CÁLCULO II

06/06/25 - AULA 12

AULA DE EXERCÍCIOS.

Estudamos na aula passada a integração por partes:

$$\int u\,dv = u\cdot v - \int v\,du$$

Ex.: $\int e^x\cdot x\,dx = ?$

$\cancel{\int e^v\,dv = e^v + C}$

$\cancel{v = x \Rightarrow dv = dx}$

o "x" "atropela"

$$\int e^x\cdot x\,dx = \int u\,dv = u\cdot v - \int v\,du$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^x\,dx \Rightarrow v = \int e^x\,dx = e^x \\ \Rightarrow v = e^x \end{cases}$$

$$\Rightarrow \int e^x\cdot x\,dx = x\cdot e^x - \int e^x\,dx$$

$$= x\cdot e^x - e^x + C$$

LISTAS:

LISTA 04:

3. Calcule cada integral a seguir, usando a integração por partes.

(a) $\int x \sin x \, dx$ (b) $\int x \sec^2 x \, dx$ (c) $\int \operatorname{arccot} x \, dx$

(d) $\int \arccos 2x \, dx$ (e) $\int x^2 e^{-x} dx$ (f) $\int \ln x \, dx$

(g) $\int x^2 \arcsin x \, dx$ (h) $\int \frac{\ln x \, dx}{(x+1)^2}$ (i) $\int \arctan \sqrt{x} \, dx$

(j) $\int e^{2x} \cos 3x \, dx$ (k) $\int (x^3 - 3x + 1) \ln x \, dx$

(e) $\int x^2 e^{-x} dx.$

$$\begin{cases} u = x^2 \Rightarrow du = 2x\,dx \\ dv = e^{-x} dx \Rightarrow v = -\int e^{-x}(-dx) \end{cases}$$

$$\begin{cases} w = -x \\ dw = -dx \end{cases}$$

$\Rightarrow v = -e^{-x}$

Disso, obtemos:

$$\int x^2 \cdot e^{-x} dx = \int u\,dv = u \cdot v - \int v\,du$$

$$= x^2 \cdot (-e^{-x}) - \int -e^{-x} \cdot 2x\,dx$$

$$= -x^2 e^{-x} + 2 \cdot \underbrace{\int e^{-x} \cdot x\,dx}_{\int u\,dv = u \cdot v - \int v\,du} =$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^{-x} \Rightarrow v = -e^{-x} \end{cases}$$

$$= -x^2 \cdot e^{-x} + 2 \cdot \left(x \cdot (-e^{-x}) - \int -e^{-x} \cdot dx \right)$$

$$= -x^2 e^{-x} - 2x e^{-x} + 2 \cdot \left(-\int e^{-x} (-dx) \right)$$

$\int e^w dw$

$w = -x$

$dw = -dx$

$$= -x^2 e^{-x} - 2x e^{-x} - 2 \cdot e^{-x} + C$$

$$= -(x^2 + 2x + 2)\, e^{-x} + C$$

h) $$\int \frac{\ln x \, dx}{(x+1)^2} = \int u \, dv = u \cdot v - \int v \cdot du$$

$$\left[\begin{array}{l} u = \ln x \Rightarrow du = \frac{1}{x} \cdot dx \\ dv = \frac{dx}{(x+1)^2} \Rightarrow v = \int \frac{dx}{(x+1)^2} \end{array} \right.$$

$$= \int (x+1)^{-2} dx$$

$\int w^k dw$

$w = x+1$

$dw = dx$

$$= \frac{w^{-1}}{-1} = -\frac{1}{w} = -\frac{1}{x+1}$$

$$\Rightarrow v = -\frac{1}{x+1}$$

Dessa forma, obtemos:

$$\int \frac{\ln x}{(x+1)^2}\,dx = \ln x\cdot\left(-\frac{1}{x+1}\right) - \int -\frac{1}{x+1}\cdot\frac{dx}{x}$$

$$= -\frac{1}{x+1}\cdot\ln x + \int \frac{dx}{x^2+x}$$

$\cancel{\int \frac{dw}{w}}$

$\cancel{w = x^2+x}$

$\cancel{dw = (2x+1)dx}$

COMPLETAR UM QUADRADO PERFEITO.

