GABARITO P1 – T2

01) (a) $y = x \cdot |x-1|$

Note que

$$|x-1| = \begin{cases} x-1, & \text{se } x-1 \geq 0 \\ -(x-1), & \text{se } x-1 < 0 \end{cases}$$

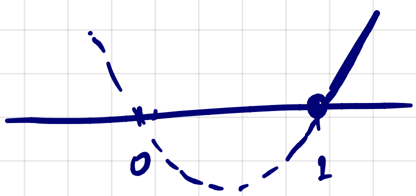
$$= \begin{cases} x-1, & \text{se } x \geq 1 \\ -x+1, & \text{se } x < 1. \end{cases}$$

Assim, teremos:

$$y = x \cdot |x-1| = \begin{cases} x \cdot (x-1), & \text{se } x \geq 1 \\ x(1-x), & \text{se } x < 1. \end{cases}$$

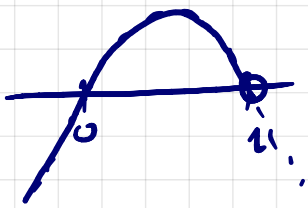
•) $y = x(x-1)$ $(x \geq 1)$

zeros: $x = 0$ e $x = 1$.

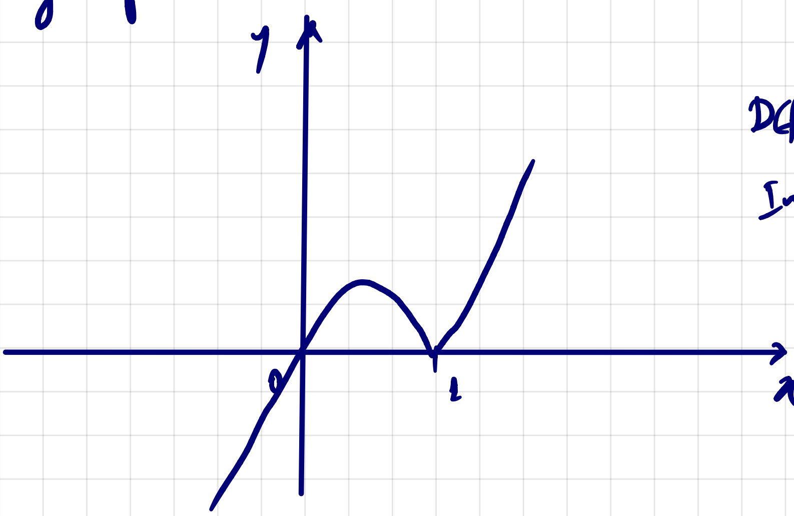



••) $y = x(1-x)$ $(x<1)$

zeros: $x = 0$; $x = 1$.




esboço gráfico:




$D(f) = \mathbb{R}$

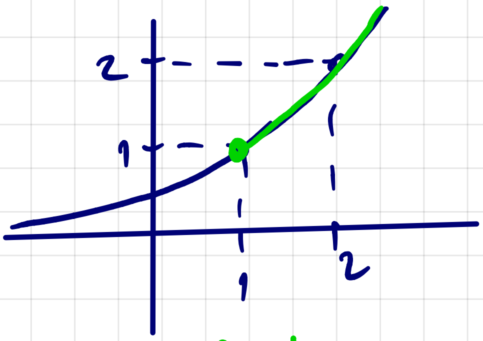
$Im(f) = \mathbb{R}$.

(b) $y = 2^{|x-1|}$

$$|x-1| = \begin{cases} x-1, & \text{se } x \geq 1 \\ 1-x, & \text{se } x < 1 \end{cases}$$

(•) $y = \underbrace{2^{x-1}}_{>0}$ ; $x \geq 1$

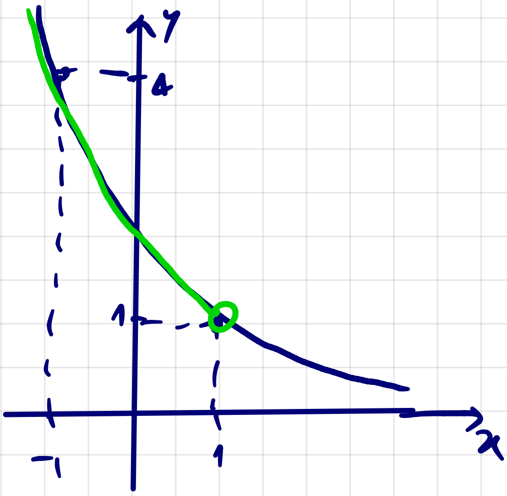
| $x$ | $y = 2^{x-1}$ |
|---|---|
| 1 | 1 |
| 2 | 2 |



