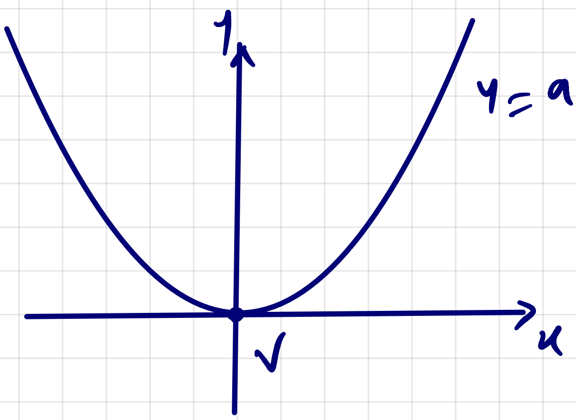
CÁLCULO, T2

02/07/24 - AULA 07

Obs.: AULÃO DE CÁLCULO 1. DIA 06/07; 9h → 11h
(Sala 206 — CAMPUS II na BARROSO) GAMA
Conteúdo: funções e limites.

Na aula passada, começamos estudando o esboço gráfico da função quadrática $y=ax^2$; $a>0$

C.P.C.

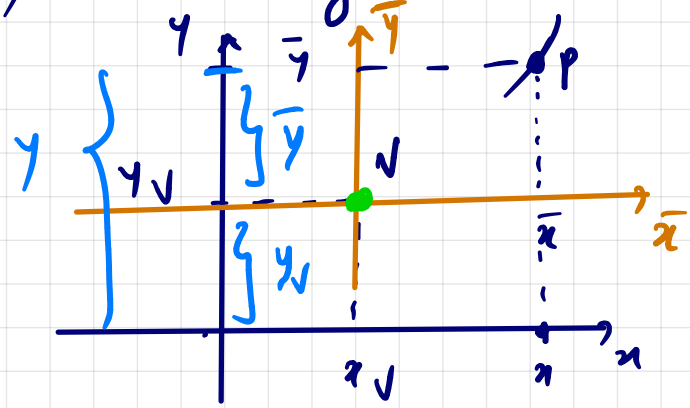


$V(0,0)$

CASO GERAL: $y = ax^2+bx+c$.

$V=(x_V, y_V)$; onde $x_V = -\dfrac{b}{2a}$.

Vamos considerar um novo sistema cartesiano $\bar{x}\bar{y}$, com eixos paralelos aos respectivos eixos x e y, com origem no vértice V.



Neste caso, faremos a mudança de sistema de coordenadas, escrevendo:

$$\begin{cases} x = x_V + \bar{x} = -\dfrac{b}{2a} + \bar{x} \\ y = y_V + \bar{y} = -\dfrac{\Delta}{4a} + \bar{y} \end{cases}$$

$$y = ax^2 + bx + c$$

$$-\frac{\Delta}{4a} + \bar{y} = a.\left(-\frac{b}{2a} + \bar{x}\right)^2 + b.\left(-\frac{b}{2a} + \bar{x}\right) + c$$

$$-\frac{\Delta}{4a} + \bar{y} = a.\left(\frac{b^2}{4a^2} - \cancel{2}.\frac{b}{\cancel{2}a}.\bar{x} + \bar{x}^2\right) - \frac{b^2}{2a} + b.\bar{x} + c$$

