GABARITO DA PROVA 3 DE C2:

01) $$\int_0^{+\infty} x \cdot e^{-x^2}\,dx = ?$$

1,0

- $$\int x \cdot e^{-x^2}\,dx = \int e^{u} \left(-\frac{du}{2}\right) = -\frac{1}{2} e^{u} + c = -\frac{1}{2} e^{-x^2} + c$$

$$u = -x^2 \Rightarrow du = -2x\,dx \Rightarrow x\,dx = -\frac{du}{2}$$

Assim:

$$\int_0^{+\infty} x\, e^{-x^2}\,dx = \lim_{b \to +\infty} \int_0^{b} x\, e^{-x^2}\,dx = \lim_{b \to +\infty} \left(-\frac{1}{2} e^{-x^2}\right)\Bigg|_0^{b}$$

$$= \lim_{b \to +\infty} \left(-\frac{1}{2} e^{-b^2} + \frac{1}{2} e^{0}\right) = \lim_{b \to +\infty} \left(\underbrace{-\frac{1}{2e^{b^2}}}_{\to 0} + \frac{1}{2}\right) = \frac{1}{2}$$

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02) $$l = \int_0^{\frac{\pi}{3}} \sqrt{1 + [f'(x)]^2}\,dx$$

$$f(x) = \ln \sec x \Rightarrow f'(x) = \frac{\cancel{\sec x} \cdot \tan x}{\cancel{\sec x}}$$

$$\Rightarrow f'(x) = \tan x.$$

Assim, teremos:

$$\underset{\sim}{l} = \int_0^{\frac{\pi}{3}} \sqrt{1+\tan^2 x}\, dx = \int_0^{\frac{\pi}{3}} \sqrt{\cancel{\sec^2 x}}\, . dx$$

$$= \int_0^{\frac{\pi}{3}} \sec x\, dx = \ln|\sec x + \tan x|\Big|_0^{\frac{\pi}{3}} =$$

1,0

$$= \ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln|2+\sqrt{3}| - \underbrace{\ln|1+0|}_{=0} = \underset{\sim}{\ln(2+\sqrt{3})}$$

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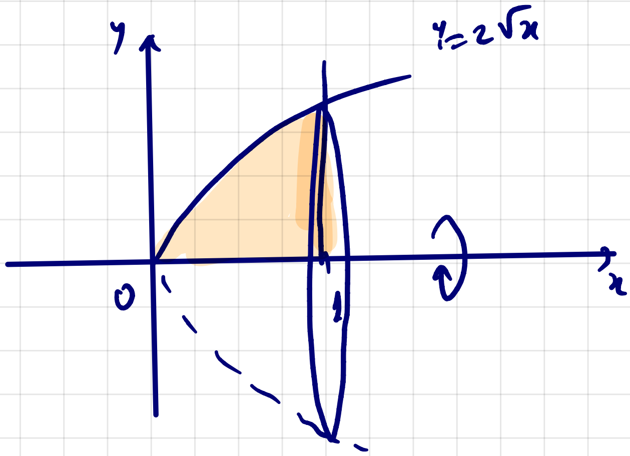
03)



$$V = \pi . \int_0^1 [f(x)]^2\, dx =$$

$$= \pi . \int_0^1 (2\sqrt{x})^2 dx$$

$$= \pi \int_0^1 4x\, dx = 4\pi \frac{x^2}{2}\Big|_0^1$$

$$= 4\pi . \left(\frac{1}{2} - 0\right) = \underset{\sim}{2\pi\, u.a.}$$

1.0

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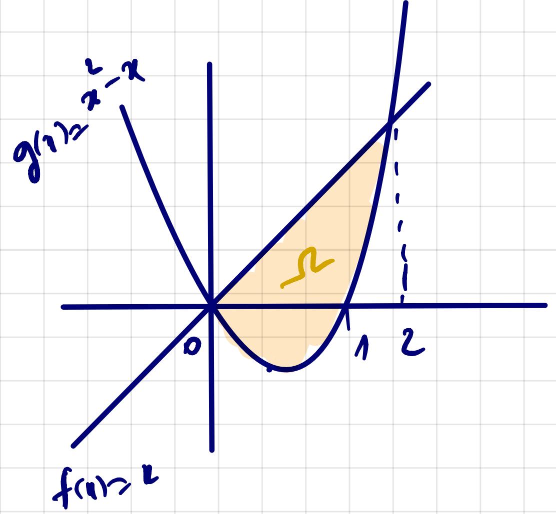
04)



interceptos:

$$x^2 - x = x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$\left[\begin{array}{l} x = 0 \\ x = 2 \end{array}\right.$$