$$x^2+x = (x+k)^2+l$$

$$= x^2+2kx+k^2+l$$

$$\begin{cases} 2k = 1 \rightarrow k = \frac{1}{2} \\ k^2+l = 0 \end{cases}$$

$$\left(\frac{1}{2}\right)^2 + l = 0 \rightarrow l = -\frac{1}{4}$$

Assim, teremos:

$$x^2+x = \left(x+\frac{1}{2}\right)^2 - \frac{1}{4}$$

$$\int \frac{\ln x}{(x+1)^2} = -\frac{1}{x+1}\cdot \ln x + \int \frac{dx}{\left(x+\frac{1}{2}\right)^2 - \frac{1}{4}} \circledast$$

$$\left[\int \frac{dv}{v^2-a^2} = \frac{1}{a} \ln \left| \frac{v-a}{\sqrt{v^2-a^2}} \right| + c \right.$$

$$v = x+\frac{1}{2} \Rightarrow dv = dx$$

$$a^2 = \frac{1}{4} \Rightarrow a = \frac{1}{2}$$

$$\circledast \; -\frac{1}{x+1}\cdot \ln x + \frac{1}{\frac{1}{2}}\cdot \ln \left| \frac{x+\cancel{\frac{1}{2}} - \cancel{\frac{1}{2}}}{\sqrt{\underbrace{\left(x+\frac{1}{2}\right)^2 - \frac{1}{4}}_{= x^2+x}}} \right| + c$$

$$= -\frac{1}{x+1}\cdot \ln x + 2\cdot \ln \left| \frac{x}{\sqrt{x^2+x}} \right| + c$$

(j) $\int e^{2x}\cdot \cos 3x \, dx = \int u \, dv = u\cdot v - \int v \, du$

$$u = e^{2x} \Rightarrow du = 2\cdot e^{2x}\cdot dx$$

$$dv = \cos 3x \, dx \Rightarrow v = \frac{1}{3}\int \cos \underbrace{3x}_{w} \, 3 \, dx$$

$$w = 3x \Rightarrow dw = 3dx$$

$$v = \frac{1}{3}\cdot \operatorname{sen} 3x$$

$$\int e^{2x} \cdot \cos 3x \, dx = e^{2x} \cdot \frac{1}{3} \cdot \operatorname{sen} 3x - \int \frac{1}{3} \operatorname{sen} 3x \cdot (2e^{2x} dx)$$

$$= \frac{1}{3} e^{2x} \cdot \operatorname{sen} 3x - \frac{2}{3} \underbrace{\int \operatorname{sen} 3x \cdot e^{2x} dx}_{u \cdot v - \int v du}$$

$$\begin{cases} u = e^{2x} \Rightarrow du = 2 \cdot e^{2x} dx \\ dv = \operatorname{sen} 3x \, dx \Rightarrow \\ \quad v = \frac{1}{3} \int \operatorname{sen} 3x (3 dx) \\ \quad v = \frac{1}{3} \cdot (-\cos 3x) \end{cases}$$

Disso, obtemos:

$$\int e^{2x} \cdot \cos 3x \, dx = \frac{1}{3} e^{2x} \cdot \operatorname{sen} 3x - \frac{2}{3} \cdot \left(e^{2x} \cdot \left(-\frac{1}{3} \cos 3x\right) - \int -\frac{1}{3} \cos 3x \cdot (2e^{2x} dx) \right)$$

$$1. \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \cdot \operatorname{sen} 3x + \frac{2}{9} e^{2x} \cdot \cos 3x - \frac{4}{9} \int e^{2x} \cdot \cos 3x \, dx$$

$$\left(1 + \frac{4}{9}\right) \int e^{2x} \cos 3x \, dx = \frac{1}{3} e^{2x} \cdot \operatorname{sen} 3x + \frac{2}{9} \cdot e^{2x} \cos 3x + c$$

$$\int e^{2x}\cdot\cos 3x\,dx = \frac{9}{13}\left(\frac{1}{3}\cdot e^{2x}\cdot \operatorname{sen} 3x + \frac{2}{9}\cdot e^{2x}\cdot\cos 3x + c\right)$$

$$= \frac{3}{13}\cdot e^{2x}\cdot \operatorname{sen} 3x + \frac{2}{13}\cdot e^{2x}\cdot\cos 3x + c$$

$$= \frac{1}{13}\cdot e^{2x}\cdot\left(3\operatorname{sen} 3x + 2\cos 3x\right) + c$$

LISTA 02:

4. De acordo com o Teorema Fundamental do Cálculo, calcule cada integral definida a seguir[1]:

(a) $\int_0^2 x^2 dx$ (b) $\int_{-2}^2 (x^3+1)dx$ (c) $\int_1^4 (x^2+4x+5)dx$

(d) $\int_1^4 |x-2|dx$ (e) $\int_{-2}^2 x|x-3|dx$ (f) $\int_{-3}^3 \sqrt{3+|x|}dx$

