=\frac{\pi}{4}+k\pi+\frac{\pi}{2}, \quad \forall k\in\mathbb{Z}$$

$$\Leftrightarrow \boxed{x=\frac{3\pi}{4}+k\pi, \quad \forall k\in\mathbb{Z}.}$$

Solução: $S=\left\{k\pi \text{ ou } \frac{3\pi}{4}+k\pi, \ \forall k\in\mathbb{Z}\right\}$.

---

f) $\sqrt{3}\cdot \operatorname{sen}x-\cos x=-\sqrt{3}$

$$\sqrt{3}\operatorname{sen}x+\sqrt{3}=\cos x$$

$$\sqrt{3}(\operatorname{sen}x+1)=\cos x$$

Elevando ao quadrado, vem:

$$\left[\sqrt{3}.(\operatorname{sen}x+1)\right]^2=\cos^2x$$

$\operatorname{sen}^2x+\cos^2x=1$

$\cos^2x=1-\operatorname{sen}^2x$

$$3.(\operatorname{sen}^2x+2\operatorname{sen}x+1)=\cos^2x$$

$$3\operatorname{sen}^2x+6\operatorname{sen}x+3=1-\operatorname{sen}^2x$$

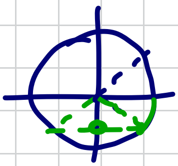
$$3\,\text{sen}^2 x + 6\,\text{sen}\,x + 3 - 1 + \text{sen}^2 x = 0$$

$$4\,\text{sen}^2 x + 6\,\text{sen}\,x + 2 = 0 \quad (\div 2)$$

$$2\,\text{sen}^2 x + 3\,\text{sen}\,x + 1 = 0$$

$$\text{sen}\,x = \frac{-3 \pm \sqrt{9-8}}{2 \cdot 2} \;\begin{cases} \nearrow \ \text{sen}\,x = \dfrac{-3+1}{4} = -\dfrac{1}{2} \\[2ex] \searrow \ \text{sen}\,x = \dfrac{-3-1}{4} = -1 \end{cases}$$

• $\text{sen}\,x = -\frac{1}{2}$: $\left(1^{\underline{o}}\,q: \frac{\pi}{6}\right)$



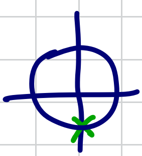
$\rightarrow$ $x \in 3^{\underline{o}}\,q$ ou $x \in 4^{\underline{o}}\,q$.

$\underline{3^{\underline{o}}\,q}$: $x = \pi + \frac{\pi}{6} + 2k\pi, \ \forall k \in \mathbb{Z}.$

$4^{\underline{o}}\,q$: $x = 2\pi - \frac{\pi}{6} + 2k\pi, \ \forall k \in \mathbb{Z}$

$$x = \frac{7\pi}{6} + 2k\pi \quad \text{ou} \quad x = \frac{11\pi}{6} + 2k\pi, \ \forall k \in \mathbb{Z}.$$

• $\text{sen}\,x = -1$: $\Leftrightarrow x = \frac{3\pi}{2} + 2k\pi, \ \forall k \in \mathbb{Z}.$



Solução: $\left\{ \frac{7\pi}{6} + 2k\pi \ \text{ou} \ \frac{11\pi}{6} + 2k\pi \ \text{ou} \ \frac{3\pi}{2} + 2k\pi, \ \forall k \in \mathbb{Z} \right\}.$

(h) $\operatorname{sen} 7x + \operatorname{sen} 5x = 0$

$$\operatorname{sen} p + \operatorname{sen} q = 2 \cdot \operatorname{sen}\left(\frac{p+q}{2}\right) \cdot \cos\left(\frac{p-q}{2}\right)$$

$p = 7x$

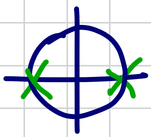
$q = 5x$

$$2 \cdot \operatorname{sen}\left(\frac{7x+5x}{2}\right) \cdot \cos\left(\frac{7x-5x}{2}\right) = 0$$

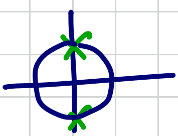
$$2 \cdot \operatorname{sen} 6x \cdot \cos x = 0$$

$$\operatorname{sen} 6x = 0 \quad \text{ou} \quad \cos x = 0$$

• $\operatorname{sen} 6x = 0 \Longleftrightarrow 6x = k\pi \Leftrightarrow x = \frac{k\pi}{6}, \ \forall k \in \mathbb{Z}.$



• $\cos x = 0 \Longleftrightarrow x = \frac{\pi}{2} + k\pi, \ \forall k \in \mathbb{Z}.$



$$S = \left\{ k \cdot \frac{\pi}{6} \ \text{ou} \ \frac{\pi}{2} + k\pi, \ \forall k \in \mathbb{Z} \right\}$$