$x \geq 1$

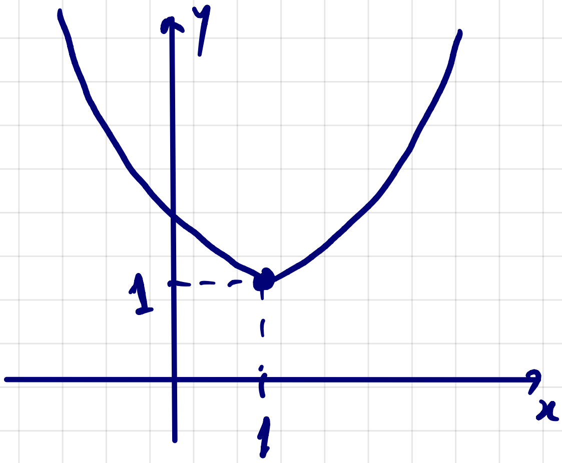
(••) $y = \underbrace{2^{1-x}}_{>0}$ ; $x < 1$.

| $x$ | $y = 2^{1-x}$ |
|---|---|
| 1 | 1 |
| $-1$ | 4 |



$x < 1$

COMBINANDO OS DOIS CASOS, OBTEMOS:



$D(f) = \mathbb{R}$

$Im(f) = [1, +\infty)$

(c) $y = 1 + \log_2(1-2x)$.

$y - 1 = \log_2(1-2x) \Leftrightarrow 1 - 2x = \underbrace{2^{y-1}}_{>0}$.

$\Leftrightarrow 2x = 1 - 2^{y-1} \Leftrightarrow x = \frac{1}{2} - \frac{1}{2} \cdot 2^{y-1}$

$\Leftrightarrow x = \frac{1}{2} - 2^{y-2}$

$1 - 2x > 0$

$-2x > -1$

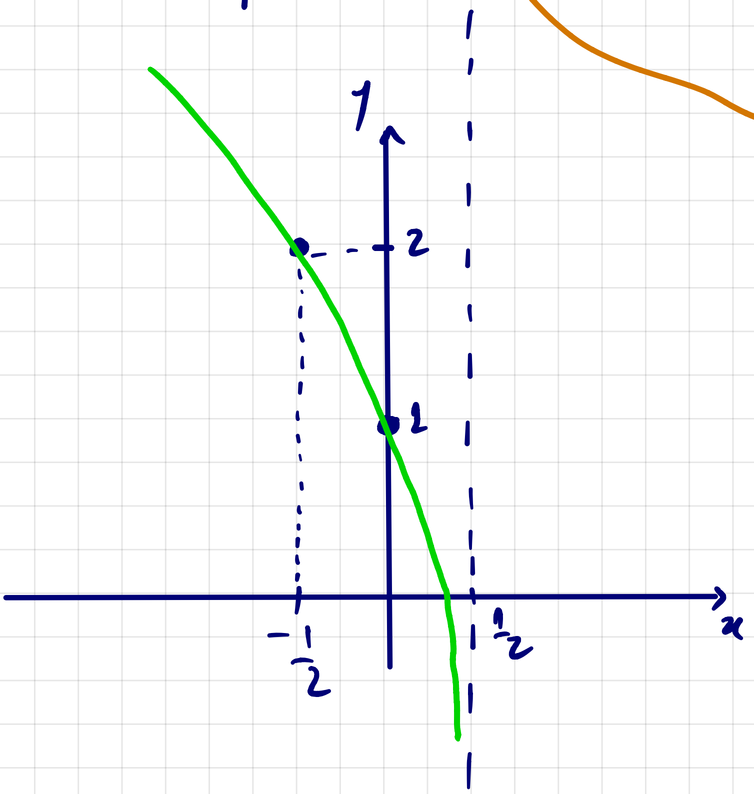
$x < \frac{1}{2}$

$D(f) = (-\infty, \frac{1}{2})$

| $x = \frac{1}{2} - 2^{y-2}$ | $y$ |
| --- | --- |
| $-\frac{1}{2}$ | 2 |
| 0 | 1 |

$x = \frac{1}{2}$ é A

ASSÍNTOTA VERTICAL.