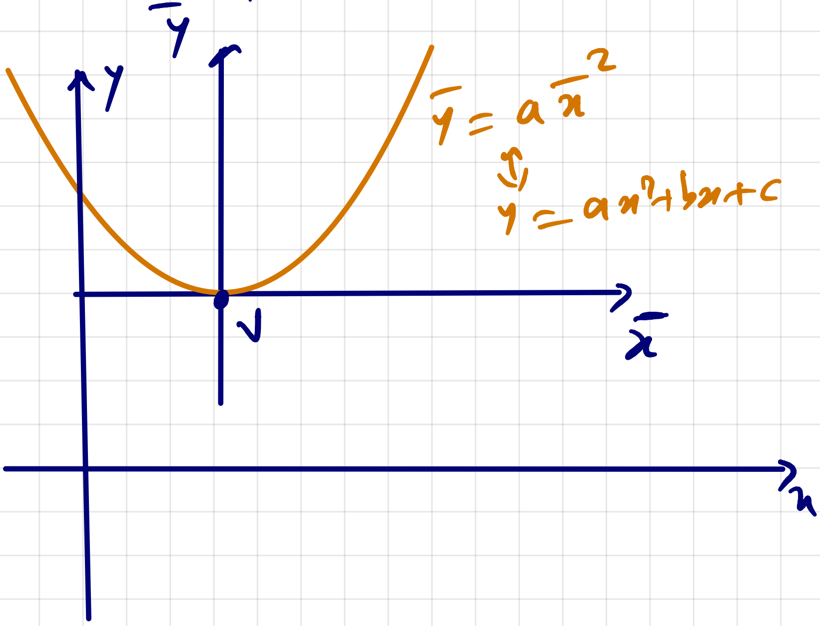
$$-\frac{\Delta}{4a} + \bar{y} = \frac{b^2}{4a} - b\,\bar{x} + a\bar{x}^2 - \frac{b^2}{2a} + b\bar{x} + c$$

$$-\frac{(b^2 - 4ac)}{4a} + \bar{y} = \frac{b^2}{4a} - b\,\bar{x} + a\bar{x}^2 - \frac{b^2}{2a} + b\bar{x} + c$$

$$-\frac{b^2}{4a} + \cancel{c} + \bar{y} = \frac{b^2 - 2b^2}{4a} - \cancel{b\,\bar{x}} + a\bar{x}^2 + \cancel{b\bar{x}} + \cancel{c}$$

$$-\cancel{\frac{b^2}{4a}} + \bar{y} = -\cancel{\frac{b^2}{4a}} + a\bar{x}^2 \Rightarrow \bar{y} = a\bar{x}^2$$

Ou seja, $y = ax^2 + bx + c$, no plano $\bar{x}\,\bar{y}$ reduz-se à eq. $\bar{y} = a\bar{x}^2$, que já sabemos o esboço:



Assim, o esboço gráfico de $y = ax^2 + bx + c$ é o mesmo feito na aula passada, para $y = ax^2$.

<u>Ex.:</u> 01) $y = x^2 - 4x + 3$. gráfico?

zeros: $f(x) = 0 \iff x^2 - 4x + 3 = 0$

$$\iff x = \frac{4 \pm \sqrt{16 - 12}}{2}$$

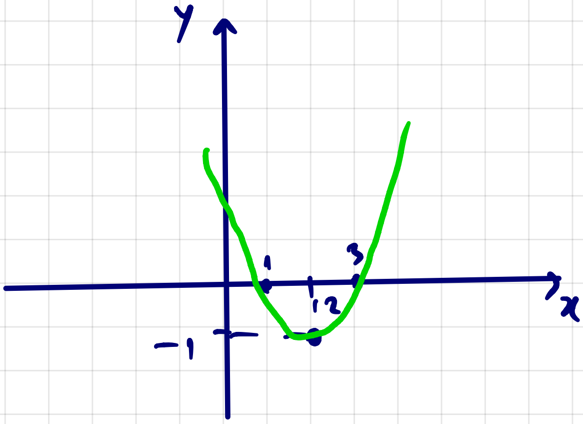
$$x = \frac{4 \pm 2}{2} \begin{cases} \nearrow x = 3 \\ \searrow x = 1. \end{cases}$$

$V(x_v, y_v)$

$$x_v = -\frac{b}{2a} = -\frac{(-4)}{2 \cdot 1} = 2$$

$$y_v = f(x_v) = (2)^2 - 4 \cdot (2) + 3 = 4 - 8 + 3 = -1.$$

$$V(2, -1)$$



$D(f) = \mathbb{R}$.

$Im(f) = [-1, +\infty)$.

