CÁLCULO 1, T2:

16/07/24 – AULA 15

L2

25. Mostre que

(a) $\dfrac{\sec\alpha - \csc\alpha}{\sec\alpha + \csc\alpha} = \dfrac{\tan\alpha - 1}{\tan\alpha + 1}$.

$$\frac{\sec\alpha - \csc\alpha}{\sec\alpha + \csc\alpha} = \frac{\frac{1}{\cos\alpha} - \frac{1}{\operatorname{sen}\alpha}}{\frac{1}{\cos\alpha} + \frac{1}{\operatorname{sen}\alpha}} = \frac{\frac{\operatorname{sen}\alpha - \cos\alpha}{\cancel{\cos\alpha\operatorname{sen}\alpha}}}{\frac{\operatorname{sen}\alpha + \cos\alpha}{\cancel{\cos\alpha\operatorname{sen}\alpha}}}$$

$$= \frac{\operatorname{sen}\alpha - \cos\alpha}{\operatorname{sen}\alpha + \cos\alpha} = \frac{\frac{\operatorname{sen}\alpha - \cos\alpha}{\cos\alpha}}{\frac{\operatorname{sen}\alpha + \cos\alpha}{\cos\alpha}} =$$

$$= \frac{\frac{\operatorname{sen}\alpha}{\cos\alpha} - \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}}{\frac{\operatorname{sen}\alpha}{\cos\alpha} + \frac{\cancel{\cos\alpha}}{\cancel{\cos\alpha}}} = \frac{\tan\alpha - 1}{\tan\alpha + 1}.$$

L2:

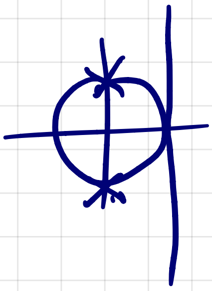
15. Determine todos os valores de x para os quais $\tan\left(\pi x - \dfrac{\pi}{2}\right)$ não exista.

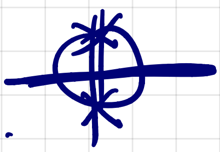
$$\tan\alpha = \frac{\operatorname{sen}\alpha}{\cos\alpha}.$$

$\nexists \tan\alpha$ onde $\cos\alpha = 0$

$\Updownarrow$

$\alpha = k\pi + \frac{\pi}{2}, \; k \in \mathbb{Z}.$





Assim, $\nexists \tan\left(\pi x - \frac{\pi}{2}\right) \Leftrightarrow$

$$\pi x - \frac{\pi}{2} = k\pi + \frac{\pi}{2}, \quad \forall k \in \mathbb{Z}.$$

$$\Leftrightarrow \pi x = k\pi + \frac{\pi}{2} + \frac{\pi}{2}, \quad \forall k \in \mathbb{Z}$$

$$\Leftrightarrow \pi x = k\pi + \pi, \quad \forall k \in \mathbb{Z}.$$

$\div \pi$

$$\Leftrightarrow x = k+1, \quad \forall k \in \mathbb{Z}.$$

Conclusão: $\nexists \tan\left(\pi x - \frac{\pi}{2}\right) \Leftrightarrow x = k+1, k \in \mathbb{Z}.$

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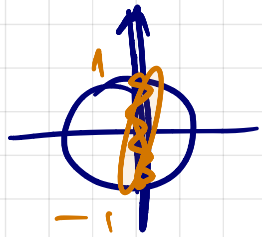
16. Determine os valores de $k \in \mathbb{R}$ para os quais temos

(a) $\operatorname{sen} x = 3k - 2$ (b) $\cos x = \dfrac{k+1}{k-1}$

(a) $-1 \leq \operatorname{sen} x \leq 1.$



eixo dos senos

$$\Updownarrow$$

$$-1 \leq 3k-2 \leq 1 \qquad +2:$$

$$-1+2 \leq 3k \leq 1+2$$

$$1 \leq 3k \leq 3 \qquad \div 3$$

$$\Leftrightarrow \frac{1}{3} \leq k \leq 1. \qquad k \in \left[\frac{1}{3}, 1\right].$$

(b) $-1 \le \cos n \le 1$. $\iff \underbrace{-1 \le \overbrace{\frac{k+1}{k-1} \le 1}^{(\mathrm{I})}}_{(\mathrm{II})}$