$Im(f) = \mathbb{R}$.

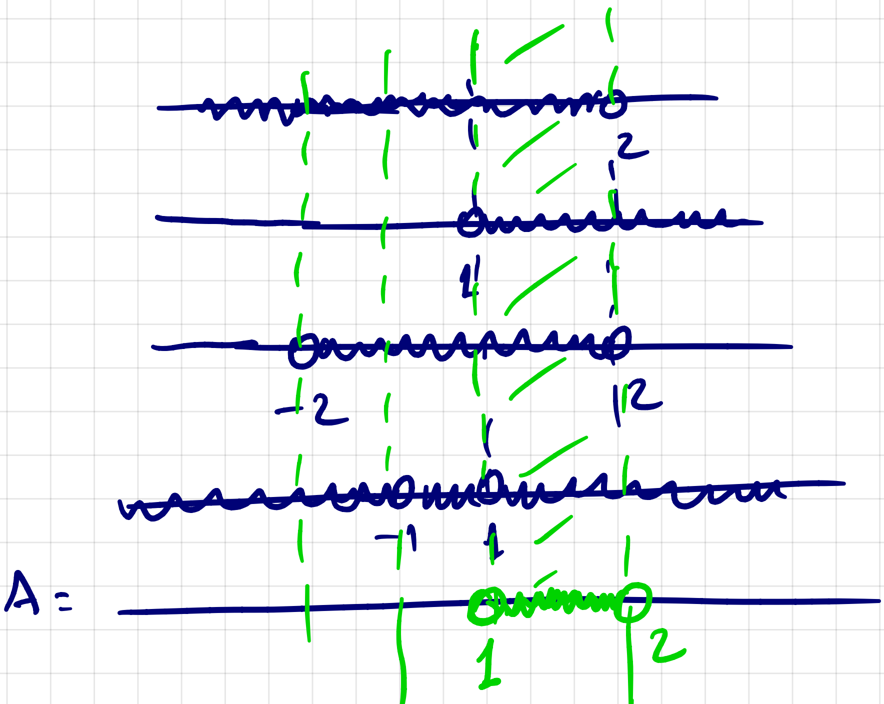
02) Primeiramente, note que podemos reescrever a $f$ do seguinte modo, em virtude da propriedade dos logaritmos:

$$f(x) = \frac{x^2+2x+1}{\sqrt{2-x}} + \ln(x+1) - \ln(4-x^2) + 3^{\frac{x}{1-x^2}}.$$

Condições de existência para $f$:

$$\begin{cases} 2-x>0 \longrightarrow x<2 \\ x+1>0 \longrightarrow x>1 \\ 4-x^2>0 \longrightarrow \text{zeros: } x^2-4=0 \Leftrightarrow x=\pm 2 \\ 1-x^2 \neq 0 \longrightarrow x \neq \pm 1. \end{cases}$$

Assim, seu domínio será a interseção de todas as condições acima, ou seja:

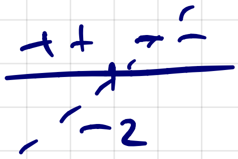


$\Rightarrow A = (1, 2).$

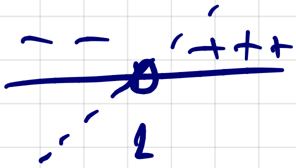
Domínio de $g(x) = \sqrt{\dfrac{x+2}{x-1}}$ :

condição de existência: $\dfrac{x+2}{x-1} \geq 0$.