02) $y = -x^2 + 3x - 2$. grafico?

zeri: $f(x) = 0 \iff -x^2+3x-2=0 \quad x(-1)$

$a = -1 < 0 \Rightarrow$ c.p.B

$x^2 - 3x + 2 = 0$

$$x = \frac{+3 \pm \sqrt{9-8}}{2}$$

$$x = \frac{3 \pm 1}{2} \quad \begin{matrix} \nearrow x = 2 \\ \searrow x = 1 \end{matrix}$$

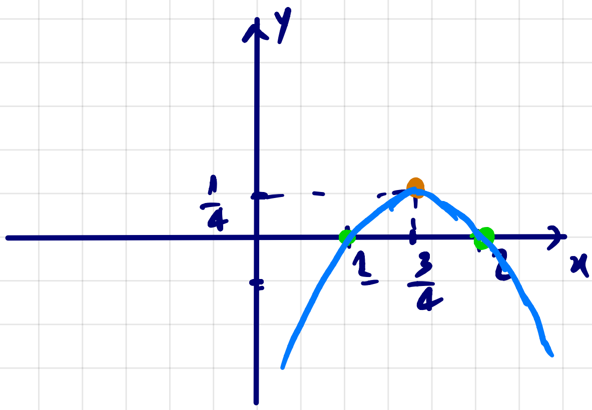
$V(x_v, y_v)$

$$x_v = -\frac{b}{2a} = \frac{-3}{2\cdot(-1)} = +\frac{3}{2}$$

$$y_v = f(x_v) = -\left(\frac{3}{2}\right)^2 + 3\cdot\left(\frac{3}{2}\right) - 2$$

$$= -\frac{9}{4} + \frac{9}{2} - 2 = \frac{-9+18-8}{4} = \frac{1}{4}$$

$$V = \left(\frac{3}{2}, \frac{1}{4}\right)$$



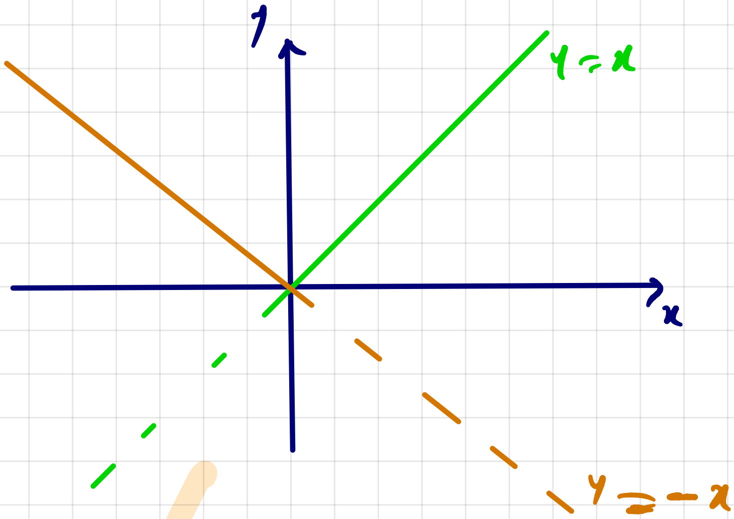
$D(f) = \mathbb{R}$

$Im(f) = (-\infty, \frac{1}{4}]$.

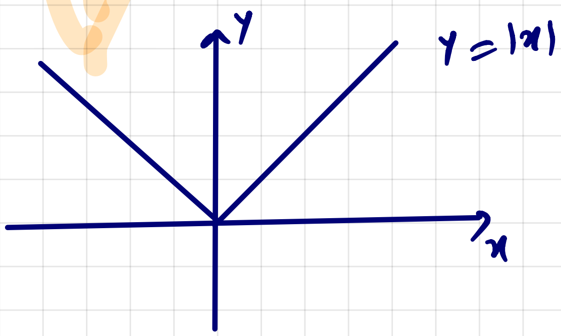
03) FUNÇÃO MODULAR: É a função $f: \mathbb{R} \to \mathbb{R}$ dada por

$$f(x) = |x|.$$

Lembre que $f(x) = |x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$



RESUMINDO, A PARTE NEGATIVA FICA "REBATIDA" PARA CIMA DO EIXO OX.