(I): $\frac{k+1}{k-1} \le 1 \iff \frac{k+1}{k-1} - 1 \le 0$

$$\iff \frac{k+1-(k-1)}{k-1} \le 0$$

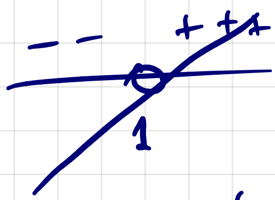
$$\iff \frac{\cancel{k}+1-\cancel{k}+1}{k-1} \le 0$$

$$\iff \frac{2}{k-1} \le 0$$

• SINAL DO NUMERADOR: $2 \cdot +$

$+ + + +$

• SINAL DO DENOM.: $k-1=0 \iff k=1 \quad (\neq 0)$



$\forall k<1 \Rightarrow k-1<0$

$\forall k>1 \Rightarrow k-1>0$

$- \ + + +$

$- - \ | \ + +$

1

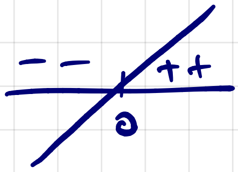
$+ + +$ P_1.

1

(II): $\frac{k+1}{k-1} \geq -1$.

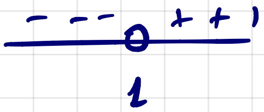
$$\frac{k+1}{k-1}+1 \geq 0 \Leftrightarrow \frac{k\cancel{+1}+k\cancel{-1}}{k-1} \geq 0 \Leftrightarrow \boxed{\frac{2k}{k-1} \geq 0}$$

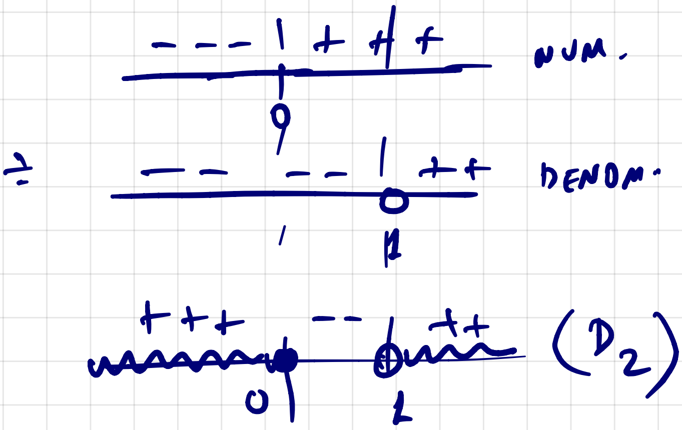
• ZEROS DO NUM: $2k=0 \Leftrightarrow k=0$



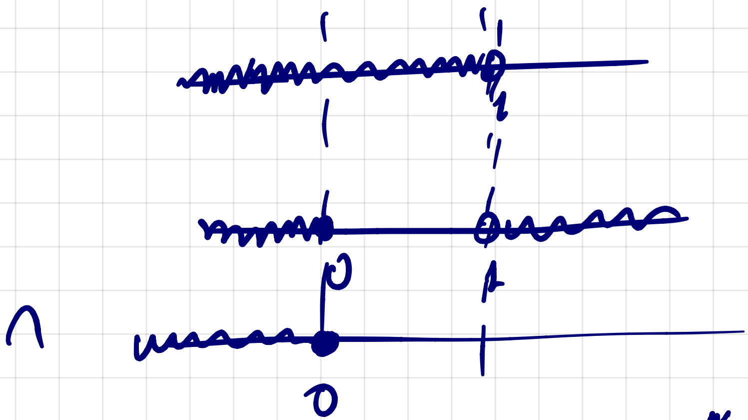
$$\begin{cases} \forall k>0 \Rightarrow 2k>0 \\ \forall k<0 \Rightarrow 2k<0 \end{cases}$$

• ZEROS DO DENOM.: (igual ao caso I)





SOLUÇÃO FINAL:



$k \in (-\infty, 0]$.

L1

3. As funções f e g cujas leis são

$$f(x) = \sqrt{\frac{x-1}{x+1}} \quad \text{e} \quad g(x) = \frac{\sqrt{x-1}}{\sqrt{x+1}}$$

são iguais? Justifique.

Depende dos domínios. Em geral, são diferentes.

A propriedade
$$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$$
é verdadeira, se $a \geq 0$ e $b > 0$

Ex: $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}}$.

Porém, para funções:

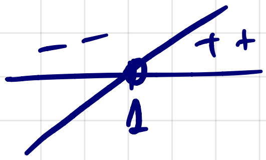
$\sqrt{\frac{f(x)}{g(x)}}$ tem sentido se $f(x), g(x) < 0$,

mas $\frac{\sqrt{f(x)}}{\sqrt{g(x)}}$ não, se $f(x), g(x) < 0$.

