GABARITO DA 1ª PROVA

01)

(a) $$\left(\frac{1}{2}\right)^{2x-5} \geq \left(\operatorname{sen}\frac{3\pi}{4}\right)^{2-2x}$$

Como $\underbrace{\operatorname{sen}\frac{3\pi}{4}}_{2^{\circ}q} = +\operatorname{sen}\left(\pi - \frac{3\pi}{4}\right) = \operatorname{sen}\frac{\pi}{4} = \frac{\sqrt{2}}{2}$;

temos:

$$\left(\frac{1}{2}\right)^{2x-5} \geq \left(\frac{\sqrt{2}}{2}\right)^{2-2x}$$

$$\left(2^{-1}\right)^{2x-5} \geq \left(2^{\frac{1}{2}-1}\right)^{2-2x}$$

$$2^{5-2x} \geq 2^{-1+x} \iff 5-2x \geq -1+x$$

$$\iff -2x - x \geq -1-5$$

$$\iff -3x \geq -6$$

$$\iff 3x \leq 6$$

$$\iff x \leq 2$$

$$S = (-\infty, 2].$$

(b) $\log_{\frac{1}{2}}(x-1) < \log_2 8 = \log_2 2^3 = 3 = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^3$

$$\iff \log_{\frac{1}{2}}(x-1) < \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^3$$

$$\iff x-1 > \left(\frac{1}{2}\right)^3 \iff \underline{x} > \frac{1}{8}+1 = \underline{\frac{9}{8}}$$

$$S = \left(\frac{9}{8}, +\infty\right)$$

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02) $\log_3(\sqrt{27} \cdot \tan 1470^\circ) = ?$

Note que

$$\begin{array}{r|l} 1470^\circ & 360 \\ \hline -1440^\circ & 4 \\ \hline 30^\circ & \end{array}$$

Então: $\tan 1470^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}$. Assim:

$$\log_3(\sqrt{27} \cdot \tan 1470^\circ) = \log_3\left(\sqrt{3^3} \cdot \frac{\sqrt{3}}{3}\right)$$

$$= \log_3 \left(3^{\frac{3}{2}} \cdot 3^{\frac{1}{2}} \cdot 3^{-1} \right) = \log_3 \left(3^{\frac{3}{2} + \frac{1}{2} - 1} \right)$$

$$= \frac{3}{2} + \frac{1}{2} - 1 = 2 - 1 = 1$$

03)

$t = 0 \; ; \quad m = m_0$

$t = 5730 \; ; \quad m = \dfrac{m_0}{2}$

$t = 2 \times 5730 \; ; \quad m = \dfrac{m_0}{2^2}$

$\vdots$

$t = k \cdot 5730 \; ; \quad m = \dfrac{m_0}{2^k}$

$\vdots$

Assim, a função $m(t)$ será dada por:

$$m(t) = m_0 \cdot \left(\frac{1}{2}\right)^k \; ;$$

e como $t = k \cdot 5730 \Longrightarrow k = \dfrac{t}{5730}$.

Portanto, obtemos:

$$m(t) = m_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

Queremos obter o valor de t para o qual

$m(t) = 0{,}145\, m_0$. Assim:

$$0{,}145\,\cancel{m_0} = \cancel{m_0} \cdot \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$0{,}145 = \left(\frac{1}{2}\right)^{\frac{t}{5730}}$$

$$\Rightarrow \log 0{,}145 = \frac{t}{5730} \cdot \log \frac{1}{2}$$

$$\Rightarrow t = \frac{5730 \cdot \log 0{,}145}{\log 0{,}5} \text{ anos}$$

$$\boxed{t \cong 15.963{,}0649 \text{ anos.}}$$

04) Note que:

- $\cot(\underbrace{270^\circ+x}_{4^\text{o}q}) = -\cot(360^\circ-(270^\circ+x)) = -\cot(90^\circ-x)$

$$= -\tan x$$

- $\operatorname{sen}(\underbrace{180^\circ+x}_{3^\text{o}q}) = -\operatorname{sen}(\cancel{180^\circ}+x-\cancel{180^\circ}) = -\operatorname{sen} x.$