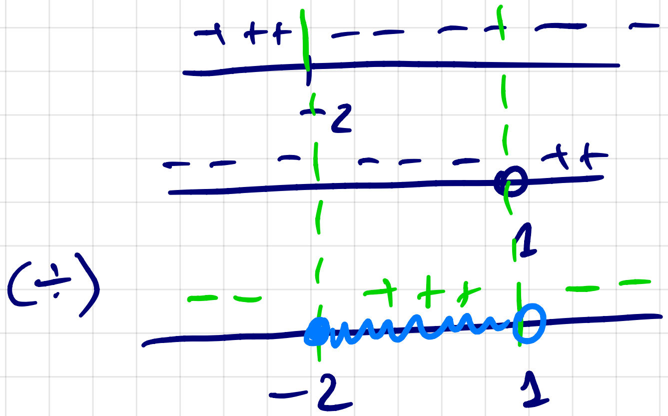
• zeros do numerador: $x+2=0 \Leftrightarrow x=-2$



• zeros do denominador ($\neq 0$): $x-1=0 \Leftrightarrow x=1$

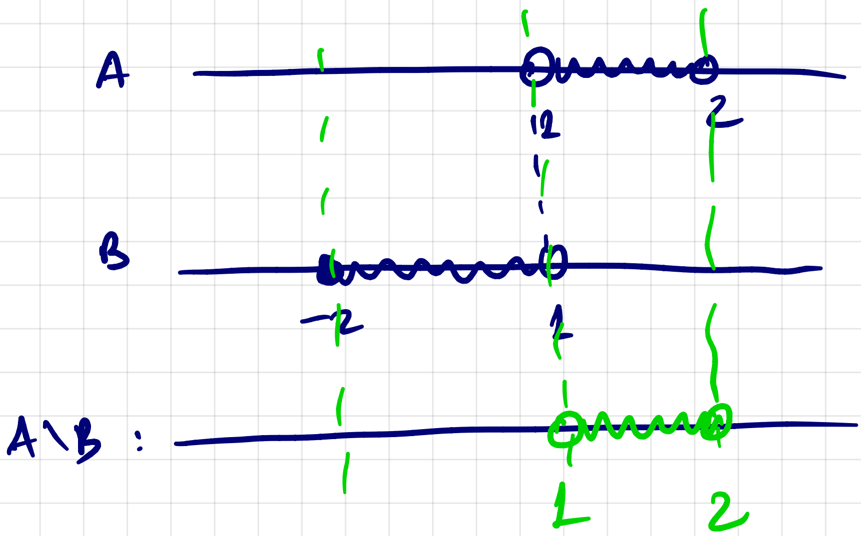


Efetuando a divisão de sinais, vem:



$\Rightarrow B = [-2, 1)$.

Por fim, calculando $A \setminus B$:



$= (1, 2) = A$.

(pois são disjuntos)

03) $\cot x = -\dfrac{2}{3}$ , $x \in 2^{\circ}q.$

$$\tan x = \frac{1}{\cot x} = -\frac{3}{2} \Rightarrow \boxed{\tan x = -\frac{3}{2}}$$

$$1 + \tan^2 x = \sec^2 x \Rightarrow \sec = \pm\sqrt{1+\tan^2 x}$$

$$\Rightarrow \sec x = -\sqrt{1 + \left(-\frac{3}{2}\right)^2}$$

$x \in 2^{\circ}q$

$$\Rightarrow \sec x = -\sqrt{1+\frac{9}{4}} =$$

$$= -\sqrt{\frac{4+9}{4}} = -\frac{\sqrt{13}}{2}$$

$$\Rightarrow \boxed{\sec x = -\frac{\sqrt{13}}{2}}$$

$$\cos x = \frac{1}{\sec x} \Rightarrow \cos x = -\frac{2}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}}$$

$$\Rightarrow \boxed{\cos x = -\frac{2\sqrt{13}}{13}}$$

$$\operatorname{sen}^2 x + \cos^2 x = 1 \Rightarrow \operatorname{sen}^2 x = 1 - \cos^2 x$$

$$\Rightarrow \operatorname{sen} x = \pm\sqrt{1-\cos^2 x}$$

$$\Rightarrow \operatorname{sen} x = +\sqrt{1-\cos^2 x}$$

$x \in 2^{\circ}q.$

$$\Rightarrow \operatorname{sen} x = \sqrt{1 - \left(-\frac{2}{\sqrt{13}}\right)^2} = \sqrt{1 - \frac{4}{13}} = \sqrt{\frac{13-4}{13}}$$

$$= \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}.$$

$$\Rightarrow \boxed{\operatorname{sen} x = \frac{3}{\sqrt{13}}}$$

$$\csc x = \frac{1}{\operatorname{sen} x} \Rightarrow \boxed{\csc x = \frac{\sqrt{13}}{3}}.$$