$D(f) = \mathbb{R}.$

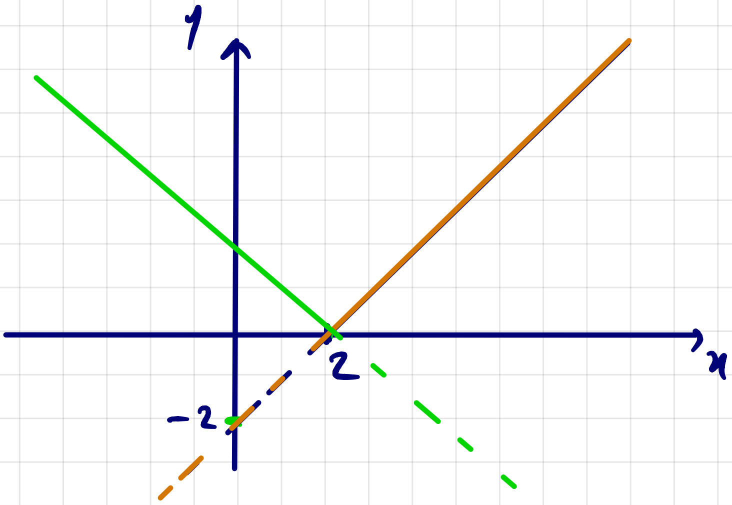
$Im(f) = [0, +\infty).$

EXEMPLOS:

01) $y = |x - 2|.$ gráfico?

$$y = |x-2| = \begin{cases} x-2, & \text{se } x-2 \geq 0 \\ -(x-2), & \text{se } x-2 < 0 \end{cases}$$

$$= \begin{cases} x-2, & \text{se } x \geq 2 \\ -x+2, & \text{se } x < 2 \end{cases}$$



$y = x - 2$

$y = -x + 2$

$D(f) = \mathbb{R}.$

$Im(f) = [0, +\infty)$

02) $y = x^2 - 5|x| + 4.$ gráfico?

Note que $|x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$

Então:

$$f(x) = x^2 - 5 \cdot |x| + 4 = \begin{cases} x^2 - 5x + 4, & \text{se } x \geq 0 \\ x^2 - 5(-x) + 4, & \text{se } x < 0 \end{cases}$$

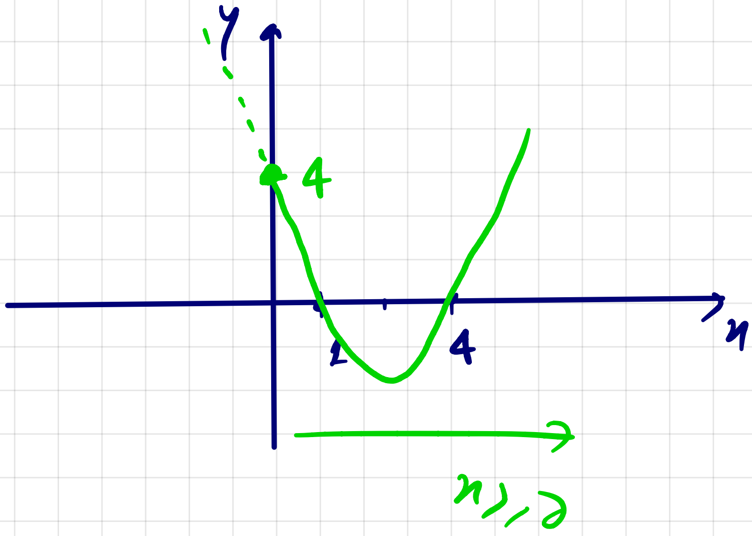
$$= \begin{cases} x^2 - 5x + 4, & \text{se } x \geq 0 \\ x^2 + 5x + 4, & \text{se } x < 0 \end{cases}$$

$y_1 = x^2 - 5x + 4$:

zeros: $x^2 - 5x + 4 = 0 \iff x = \dfrac{5 \pm \sqrt{25 - 16}}{2}$

$\iff x = \dfrac{5 \pm 3}{2}$ $\nearrow x = 4$, $\searrow x = 1$

$x_V = \dfrac{5}{2}$ c.p.c.