Pois para $f(x) = \sqrt{\frac{x-1}{x+1}}$, seu domínio é

onde $\frac{x-1}{x+1} \boxed{\geq 0}$.

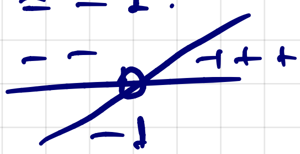
- zeros do num: $x - 1 = 0 \Leftrightarrow x = 1$.

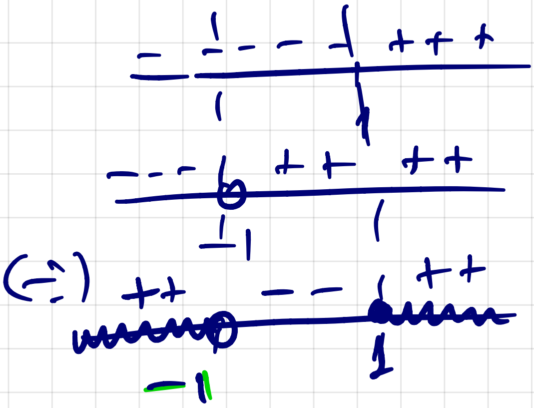


- zeros do denom:

$x + 1 = 0 \quad (\neq 0)$

$\Leftrightarrow x = -1$.





$D(f) = (-\infty, -1) \cup [1, +\infty)$

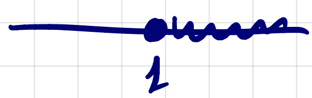
Por outro lado:

$$g(x) = \frac{\sqrt{x-1}}{\sqrt{x+1}}.$$

Neste caso, temos as condições:

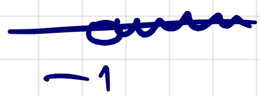
- $x - 1 \geq 0$

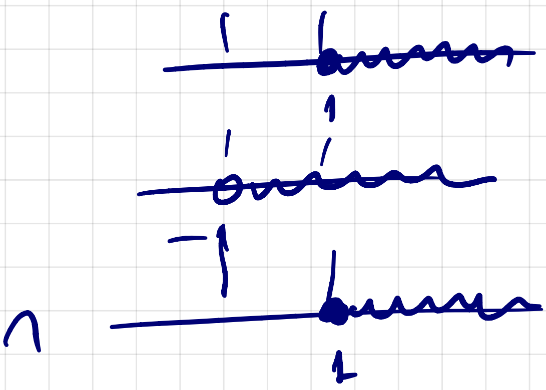
  $\hookrightarrow x \geq +1$



- $x + 1 > 0$

  $\hookrightarrow x > -1.$





$D(g) = [1, +\infty)$

LISTA 02

10. Determine o valor numérico de

(a) $y = \dfrac{\csc\frac{\pi}{6} + \operatorname{sen}\frac{\pi}{6}}{\operatorname{sen}\frac{\pi}{4} - \sec\frac{\pi}{3}}$ (b) $y = \dfrac{\cot^2\frac{\pi}{6} - \sqrt{2}\cdot\cos\frac{\pi}{4}}{\tan\frac{\pi}{3}\cdot\csc\frac{\pi}{6}}$

(b)

Temos:

$$\begin{cases} \bullet\ \cot\frac{\pi}{6} = \dfrac{1}{\tan\frac{\pi}{6}} = \dfrac{1}{\frac{\sqrt{3}}{3}} = \dfrac{3}{\sqrt{3}} \\ \bullet\ \cos\frac{\pi}{4} = \dfrac{\sqrt{2}}{2} \\ \bullet\ \tan\frac{\pi}{3} = \sqrt{3} \\ \bullet\ \csc\frac{\pi}{6} = \dfrac{1}{\operatorname{sen}\frac{\pi}{6}} = \dfrac{1}{\frac{1}{2}} = 2 \end{cases}$$

Assim:

$$y = \frac{\left(\frac{3}{\sqrt{3}}\right)^2 - \sqrt{2}\cdot\frac{\sqrt{2}}{2}}{\sqrt{3}\cdot 2} = \frac{\frac{9}{3} - \frac{2}{2}}{2\sqrt{3}} = \frac{3-1}{2\sqrt{3}}$$

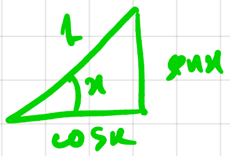
$$= \frac{\cancel{2}}{\cancel{2}\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