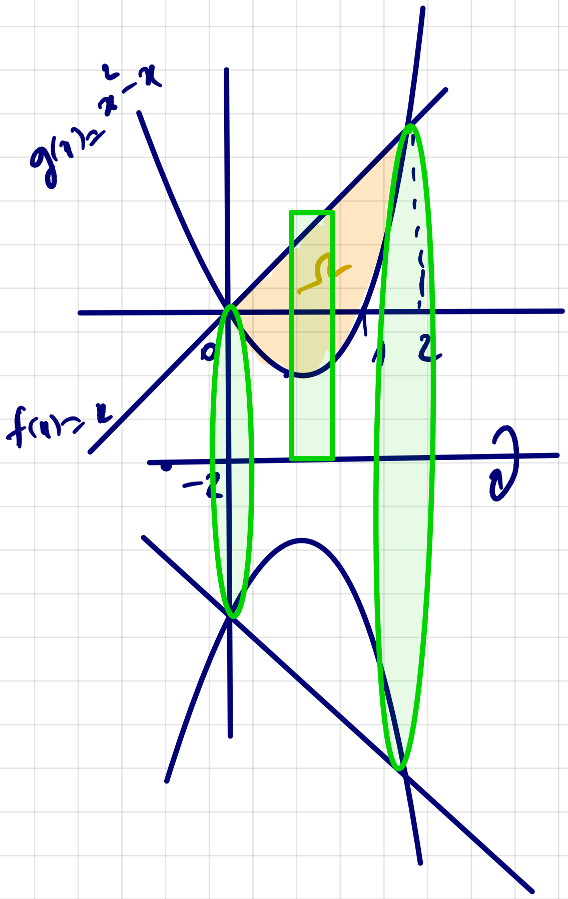
(a)

$$A = \int_0^2 (f(x) - g(x))\,dx = \int_0^2 (x - (x^2 - x))\,dx =$$

1.0

$$= \int_0^2 (2x - x^2)\,dx = \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = 4 - \frac{8}{3} - 0 = \frac{4}{3}\ u.a.$$

(b)



$V = V_1 - V_2$; onde

$$V_1 = \pi \int_0^2 [f(x) - (-2)]^2\,dx$$

$$= \pi \int_0^2 [x+2]^2\,dx =$$

$$= \pi \left.\frac{(x+2)^3}{3}\right|_0^2 =$$

1,5

$$= \pi \cdot \frac{(4)^3}{3} - \pi \cdot \frac{(2)^3}{3} = \frac{64\pi}{3} - \frac{8\pi}{3} = \frac{56\pi}{3}.$$

$$V_2 = \pi \int_0^2 [g(x) - (-2)]^2 dx = \pi \int_0^2 [x^2 - x + 2]^2 dx$$

$$= \pi \int_0^2 (x^2 + (2-x))^2 dx = \pi \int_0^2 \left[x^4 - 2x^2(2-x) + (2-x)^2\right] \cdot dx$$

$$= \pi \cdot \int_0^2 (x^4 - 2x^2 + 2x^3 + 4 - 4x + x^2) dx$$

$$= \pi \int_0^2 (x^4 + 2x^3 - x^2 - 4x + 4) dx = \pi \cdot \left(\frac{x^5}{5} + \frac{2x^4}{4} - \frac{x^3}{3} - \frac{4x^2}{2} + 4x\right)\Big|_0^2$$

$$= \pi \cdot \left(\frac{x^5}{5} + \frac{x^4}{2} - \frac{x^3}{3} - 2x^2 + 4x\right)\Big|_0^2 =$$

$$= \pi \cdot \left[\frac{32}{5} + \frac{16}{2} - \frac{8}{3} - \cancel{8} + \cancel{8} - 0\right] =$$

$$= \pi \cdot \left(\frac{32}{5} + 8 - \frac{8}{3}\right) = \pi \cdot \frac{96 + 120 - 40}{15} = \frac{176\pi}{15}$$

Luego, obtenemos:

$$V = V_1 - V_2 = \frac{56\pi}{3} - \frac{176\pi}{15} = \frac{280\pi - 176\pi}{15}$$

$$V = \frac{104\pi}{15} \text{ u.m.}$$

05) $x_n = \dfrac{2^n}{1+2^n}$.