- $\csc(\underbrace{90^\circ+x}_{2^\text{o}q}) = +\csc(180^\circ-(90^\circ+x)) = \csc(90^\circ-x) =$

$$= \sec x$$

- $\cos(\underbrace{180^\circ-x}_{2^\text{o}q}) = -\cos(\cancel{180^\circ}-(\cancel{180^\circ}-x)) = -\cos x.$

Assim, obtemos:

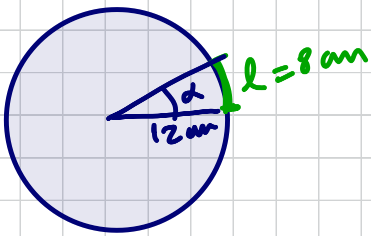
$$\frac{\cot(270^\circ+x)-\operatorname{sen}(180^\circ+x)}{\csc(90^\circ+x)+\cos(180^\circ-x)} = \frac{-\tan x-(-\operatorname{sen} x)}{\sec x+(-\cos x)} =$$

$$= \frac{-\tan x+\operatorname{sen} x}{\sec x-\cos x} = \frac{-\frac{\operatorname{sen} x}{\cos x}+\operatorname{sen} x}{\frac{1}{\cos x}-\cos x} =$$

$$= \frac{\dfrac{-\operatorname{sen}x + \operatorname{sen}x \cdot \cos x}{\cancel{\cos x}}}{\dfrac{1 - \cos^2 x}{\cancel{\cos x}}} = \frac{-\cancel{\operatorname{sen}x}(1 + \cos x)}{\operatorname{sen}^{\cancel{2}} x} =$$

$$= \frac{1 + \cos x}{\operatorname{sen}x} = -\frac{1}{\operatorname{sen}x} + \frac{\cos x}{\operatorname{sen}x} = -\csc x + \cot x$$

05)



$l = 8$ cm
$R = 12$ cm

(a) $$\alpha = \frac{l}{R} = \frac{8}{12} = \frac{\cancel{4} \cdot 2}{\cancel{4} \cdot 3} = \underline{\frac{2}{3} \text{ rad.}}$$

(b) 2π rad — $\pi \cdot (12)^2$

$\frac{2}{3}$ rad. — A

$$2\cancel{\pi} \cdot A = (12)^2 \cdot \cancel{\pi} \cdot \frac{2}{3}$$

$$A = \frac{12 \cdot 12 \cdot \cancel{2}}{3 \cdot \cancel{2}}$$

$$A = \frac{12 \cdot 4 \cdot \cancel{3}}{\cancel{3}}$$

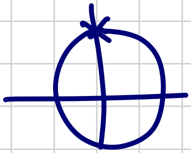
$$\boxed{A = 48 \text{ cm}^2}$$

(c) $\csc(2x + \alpha\pi) = 1$

onde: $\boxed{\alpha = \frac{2}{3} \text{ rad!}}$

$$\Updownarrow$$

$\csc\left(2x + \frac{2\pi}{3}\right) = 1$



$$\Updownarrow$$

$$2x + \frac{2\pi}{3} = 2k\pi + \frac{\pi}{2}$$

$$\Leftrightarrow \quad 2x = 2k\pi + \frac{\pi}{2} - \frac{2\pi}{3}$$

$$\Leftrightarrow \quad 2x = 2k\pi + \frac{3\pi - 4\pi}{6}$$

$$\Leftrightarrow \quad 2x = 2k\pi - \frac{\pi}{6} \quad \Leftrightarrow \quad x = k\pi + \frac{\pi}{12}$$

Sol: $\boxed{x = k\pi + \frac{\pi}{12}}$

06) $y = 1 + 2\log_2(x-1)$.

Cond. de existência: $x-1>0 \Leftrightarrow x>1$.

Logo: $D(f) = (1, +\infty)$.