(c) $$\int_1^4 (x^2+4x+5)dx = \left(\frac{x^3}{3} + \frac{\overset{2}{\cancel{4}}x^2}{\cancel{2}} + 5x + c\right)\Bigg|_1^4 =$$

$$\left(\frac{x^3}{3} + 2x^2 + 5x + c\right)\Bigg|_1^4 =$$

$$= \frac{(4)^3}{3} + 2(4)^2 + 5\cdot(4) + C - \left(\frac{(1)^3}{3} + 2(1)^2 + 5\cdot(1) + C\right)$$

$$= \frac{64}{3} + 32 + 20 + \cancel{C} - \frac{1}{3} \underbrace{-2 - 5} - \cancel{C}$$

$$= \frac{63}{3} + 45 = 21 + 45 = 66$$

(f) $$\int_{-3}^{3} \sqrt{3+|x|}\, dx =$$

$$|x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$$

$$= \underbrace{\int_{-3}^{0} \sqrt{3+|x|}\, dx}_{x<0} + \underbrace{\int_{0}^{3} \sqrt{3+|x|}\, dx}_{x \geq 0} =$$

$$\int_{-3}^{0} \sqrt{3-x}\, dx + \int_{0}^{3} \sqrt{3+x}\, dx$$

Temos duas integrais indefinidas a calcular:

- $\int \sqrt{3-x}\,dx = \int (3-x)^{\frac{1}{2}}\,dx = \int r^{\frac{1}{2}} \cdot (-dr)$

$r = 3-x \Rightarrow dr = -dx$

$dx = -dr$

$$= -\int r^{\frac{1}{2}}\,dr = -\frac{r^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$$

$$= -\frac{r^{\frac{3}{2}}}{\frac{3}{2}} + c = -\frac{2}{3}\cdot(3-x)^{\frac{3}{2}} + c$$

- $\int \sqrt{3+x}\,dx = \int (3+x)^{\frac{1}{2}}\,dx = \frac{r^{\frac{3}{2}}}{\frac{3}{2}} + c$

$r = 3+x \Rightarrow dr = dx$

$$= \frac{2}{3}\cdot(3+x)^{\frac{3}{2}} + c$$

Assim:

$$\int_{-3}^{0} \sqrt{3-x}\,dx = \left(-\frac{2}{3}(3-x)^{\frac{3}{2}}\right)\Bigg|_{-3}^{0} =$$

$$= -\frac{2}{3}\cdot(3)^{\frac{3}{2}} - \left(-\frac{2}{3}\cdot(6)^{\frac{3}{2}}\right) =$$

$$= -2 \cdot 3^{\frac{3}{2}-1} + \frac{2}{3} \cdot \sqrt{6^{\cancel{2}} \cdot 6}$$

$$= -2 \cdot 3^{\frac{1}{2}} + \frac{2}{\cancel{3}} \cdot \overset{2}{\cancel{6}}\sqrt{6}$$

$$= \underbrace{-2\sqrt{3} + 4\sqrt{6}}$$

Analogamente para calcular $\int_0^3 \sqrt{3+x}\, dx$:

$$\int_0^3 \sqrt{3+x}\, dx = \left(\frac{2}{3} \cdot (3+x)^{\frac{3}{2}} \right) \Big|_0^3 = \dots = \underline{\underline{\alpha}}.$$

resposta final:

$$\int_3^3 \sqrt{3+|x|}\, dx = -2\sqrt{3} + 4\sqrt{6} + \underline{\underline{\alpha}}$$

LISTA 03:

1. Podemos integrar $\int \sec^2 x \tan x \, dx$ de duas maneiras como segue:

$\int u^2 \, du$

(a) $\displaystyle\int \sec^2 x \tan x \, dx = \int \underbrace{\tan x}_{u} (\underbrace{\sec^2 x \, dx}_{du}) = \frac{1}{2}\tan^2 x + c,$

(b) $\displaystyle\int \sec^2 x \tan x \, dx = \int \sec x (\sec x \tan x \, dx) = \frac{1}{2}\sec^2 x + c.$

Explique a diferença aparente entre as duas respostas.

No item (a):

$$\int \sec^2 x \tan x \, dx = \int \underbrace{\tan x}_{u} \cdot (\underbrace{\sec^2 x \, dx}_{du}) = \int u^2 \, du$$

$$= \frac{u^2}{2} + c = \frac{\tan^2 x}{2} + c$$

No item (b):

$$\int \sec^2 x \tan x \, dx = \int \underbrace{\sec x}_{u} \, (\underbrace{\sec x \cdot \tan x \, dx}_{du}) = \int u^1 \, du$$

$$= \frac{u^2}{2} + c = \frac{\sec^2 x}{2} + c$$

Lembramos que, se $F(x)$ for uma antiderivada para $f(x)$, então a antiderivada mais geral deverá $F(x) + c$.