AF-01: (x_n) é monótona. De fato:

$$\frac{x_{n+1}}{x_n} = \frac{2^{n+1}}{1+2^{n+1}} \times \frac{1+2^n}{2^n} = \frac{\cancel{2^n}.2.(1+2^n)}{(1+2^{n+1}).\cancel{2^n}}$$

$$= \frac{2+2^n.2}{1+2^{n+1}} = \frac{2+2^{n+1}}{1+2^{n+1}} = \frac{1}{1+2^{n+1}} + \frac{\cancel{1+2^{n+1}}}{\cancel{1+2^{n+1}}}$$

$$= \frac{1}{1+2^{n+1}} + 1 > 1.$$

1,5

$$\Rightarrow \frac{x_{n+1}}{x_n} > 1,\ \forall n \Rightarrow x_{n+1} > x_n,\ \forall n.$$

Logo, a seq. (x_n) é monótona, mais precisamente, é crescente.

AF02: (x_n) é limitada (superiormente)

De fato; como $2^n < 1+2^n$, então

$$\frac{1}{2^n} > \frac{1}{1+2^n}\text{; e assim:}$$

$$x_n = \frac{2^n}{1+2^n} = 2^n.\frac{1}{1+2^n} < \cancel{2^n}.\frac{1}{\cancel{2^n}} = 1$$

$$\Rightarrow x_n < 1,\ \forall n.$$

Assim, pelos AF. 01 e 02 segue que (a_n) é convergente.

06) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+2)} = \frac{3}{4}$:

$$\frac{1}{n(n+2)} = \frac{A}{n} + \frac{B}{n+2} = \frac{A(n+2)+B.n}{n(n+2)}$$

$$\Leftrightarrow \begin{cases} A+B=0 \\ 2A = 1 \end{cases} \Rightarrow A = \frac{1}{2}$$

$$B = -\frac{1}{2}$$

Disso, obtemos: $\displaystyle\frac{1}{n(n+2)} = \frac{1}{2n} - \frac{1}{2n+4} = a_n$

Seja s_n a soma parcial da série:

$$s_n = \sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{1}{2k} - \frac{1}{2k+4}$$

Então:

$$s_n = \underbrace{\frac{1}{2} - \cancel{\frac{1}{6}}}_{k=1} + \underbrace{\frac{1}{4} - \cancel{\frac{1}{8}}}_{k=2} + \underbrace{\cancel{\frac{1}{6}} - \cancel{\frac{1}{10}}}_{k=3} + \underbrace{\cancel{\frac{1}{8}} - \cancel{\frac{1}{12}}}_{k=4} + \cdots$$

$$\cdots + \underbrace{\cancel{\frac{1}{2n}} - \frac{1}{2n+4}}_{k=n}$$

$\Lambda_c 0$

$$\Rightarrow s_n = \frac{1}{2} + \frac{1}{4} - \frac{1}{2n+4} = \frac{3}{4} - \frac{1}{2n+4}$$

Então, a soma s da série será:

$$s = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{3}{4} - \frac{1}{2n+4} = \frac{3}{4}$$

07) a) $\displaystyle\sum_{n=1}^{+\infty} \frac{3^n}{n!\,n}$

Pelo teste da razão temos:

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{3^{n+1}}{(n+1)!\,.(n+1)} \cdot \frac{n!\,.n}{3^n} =$$

$$= \lim_{n\to+\infty} \frac{\cancel{3^n}.\,3\,.\,\cancel{n!}\,n}{(n+1).\cancel{n!}\,(n+1).\cancel{3^n}} = \lim_{n\to\infty} \frac{3n}{n^2+2n+1}$$

1.0

$$= \lim_{n\to\infty} \frac{3n}{n^2} = \lim_{n\to\infty} \frac{3}{n} = 0 < 1.$$

Logo a série dada converge.

(b) $\sum_{n=1}^{\infty} \left(\frac{2n+3}{3n+2}\right)^n$.

Pelo teste da raiz:

$$\lim_{n\to+\infty} \sqrt[n]{a_n} = \lim_{n\to+\infty} \sqrt[\cancel{n}]{\left(\frac{2n+3}{3n+2}\right)^{\cancel{n}}} = \lim_{n\to+\infty} \frac{2n+3}{3n+2} = \frac{2}{3} < 1.$$

Logo, a série dada converge.

(c) $\sum_{n=2}^{+\infty} \frac{1}{\sqrt{n^2-1}}$. Note que

$$n = \sqrt{n^2} > \sqrt{n^2-1}$$

$$\Rightarrow \frac{1}{n} < \frac{1}{\sqrt{n^2-1}}, \quad \forall n \geq 2$$

1.0

DIV.

Como a série harmônica $\sum \frac{1}{n}$ é divergente, segue pelo teste da comparação que a série $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^2-1}}$ também é divergente.