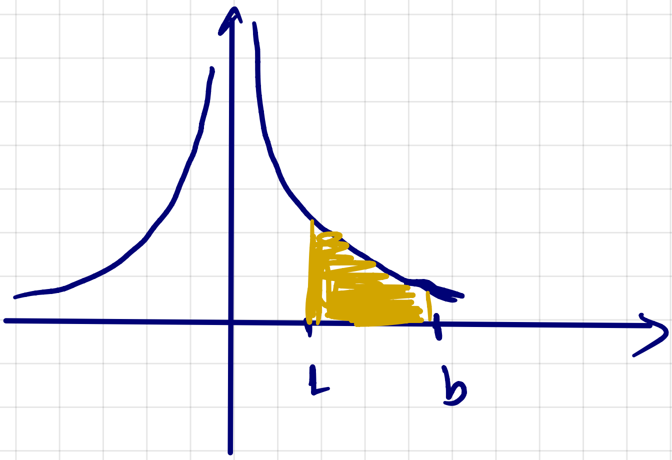
Cálculo 2

08/03/24 - AULA 24 (EXTRA)

Lista 09

2. Calcule a área acima do eixo x limitada por $y = \frac{1}{x^2}$, $x = 1$ e $x = b$, onde b é algum número maior do que 1. O resultado dependerá do valor de b. O que acontece com essa área quando $b \to +\infty$?



$, b > 1$

$$A = \int_1^b f(x)\,dx = \int_1^b \frac{1}{x^2}\,dx =$$

$$= \int_1^b x^{-2}\,dx = ?$$

- $\int x^{-2}\,dx = \frac{x^{-2+1}}{-2+1} + C = \frac{x^{-1}}{-1} + C = -\frac{1}{x} + C$

$$\int r^k\,dr = \frac{r^{k+1}}{k+1} + C$$

Dessa forma, obtemos:

$$\int_1^b \frac{dx}{x^2} = \left(-\frac{1}{x}\right)\Bigg|_1^b = -\frac{1}{b} - \left(-\frac{1}{1}\right) =$$

$$= 1 - \frac{1}{b}.$$

$$\lim_{b \to +\infty} A = ?$$

$$\lim_{b \to +\infty} \left(1 - \underbrace{\frac{1}{b}}_{\to 0}\right) = 1.$$

Ou seja, a área se aproxima de 1 quando $b \to +\infty$.

3. Esboçar a região entre as curvas e ache as áreas compreendidas:

(a) $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$.

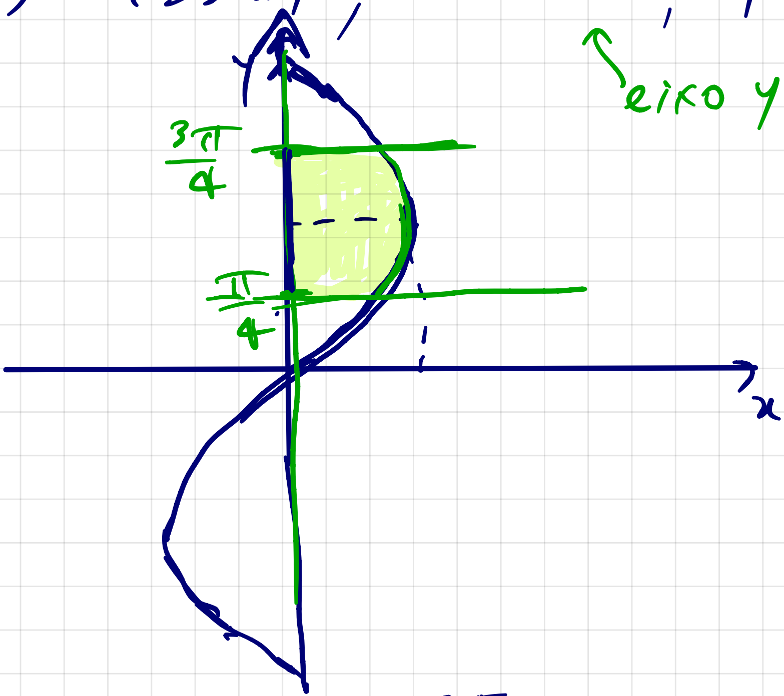
(b) $y = x^3 - 4x$, $y = 0$, $x = 0$, $x = 2$.

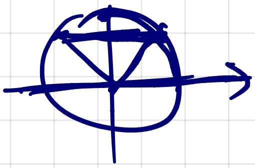
(c) $x = \operatorname{sen} y$, $x = 0$, $y = \frac{\pi}{4}$, $y = \frac{3\pi}{4}$.

(d) $y = e^x$, $y = e^{2x}$, $x = 0$, $x = \ln 2$.