(j) $\operatorname{sen}7x + \cos 3x = \cos 5x - \operatorname{sen}x$

$\operatorname{sen}7x + \operatorname{sen}x = \cos 5x - \cos 3x$

- $\operatorname{sen}p + \operatorname{sen}q = 2 . \operatorname{sen}\left(\frac{p+q}{2}\right) . \cos\left(\frac{p-q}{2}\right)$
- $\cos p - \cos q = -2 . \operatorname{sen}\left(\frac{p+q}{2}\right) . \operatorname{sen}\left(\frac{p-q}{2}\right)$

$2 . \operatorname{sen}\left(\frac{7x+x}{2}\right) . \cos\left(\frac{7x-x}{2}\right) = -2 . \operatorname{sen}\left(\frac{7x+x}{2}\right) . \operatorname{sen}\left(\frac{7x-x}{2}\right)$

$2 . \operatorname{sen}4x . \cos 3x = -2 . \operatorname{sen}4x . \operatorname{sen}3x \quad (\div 2)$

$\operatorname{sen}4x . \cos 3x = - \operatorname{sen}4x . \operatorname{sen}3x$

$\operatorname{sen}4x . \cos 3x + \operatorname{sen}4x . \operatorname{sen}3x = 0$

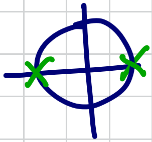
$\operatorname{sen}4x . (\cos 3x + \operatorname{sen}3x) = 0 \iff$

$\boxed{\operatorname{sen}4x = 0}$

ou

$\boxed{\cos 3x + \operatorname{sen}3x = 0}$

• $\operatorname{sen} 4x = 0 \Leftrightarrow 4x = k\pi \Leftrightarrow x = k \cdot \frac{\pi}{4}, \ \forall k \in \mathbb{Z}.$



$\boxed{\cos \alpha = \operatorname{sen}\left(\frac{\pi}{2} - \alpha\right)}$ arcos complementares.

• $\cos 3x + \operatorname{sen} 3x = 0 \Leftrightarrow$

$\operatorname{sen}\left(\frac{\pi}{2} - 3x\right) + \operatorname{sen} 3x = 0$

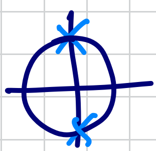
$\operatorname{sen} p + \operatorname{sen} q =$

$= 2 \cdot \operatorname{sen} \frac{p+q}{2} \cdot \cos \frac{p-q}{2}$

$\Leftrightarrow 2 \cdot \operatorname{sen}\left(\frac{\frac{\pi}{2} \cancel{-3x} \cancel{+3x}}{2}\right) \cdot \cos\left(\frac{\frac{\pi}{3} - 3x - 3x}{2}\right) = 0$

$\Leftrightarrow 2 \cdot \operatorname{sen} \frac{\pi}{4} \cdot \cos\left(\frac{\pi}{6} - 3x\right) = 0$

$\Leftrightarrow 2 \cdot \frac{\sqrt{2}}{2} \cdot \cos\left(\frac{\pi}{6} - 3x\right) = 0 \Leftrightarrow \cos\left(\frac{\pi}{6} - 3x\right) = 0$



$\Leftrightarrow \frac{\pi}{6} - 3x = \frac{\pi}{2} + k\pi$

$\Leftrightarrow -3x = \frac{\pi}{2} - \frac{\pi}{6} + k\pi$

$\Leftrightarrow -3x = \frac{\pi}{3} + k\pi$

$$\Leftrightarrow \boxed{x = -\frac{\pi}{9} - \frac{k\pi}{3}}, \quad \forall k \in \mathbb{Z}.$$

Solução: $S = \left\{ \frac{k\pi}{4} \text{ ou } -\frac{\pi}{9} - \frac{k\pi}{3}, \ \forall k \in \mathbb{Z} \right\}.$

Fim da matéria.

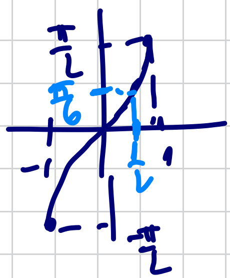
RESOLUÇÃO DE EXERCÍCIOS DA LISTA 05

2. Determine o valor principal de:

(a) $\operatorname{arcsen}\frac{1}{2}$ (b) $\arctan(-1)$ (c) $\operatorname{arccot}\sqrt{3}$ (d) $\operatorname{arcsec} 2$

Valor principal: para a função trigonométrica inversa está definida em um domínio restrito (não há quadrantes)

(a) $\operatorname{arcsen}\frac{1}{2} = y \iff \operatorname{sen} y = \frac{1}{2} \iff y = \frac{\pi}{6}.$



(c) $\operatorname{arccot}\sqrt{3} = y \iff \cot y = \sqrt{3}$

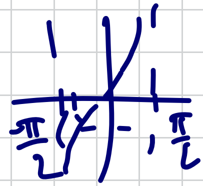
$$\Updownarrow$$

$$\frac{1}{\tan y} = \sqrt{3}$$

$$\Updownarrow \quad \tan y = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$y = \frac{\pi}{6}.$$