Ou seja, o resultado deve diferir de apenas uma constante. E é o que temos aqui, pois; sendo

$$F(x) = \frac{\tan^2 x}{2} + C \quad e$$

$$G(x) = \frac{\sec^2 x}{2} + C \quad ;$$

Da Trigonometria, temos: $1 + \tan^2 x = \sec^2 x$; e então:

$$G(x) = \frac{\sec^2 x}{2} + C = \frac{1 + \tan^2 x}{2} + C =$$

$$= \frac{1}{2} + \frac{\tan^2 x}{2} + C = \frac{\tan^2 x}{2} + \underbrace{C + \frac{1}{2}}_{\stackrel{..}{=}\, C_1}$$

$$= \frac{\tan^2 x}{2} + C_1 \quad ;$$

ou seja, as respostas diferem de apenas uma constante.

LISTA 03 :

5. Calcule cada integral indefinida a seguir.

(a) $\displaystyle\int \frac{\operatorname{sen}(\ln x)dx}{x}$ (b) $\displaystyle\int \cos\sqrt{x}\frac{dx}{\sqrt{x}}$ (c) $\displaystyle\int \frac{dx}{x\ln^2 x}$

(d) $\displaystyle\int \frac{x+3}{\sqrt{x^2-4}}dx$ (e) $\displaystyle\int \left(\frac{x}{\sqrt[3]{x}}-\frac{\ln x}{x}\right)dx$ (f) $\displaystyle\int \frac{e^{\frac{1}{x}}dx}{x^2}$

(g) $\displaystyle\int \frac{x^2dx}{1+x^6}$ (h) $\displaystyle\int 5^{\sqrt{x}}\frac{dx}{\sqrt{x}}$ (i) $\displaystyle\int \frac{e^t}{\sqrt{1-e^{2t}}}dt$

~~(j)~~ $\cancel{\displaystyle\int x\cdot 7^{x^2}dx}$ (k) $\displaystyle\int \frac{\sqrt{\tan x}}{\cos^2 x}dx$ (ℓ) $\displaystyle\int \frac{dx}{\sqrt{1+x^2}\ln(x+\sqrt{1+x^2})}$

(m) $\displaystyle\int \frac{\arctan\frac{x}{2}}{4+x^2}dx$ (n) $\displaystyle\int \sec^2(ax+b)dx$ (o) $\displaystyle\int \frac{1+\operatorname{sen}3x}{\cos^2 3x}dx$

(g) $\displaystyle\int \frac{x^2dx}{1+x^6} =$

$$\cancel{\int \frac{dm}{m} = \ln|m|+c \qquad m = 1+x^6 \Rightarrow dm = 6x^5dx}$$

$$= \int \frac{x^2\,dx}{1+(x^3)^2}$$

$$\int \frac{dm}{m^2+a^2} = \frac{1}{a}\cdot\arctan\left(\frac{m}{a}\right)+c$$

$$\Rightarrow m = x^3 \Rightarrow dm = 3x^2dx$$

$$\boxed{x^2dx = \frac{dm}{3}}$$

$$\Rightarrow \int \frac{x^2\,dx}{1+x^6} = \int \frac{x^2\,dx}{1+(x^3)^2} = \int \frac{\frac{dr}{3}}{1+r^2} =$$

$$\frac{1}{3}\int \frac{dr}{1+r^2} = \frac{1}{3}\cdot\left(\frac{1}{1}\arctan\left(\frac{r}{1}\right)+C\right)$$

$$= \frac{1}{3}\cdot \arctan(x^3) + C$$

(k) $$\int \frac{\sqrt{\tan x}}{\cos^2 x}\,dx = \int (\tan x)^{\frac{1}{2}}\cdot \sec^2 x\,dx =$$

$\sec x = \frac{1}{\cos x}$

$\int r^k\,dr.$

$r = \tan x$

$\hookrightarrow dr = \sec^2 x\,dx$

$$= \int r^{\frac{1}{2}}\cdot dr = \frac{r^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{2}{3}\, r^{3/2} + C = \frac{2}{3}\cdot(\tan x)^{\frac{3}{2}} + C$$

(l) $$\int \frac{dx}{\sqrt{1+x^2}\cdot\ln(x+\sqrt{1+x^2})} = \int \frac{\overbrace{\frac{1}{\sqrt{1+x^2}}\,dx}^{dr}}{\ln(x+\sqrt{1+x^2})} =$$

$$\int \frac{dr}{r}$$

$$r = \ln(x+\sqrt{1+x^2}).$$

$$\hookrightarrow dr = \frac{1+\frac{1}{\cancel{2}}(1+x^2)^{-\frac{1}{2}}\cdot\cancel{2}x}{x+\sqrt{1+x^2}}$$

$$dr = \frac{1+\frac{x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx = \frac{\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}}}{x+\sqrt{1+x^2}}dx$$

$$= \frac{\cancel{x+\sqrt{1+x^2}}}{\sqrt{1+x^2}}\cdot\frac{1}{\cancel{x+\sqrt{1+x^2}}}dx = \frac{1}{\sqrt{1+x^2}}dx$$

$$= \ln|r| + c = \ln|\ln(x+\sqrt{1+x^2})| + c$$