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04)

$$t = 0 \Rightarrow m = m_0$$

$$t = 1 \Rightarrow m = m_0 - \frac{4}{100} m_0 = m_0\left(1 - \frac{4}{100}\right)$$

$$t = 2 \Rightarrow m = m_0 . \left(1 - \frac{4}{100}\right) - \frac{4}{100} m_0 . \left(1 - \frac{4}{100}\right)$$

$$= m_0\left(1 - \frac{4}{100}\right) . \left(1 - \frac{4}{100}\right)$$

$$= m_0 \left(1 - \frac{4}{100}\right)^2.$$

No tempo $t$: $\quad m(t) = m_0 . \left(1 - \frac{4}{100}\right)^t$

i.e;

$$m(t) = m_0 . \left(\frac{96}{100}\right)^t$$

Quanto tempo para que $m(t) = \frac{m_0}{2}$ (meia-vida).?

Neste caso,

$$\frac{\cancel{m_0}}{2} = \cancel{m_0} \cdot \left(\frac{96}{100}\right)^t$$

$$\frac{1}{2} = \left(\frac{96}{100}\right)^t \Leftrightarrow \log 0{,}5 = \log (0{,}96)^t$$

$$\Leftrightarrow t \cdot \log (0{,}96) = \log 0{,}5$$

$$\Leftrightarrow t = \frac{\log 0{,}5}{\log 0{,}96} \cong \frac{-0{,}31102999566}{-0{,}01772876696}$$

$$t \cong 16{,}97975 \text{ anos.}$$

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05) $\log_\beta A = 3$ ; $\log_\beta B = -1$ ; $\log_\beta C = \frac{1}{2}$.

Então:

$$\log_\beta \frac{\sqrt[5]{A^3 B^2 \sqrt{C}}}{\sqrt{A \sqrt[3]{C}}} = \log_\beta \frac{(A^3 B^2 C^{\frac{1}{2}})^{\frac{1}{5}}}{(A \cdot C^{\frac{1}{3}})^{\frac{1}{2}}} =$$

$$= \log_\beta \frac{A^{\frac{3}{5}} \cdot B^{\frac{2}{5}} \cdot C^{\frac{1}{10}}}{A^{\frac{1}{2}} \cdot C^{\frac{1}{6}}} = \log_\beta \left(A^{\frac{3}{5}-\frac{1}{2}} \cdot B^{\frac{2}{5}} \cdot C^{\frac{1}{10}-\frac{1}{6}}\right)$$

$$= \log_\beta \left(A^{\frac{6-5}{10}} \cdot B^{\frac{2}{5}} \cdot C^{\frac{3-5}{30}}\right)$$

$$= \log_\beta \left(A^{\frac{1}{10}} \cdot B^{\frac{2}{5}} \cdot C^{-\frac{1}{15}}\right) =$$

$$= \log_\beta A^{\frac{1}{10}} + \log_\beta B^{\frac{2}{5}} + \log_\beta C^{-\frac{1}{15}} =$$

$$= \frac{1}{10} \underbrace{\log_\beta A}_{=3} + \frac{2}{5} \cdot \underbrace{\log_\beta B}_{=-1} - \frac{1}{15} \underbrace{\log_\beta C}_{=\frac{1}{2}} =$$

$$= \frac{3}{10} - \frac{2}{5} - \frac{1}{30} = \frac{9 - 12 - 1}{30} = -\frac{4}{30} = -\frac{2}{15}$$

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06) Provar: $\lim_{x \to -2} \frac{x^2-4}{x+2} = -4$.

Dado $\varepsilon > 0$, precisamos encontrar $\delta > 0$, tal que, $\forall x \in D(f)$, tal que $0 < |x+2| < \delta$, implique em $|f(x) + 4| < \varepsilon$.