Resp.: $\operatorname{arccot} \sqrt{3} = \frac{\pi}{6}$ (pois $\cot \frac{\pi}{6} = \sqrt{3}$)



(b) $\arctan(-1) = y$

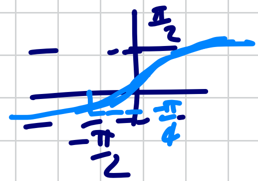
$$\Updownarrow$$

$$\tan y = -1.$$

$$\Updownarrow$$

$$y = -\frac{\pi}{4}.$$

$\arctan : \mathbb{R} \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$



3. Calcule:

(a) $\operatorname{sen}\left(\operatorname{arcsen}\left(-\frac{1}{2}\right)\right)$

(b) $\cos\left(\operatorname{arcsen}\frac{3}{4}\right)$

(c) $\tan\left(\operatorname{arcsen}\left(-\frac{2}{3}\right) + \operatorname{arcsen}\frac{1}{4}\right)$

(d) $\operatorname{sen}(\arctan 2 + \arctan 3)$

(a) seno e arco-seno são inversos um do outro.

$$\operatorname{sen}\left(\operatorname{arcsen}\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}.$$

(b) $\cos\left(\operatorname{arcsen}\frac{3}{4}\right) = ?$

$$\operatorname{arcsen}\frac{3}{4} = \theta. \qquad \cos\theta = ?$$

$$\Updownarrow$$

$\operatorname{sen}\theta = \frac{3}{4}$; e como $(1^{\underline{o}}q)$

$$\operatorname{sen}^2\theta + \cos^2\theta = 1.$$

$$\Rightarrow \cos\theta = \sqrt{1-\operatorname{sen}^2\theta} = \sqrt{1-\left(\frac{3}{4}\right)^2}$$

$$= \sqrt{1-\frac{9}{16}} = \sqrt{\frac{16-9}{16}} = \frac{\sqrt{7}}{4}.$$

conclusão:

$$\cos\left(\operatorname{arcsen}\frac{3}{4}\right) = \cos\theta = \frac{\sqrt{7}}{4}.$$

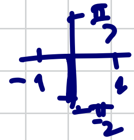
(c) $\tan\left(\operatorname{arcsen}\left(-\frac{2}{3}\right) + \operatorname{arcsen}\left(\frac{1}{4}\right)\right)$

Escreva $\operatorname{arcsen}\left(-\frac{2}{3}\right) = \alpha$ e

$$\operatorname{arc\,sen}\left(\frac{1}{4}\right) = \beta$$

Então, $\operatorname{sen}\alpha = -\frac{2}{3}$ e $\operatorname{sen}\beta = \frac{1}{4}$;

logo, $\alpha \in 4^{o}q$ e $\beta \in 1^{o}q$.



O que queremos calcular é:

$$\tan\left(\underbrace{\arcsin\left(-\frac{2}{3}\right)}_{\alpha} + \underbrace{\arcsin\frac{1}{4}}_{\beta}\right) = \tan(\alpha+\beta) =$$

$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha \cdot \tan\beta} =$$

Como $\operatorname{sen}\alpha = -\frac{2}{3}$ e $\alpha \in 4^{o}q$, temos:

$$\cos^2\alpha = 1 - \operatorname{sen}^2\alpha$$

$$\Rightarrow \cos\alpha = \underbrace{+}_{\alpha \in 4^{o}q}\sqrt{1 - \operatorname{sen}^2\alpha} = \sqrt{1 - \left(-\frac{2}{3}\right)^2}$$

$$= \sqrt{1 - \frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$$

$$\Rightarrow \tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha} = \frac{-\frac{2}{\cancel{3}}}{\frac{\sqrt{5}}{\cancel{3}}} = -\frac{2}{\sqrt{5}}$$

E como $\text{sen}\,\beta = \frac{1}{4}$; $\beta \in 1^{o}q$; temos:

$$\cos\beta = \sqrt{1-\text{sen}^2\beta} = \sqrt{1-\left(\frac{1}{4}\right)^2} = \sqrt{1-\frac{1}{16}}$$

$$= \frac{\sqrt{15}}{4}$$

Assim, $\underline{\tan\beta} = \frac{\text{sen}\,\beta}{\cos\beta} = \frac{\frac{1}{\cancel{4}}}{\frac{\sqrt{15}}{\cancel{4}}} = \underline{\frac{1}{\sqrt{15}}}$

Por fim:

$$\frac{\tan\alpha + \tan\beta}{1-\tan\alpha \cdot \tan\beta} = \frac{-\frac{2}{\sqrt{5}} + \frac{1}{\sqrt{15}}}{1-\left(-\frac{2}{\sqrt{5}}\right)\cdot\frac{1}{\sqrt{15}}} = \ldots$$

etc.