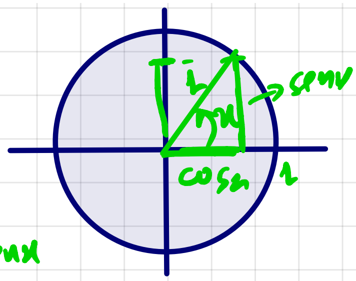
L2

22. Calcule o valor de m para que $\operatorname{sen} x = 2m + 1$ e $\cos x = 4m + 1$.

$$\operatorname{sen}^2 x + \cos^2 x = 1.$$

T. PITÁGORAS





Disso, obtemos:

$$(2m+1)^2 + (4m+1)^2 = 1.$$

$$4m^2 + 4m + 1 + 16m^2 + 8m + \cancel{1} = \cancel{1}$$

$$20m^2 + 12m + 1 = 0$$

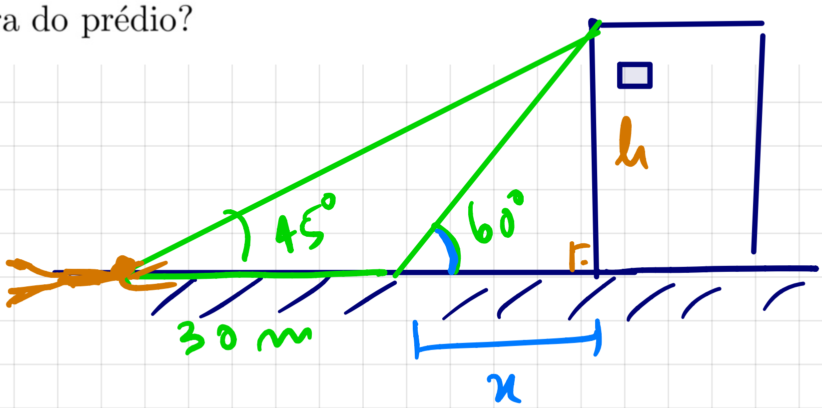
$$m = \frac{-12 \pm \sqrt{144 - 80}}{2.20}$$

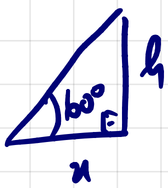
$$m = \frac{-12 \pm \sqrt{64}}{40} \quad \begin{cases} m = \dfrac{-12+8}{40} = \dfrac{1}{10} \\[2ex] m = \dfrac{-12-8}{40} = -\dfrac{1}{2} \end{cases}$$

Resp.: $m = \frac{1}{10}$ ou $m = -\frac{1}{2}$

L2:

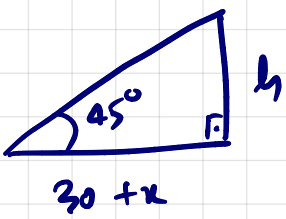
1. Um observador vê um prédio, construído em terreno plano, sob um ângulo de 60°. Afastando-se do edifício mais 30m, passa a ver o edifício sob um ângulo de 45°. Qual é a altura do prédio?





$$\tan 60° = \frac{h}{x} = h = x \cdot \tan 60°$$

$$\boxed{h = \sqrt{3}\, x.}$$



$$\underbrace{\tan 45°}_{1} = \frac{h}{30+x} \Rightarrow \boxed{h = 30 + x.}$$

$$\sqrt{3}x = 30 + x$$

$$\sqrt{3}x - x = 30$$

$$x(\sqrt{3}-1) = 30$$

$$x = \frac{30}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1} = \frac{30\cdot(1+\sqrt{3})}{3-1}$$

$$x = 15\cdot(1+\sqrt{3}) = 15 + 15\sqrt{3} \text{ m.}$$

Portanto, a altura h será:

$$h = 30 + x = 30 + 15 + 15\sqrt{3}$$

$$h = (45 + 15\sqrt{3})\text{ m}.$$

L1 || 18. As indicações R_1 e R_2, na *escala Richter*, de dois terremotos estão indicados pela fórmula

$$R_1 - R_2 = \log \frac{E_1}{E_2},$$

onde E_1 e E_2 medem a energia liberada pelos terremotos sob a forma de ondas que se propagam pela crosta terrestre.
A tabela abaixo mostra algumas medidas, onde alguns dados estão faltando. Complete a tabela, de acordo com as definições dadas e seus conhecimentos.