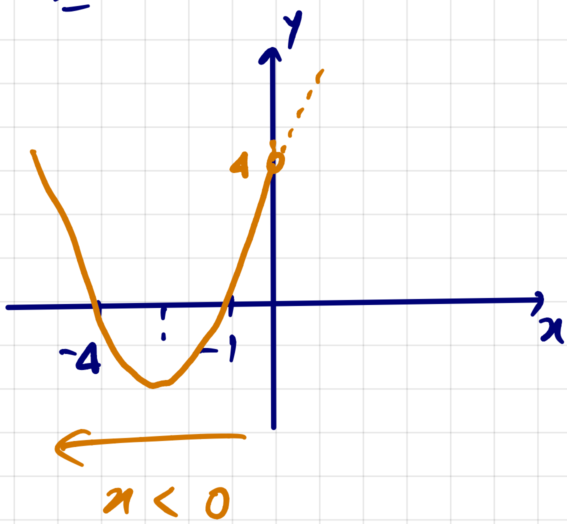


$y_2 = 1x^2 + 5x + 4$; $x < 0$.

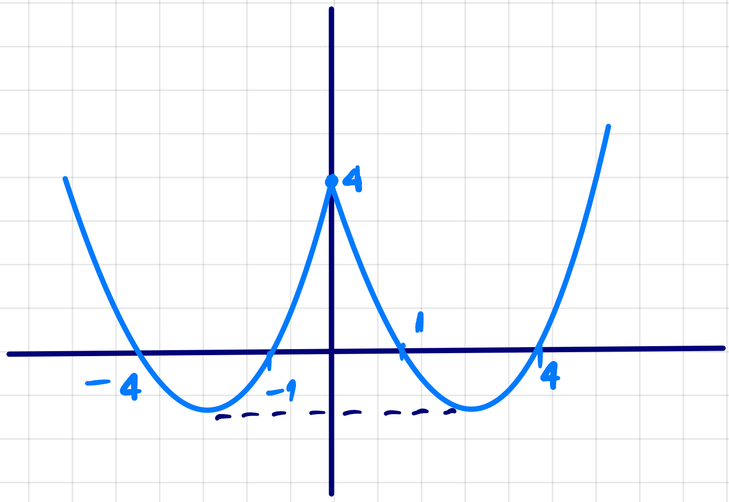
zeros : $x^2 + 5x + 4 = 0 \Leftrightarrow x = \dfrac{-5 \pm \sqrt{25 - 16}}{2}$

$\Leftrightarrow x = \dfrac{-5 \pm 3}{2}$ → $x = -1$ → $x = -4$

$x_V = -\dfrac{5}{2}$ c.q.c.



Portanto, o gráfico final será:



$D(f) = \mathbb{R}$

$Im(f) = [y_V, +\infty)$

03) $y = |x+1| + |2x-1|$. gráfico?

<u>solução</u>: Note que:

$$|x+1| = \begin{cases} x+1, & \text{se } x+1 \geq 0 \\ -(x+1), & \text{se } x+1 < 0 \end{cases}$$

$$\Rightarrow |x+1| = \begin{cases} x+1, & \text{se } x \geq -1 \\ -x-1, & \text{se } x < -1 \end{cases};$$

e

$$|2x-1| = \begin{cases} 2x-1, & \text{se } 2x-1 \geq 0 \\ -(2x-1), & \text{se } 2x-1 < 0 \end{cases}$$

$2x \geq 1$

$x \geq \frac{1}{2}$

$$\Rightarrow |2x-1| = \begin{cases} 2x-1, & \text{se } x \geq \frac{1}{2} \\ -2x+1, & \text{se } x < \frac{1}{2} \end{cases}$$

$|x+1| = -x-1$

$+$ $|2x-1| = -2x+1$

$|x+1| = x+1$

$+$ $|2x-1| = -2x+1$

$|x+1| = x+1$

$+$ $|2x-1| = 2x-1$

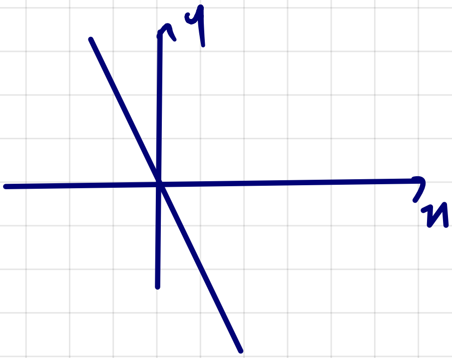
-1 $\quad$ $\frac{1}{2}$

$-3x$ $\quad$ $-x+2$ $\quad$ $3x$

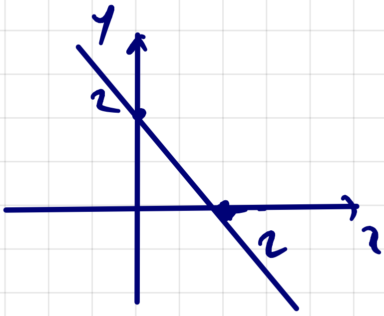
Assim, concluímos que:

$$f(x) = |x+1| + |2x-1| = \begin{cases} -3x, & \text{se } x < -1 \\ -x+2, & \text{se } -1 \leq x < \frac{1}{2} \\ 3x, & \text{se } x \geq \frac{1}{2} \end{cases}$$

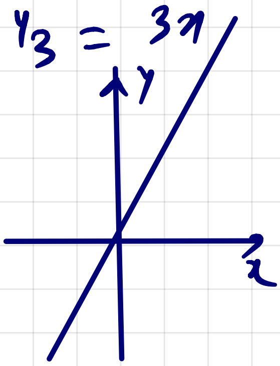
$y_1 = -3x$

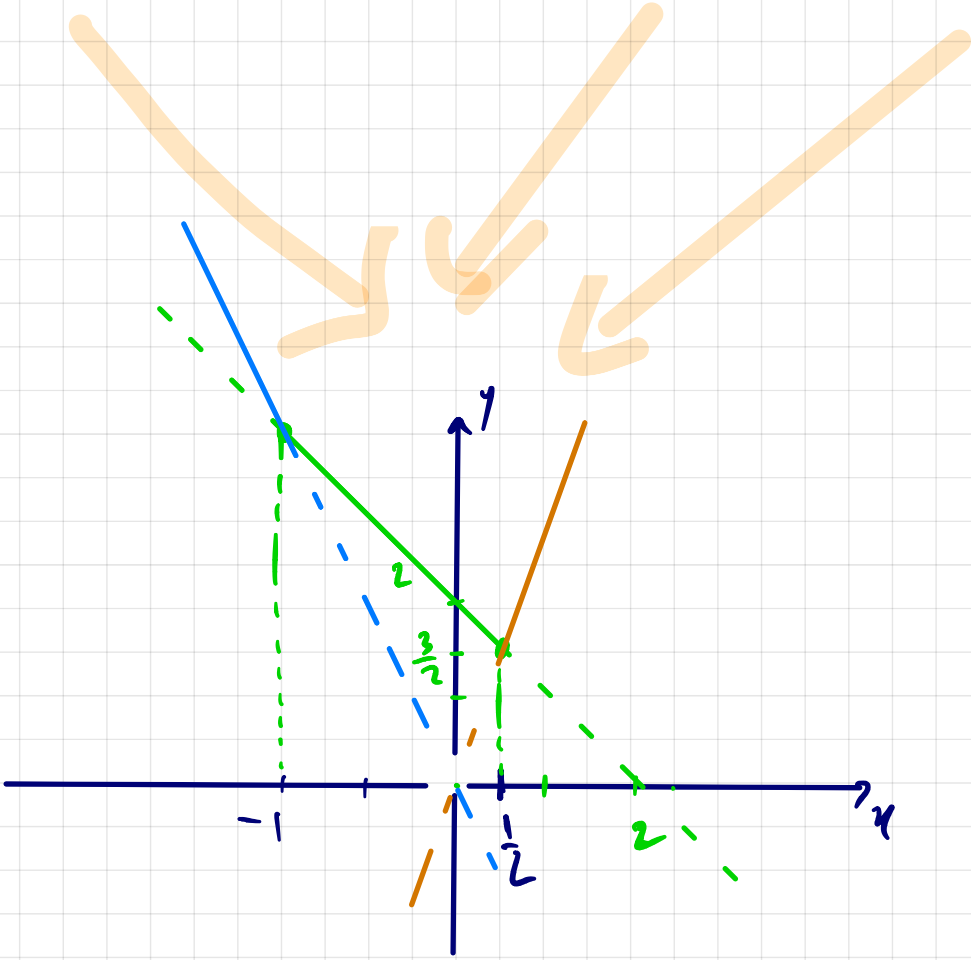


$y_2 = -x+2$

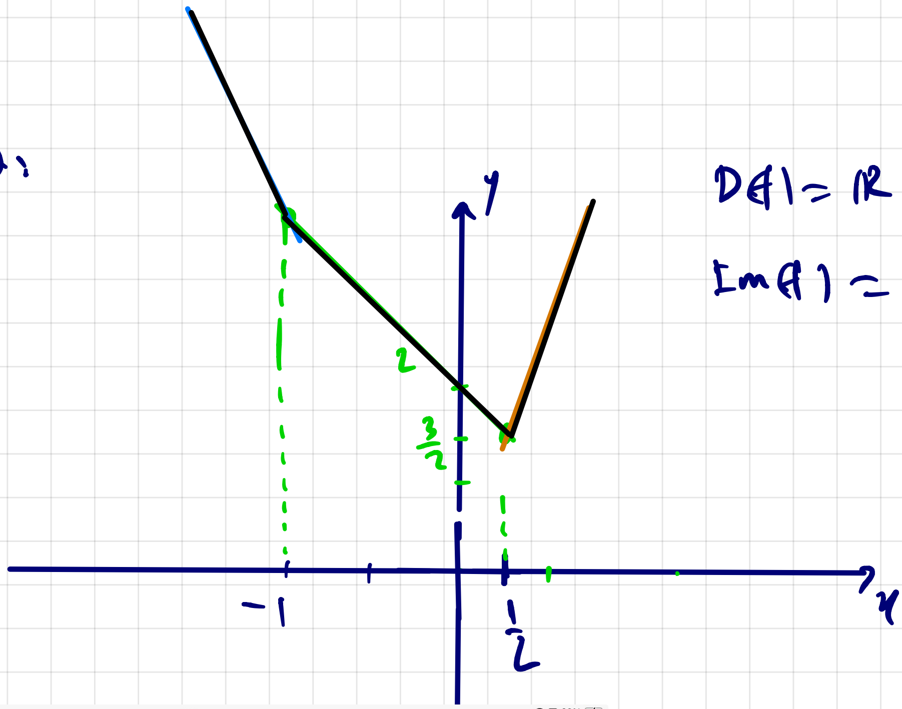


$y_3 = 3x$



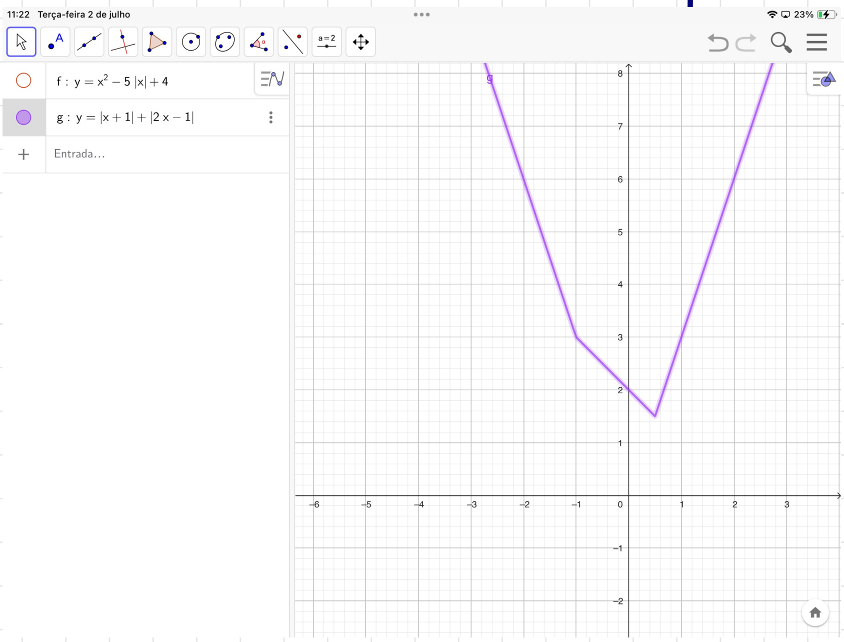


O esboço final está ao lado:



$D(f) = \mathbb{R}$

$Im(f) = [\frac{3}{2}, +\infty)$



→ Pelo geogebra.