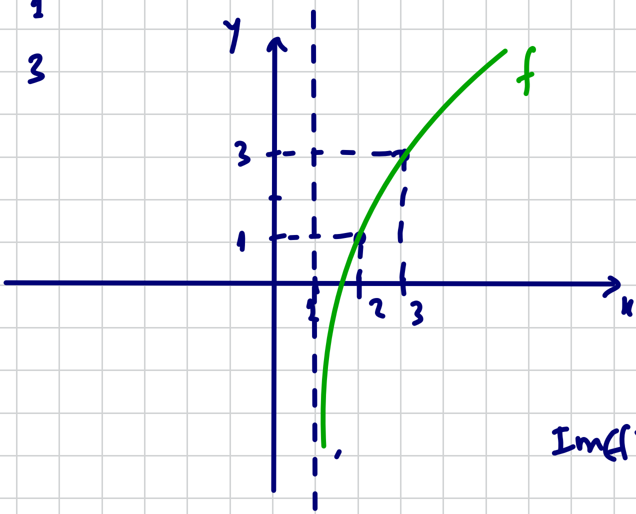
ASSÍNTOTA VERTICAL: $x=1$.

$y-1 = 2\log_2(x-1) \Leftrightarrow \log_2(x-1) = \frac{y-1}{2}$

$\Leftrightarrow 2^{\frac{y-1}{2}} = x-1$

$\Leftrightarrow \boxed{x = 1 + 2^{\frac{y-1}{2}}}$

$x = 1+2^{\frac{y-1}{2}}$	y
2	1
3	3



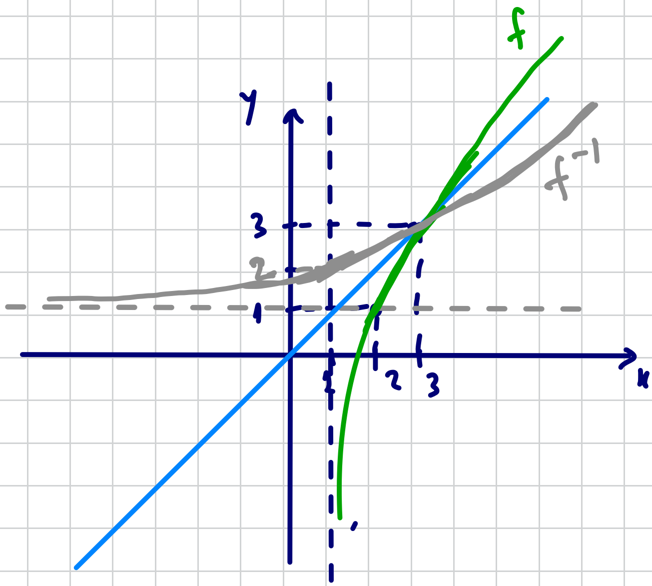
$Im(f) = \mathbb{R}$

$f: X \to \mathbb{R}$ é sobrejetiva pois $Im(f) = \mathbb{R}$.

Além disso, f é crescente; logo, é injetiva. Portanto, f é bijetiva; e com isso é inversível.

Então, $\exists\, f^{-1}: \mathbb{R} \to X = (1, +\infty)$;

$$f^{-1}(y) = 1 + 2^{\frac{y-1}{2}}$$ (é o que isolamos acima)



07) $x \in 3^{\circ}q$.

$\cot x = 2.$ $\Longrightarrow$ $\boxed{\tan x = \dfrac{1}{2}}$

$$1+\tan^2 x = \sec^2 x \Leftrightarrow \sec x = -\sqrt{1+\tan^2 x}$$

$x \in 3^{\circ}Q$

$$\Leftrightarrow \sec x = -\sqrt{1+\frac{1}{4}} = -\sqrt{\frac{5}{4}} = -\frac{\sqrt{5}}{2}$$

$$\Rightarrow \boxed{\sec x = -\frac{\sqrt{5}}{2}}$$

$$\cos x = \frac{1}{\sec x} \Rightarrow \boxed{\cos x = -\frac{2}{\sqrt{5}}}$$

$$\operatorname{sen}^2 x + \cos^2 x = 1 \Leftrightarrow \operatorname{sen} x = -\sqrt{1-\cos^2 x}$$

$x \in 3^{\circ}Q$

$$\Leftrightarrow \operatorname{sen} x = -\sqrt{1-\left(\frac{2}{\sqrt{5}}\right)^2} = -\sqrt{1-\frac{4}{5}}$$

$$= -\sqrt{\frac{5-4}{5}} = -\frac{1}{\sqrt{5}} \Rightarrow \boxed{\operatorname{sen} x = -\frac{1}{\sqrt{5}}}$$

$$\csc x = \frac{1}{\operatorname{sen} x} \Rightarrow \boxed{\csc x = -\sqrt{5}}$$