Analisando $|f(x)+4|$:

$$|f(x)+4| = \left| \frac{x^2-4}{x+2} + 4 \right| = \left| \frac{(x+\cancel{2})(x-2)}{x\cancel{+2}} + 4 \right| =$$

$$= |x-2+4| = |x+2| < \delta := \varepsilon.$$

Portanto, basta tomar $\underline{\delta = \varepsilon}$. $\square$

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07)

(a) $\lim_{x\to -3} \dfrac{x^2-5x-24}{2x^2+6x} = 0/0$ (İNDET.)

$$\begin{array}{l|l} \cancel{x^2} -5x-24 & x+3 \\ \hline -\cancel{x^2}-3x & x-8 \\ \hline -8x \cancel{-24} & \\ +8x \cancel{+24} & \\ \hline 0 & \end{array}$$

$\hookrightarrow x^2-5x-24 = (x+3)(x-8)$

$$2x^2+6x = 2x(x+3)$$

Anım:

$$\lim_{x\to -3} \frac{x^2-5x-24}{2x^2+6x} = \lim_{x\to -3} \frac{(\cancel{x+3})(x-8)}{2x(\cancel{x+3})} = \lim_{x\to -3} \frac{x-8}{2x}$$

$$= \frac{-3-8}{2\cdot(-3)} = \frac{-11}{-6} = \frac{11}{6}$$

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(b) $\lim_{x\to 5} \dfrac{\sqrt{x-1}-2}{x-5} \times \dfrac{\sqrt{x-1}+2}{\sqrt{x-1}+2} =$

$$= \lim_{x\to 5} \frac{(\sqrt{x-1})^2-(2)^2}{(x-5)(\sqrt{x-1}+2)} = \lim_{x\to 5} \frac{x-1-4}{(x-5)(\sqrt{x-1}+2)}$$

$$= \lim_{x\to 5} \frac{\cancel{x-5}}{(\cancel{x-5})(\sqrt{x-1}+2)} = \frac{1}{\sqrt{5-1}+2} = \frac{1}{2+2} = \frac{1}{4}$$

(e) $\lim_{x\to 1} \dfrac{\sqrt{x^2+3x+1}-\sqrt{6x-1}}{x^3-1} = \dfrac{0}{0}$ (INDET.)

$$\lim_{x\to 1} \frac{\sqrt{x^2+3x+1}-\sqrt{6x-1}}{x^3-1} \times \frac{\sqrt{x^2+3x+1}+\sqrt{6x-1}}{\sqrt{x^2+3x+1}+\sqrt{6x-1}} =$$

$$= \lim_{x\to 1} \frac{\left(\sqrt{x^2+3x+1}\right)^2-\left(\sqrt{6x-1}\right)^2}{(x^3-1)\cdot\left(\sqrt{x^2+3x+1}+\sqrt{6x-1}\right)} =$$

$$= \lim_{x\to 1} \frac{x^2+3x+1-6x+1}{(x^3-1)\left(\sqrt{x^2+3x+1}+\sqrt{6x-1}\right)} =$$

$$= \lim_{x\to 1} \frac{x^2-3x+2}{(x^3-1)\left(\sqrt{x^2+3x+1}+\sqrt{6x-1}\right)} =$$

$$\begin{array}{l|l} \cancel{x^2}-3x+2 & x-1 \\ \hline -\cancel{x^2}+x & x-2 \\ \hline -2x+\cancel{2} & \\ +2x-\cancel{2} & \\ \hline 0 & \end{array}$$

$\Rightarrow x^2-3x+2 = (x-1)(x-2)$

$$
\begin{array}{l|l}
\cancel{x^3} - 1 & x - 1 \\ \cline{2-2}
-\cancel{x^3} + x^2 & x^2 + x + 1 \\ \cline{1-1}
\quad \cancel{x^2} - 1 & \\
\quad -\cancel{x^2} + x & \\ \cline{1-1}
\qquad x - 1 & \\
\qquad -x + 1 & \\ \cline{1-1}
\qquad\quad 0 &
\end{array}
$$

$$\hookrightarrow \quad x^3 - 1 = (x-1)\cdot(x^2+x+1)$$

Assim, teremos:

$$= \lim_{x \to 1} \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}(x^2+x+1)\cdot\left(\sqrt{x^2+3x+1} + \sqrt{6x-1}\right)}$$

$$= \lim_{x \to 1} \frac{(x-2)}{(x^2+x+1)\left(\sqrt{x^2+3x+1} + \sqrt{6x-1}\right)}$$

$$= \frac{1-2}{(1+1+1)(\sqrt{5}+\sqrt{5})} = \frac{-1}{3\cdot 2\sqrt{5}} = -\frac{1}{6\sqrt{5}}$$