R_1	R_2	E_1	E_2
8	6	10	x
5	7	y	13
z	9	2	20
7	7	10	10

Olhando a 1ª linha, temos: $R_1 = 8$; $R_2 = 6$
$E_1 = 10$; $E_2 = x$

Neste caso,

$$R_1 - R_2 = \log \frac{E_1}{E_2}$$

$$8-6=\log\frac{10}{x} \Leftrightarrow \log_{10}\frac{10}{x}=2$$

$$\log_b a = c \Leftrightarrow b^c = a$$

$$\Leftrightarrow (10)^2=\frac{10}{x}$$

$$100=\frac{10}{x}$$

$$10\cancel{0}x = 1\cancel{0}$$

$$10x=1$$

$$x=\frac{1}{10} \Rightarrow \underline{\underline{x=0{,}1}}$$

Olhando a 2ª linha, temos:

$$R_1=5\ ;\ R_2=7\ ;\ E_1=y\ ;\ E_2=13\ :$$

Dai, obtemos:

$$R_1-R_2=\log\frac{E_1}{E_2}$$

$$5-7=\log\frac{y}{13} \Leftrightarrow \log_{10}\frac{y}{13}=-2$$

$$\therefore$$

$$\Leftrightarrow (10)^{-2}=\frac{y}{13}$$

$$\Leftrightarrow \frac{1}{100} \times \frac{y}{13}$$

$$\Leftrightarrow 100y=13$$

$$\Leftrightarrow y=\frac{13}{100}=\underline{\underline{0{,}13}}$$

Por fim, olhando a 3ª linha, temos:

$R_1 = z \quad R_2 = 9 \; ; \quad E_1 = 2 \quad E_2 = 20$

$$\Rightarrow R_1 - R_2 = \log \frac{E_1}{E_2}$$

$$z - 9 = \log_{10} \frac{2}{20}$$

$$z - 9 = \log_{10} \frac{1}{10} \Leftrightarrow z - 9 = \log_{10} 10^{-1} \quad (*)$$

$$\Leftrightarrow z - 9 = -1 .$$

$$\Leftrightarrow z = 9 - 1 \Leftrightarrow \boxed{z = 8}$$

$\log_a a^w = w$

ou: $\log_{10} 10^{-1} = z - 9$

$$10^{z-9} = 10^{-1}$$

$$\Leftrightarrow z - 9 = -1 \Leftrightarrow z = 8$$

L3 (9)

$$(\ell) \lim_{x \to -2} \frac{1 - \sqrt{x+3}}{\sqrt{x^2 + x - 1} - 1} = \frac{0}{0} \quad (\text{INDET.})$$

$$\lim_{x \to -2} \frac{1 - \sqrt{x+3}}{\sqrt{x^2+x-1} - 1} \times \frac{1 + \sqrt{x+3}}{1 + \sqrt{x+3}} \times \frac{\sqrt{x^2+x-1} + 1}{\sqrt{x^2+x-1} + 1} =$$

$$= \lim_{x \to -2} \frac{\left[(1)^2 - \left(\sqrt{x+3}\right)^2\right] \left(\sqrt{x^2+x-1} + 1\right)}{\left[\left(\sqrt{x^2+x-1}\right)^2 - (1)^2\right] \cdot \left(1 + \sqrt{x+3}\right)} =$$

$$= \lim_{x\to -2} \frac{(1-(x+3))(\sqrt{x^2+x-1}+1)}{(x^2+x-1-1)\cdot(1+\sqrt{x+3})}$$

$$= \lim_{x\to -2} \frac{(-x-2)(\sqrt{x^2+x-1}+1)}{(x^2+x-2)(1+\sqrt{x+3})} \Leftrightarrow$$

$$\begin{array}{l|l} \cancel{x^2}+x-2 & x+2 \\ \hline -x^2-2x & x-1 \\ \hline -x-\cancel{2} & \\ +x\cancel{+2} & \\ \hline 0 & \end{array} \qquad \div \; x-(-2)$$

$$\Rightarrow \quad x^2+x-2 = (x+2)(x-1)$$

$$\Leftrightarrow \lim_{x\to -2} \frac{-\cancel{(x+2)}(\sqrt{x^2+x-1}+1)}{\cancel{(x+2)}(x-1)\cdot(1+\sqrt{x+3})} =$$

$$= \lim_{x\to -2} \frac{-(\sqrt{x^2+x-1}+1)}{(x-1)\cdot(1+\sqrt{x+3})} = \frac{-(\sqrt{(-2)^2-2-1}+1)}{(-2-1)\cdot(1+\sqrt{-2+3})}$$

$$= \frac{-(1+1)}{-3\cdot(1+1)} = \frac{-\cancel{2}}{-3\cdot\cancel{2}} = +\frac{1}{3}$$