(c) $x = \operatorname{sen} y$; $x = 0$; $y = \frac{\pi}{4}$; $y = \frac{3\pi}{4}$

eixo y





$$A = \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \operatorname{sen} y \, dy = -\cos y \Big|_{\frac{\pi}{4}}^{\frac{3\pi}{4}} = -\cos\frac{3\pi}{4} - \left(-\cos\frac{\pi}{4}\right)$$

$\int \operatorname{sen} u \, du = -\cos u + C$

$u = y \Rightarrow du = dy$

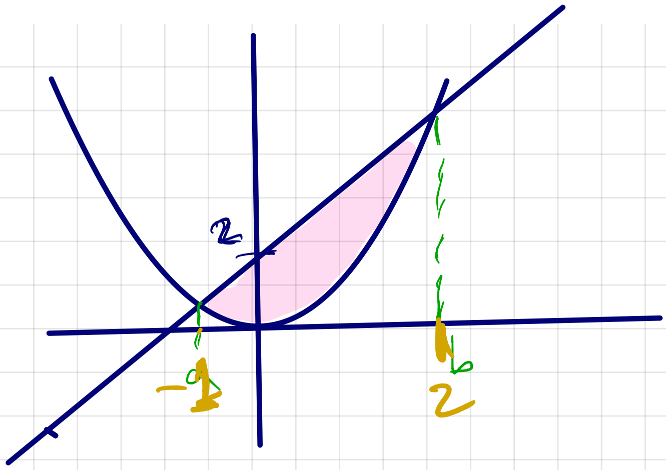
$$= -\left(-\frac{\sqrt{2}}{2}\right) + \frac{\sqrt{2}}{2} =$$

$$= \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$$

$A = \sqrt{2}$ unidades de área.

L9

4. Calcule a área formada pelas curvas $y = x^2$ e $y = x + 2$.



intersectos : $y = y$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$x = \frac{+1 \pm \sqrt{1+8}}{2} = \frac{1 \pm 3}{2}$$

$$\begin{cases} x = \frac{4}{2} = 2 \\ x = \frac{-2}{2} = -1 \end{cases}$$

$$A = \int_{-1}^{2} (x+2)\,dx - \int_{-1}^{2} x^2\,dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Bigg|_{-1}^{2}$$

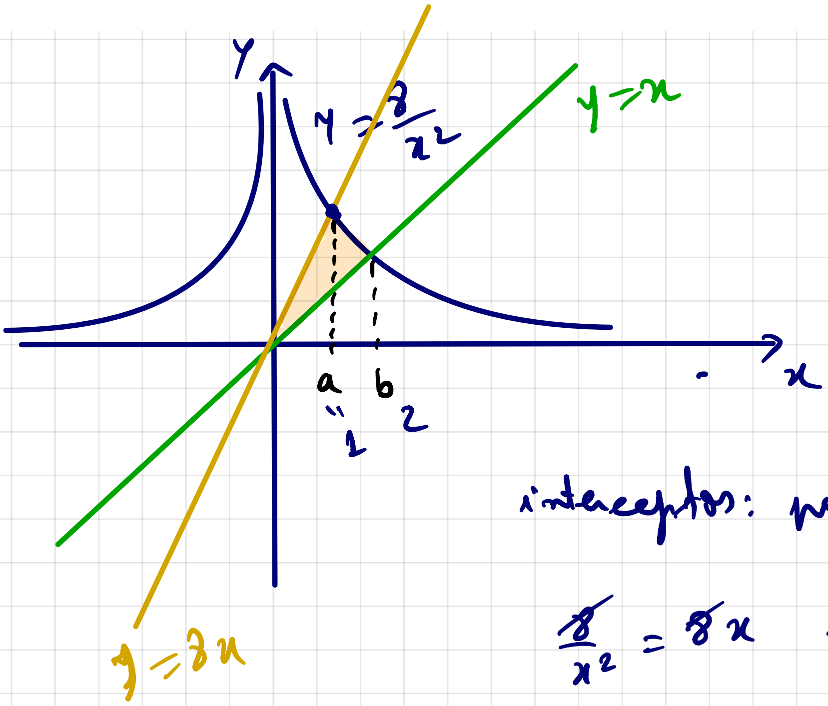
$$= \frac{(2)^{\cancel{2}}}{\cancel{2}} + 2\cdot(2) - \frac{(2)^3}{3} - \left(\frac{(-1)^2}{2} + 2\cdot(-1) - \frac{(-1)^3}{3}\right)$$

$$= 2 + 4 - \frac{8}{3} - \frac{1}{2} + 2 - \frac{1}{3}$$

$= 8 - 3 - \frac{1}{2} = 5 - \frac{1}{2} = \frac{10-1}{2} =$

$\frac{9}{2}$ unidades de área.

10. Obtenha a área da região limitada pelas curvas $y = \dfrac{8}{x^2}$, $y = 8x$ e $y = x$.



interceptos: para achar "a":

$\frac{\cancel{8}}{x^2} = \cancel{8}x \quad \div 8$

$\frac{1}{x^2} = x \iff 1 = x^3$

$\iff \boxed{x = 1}$

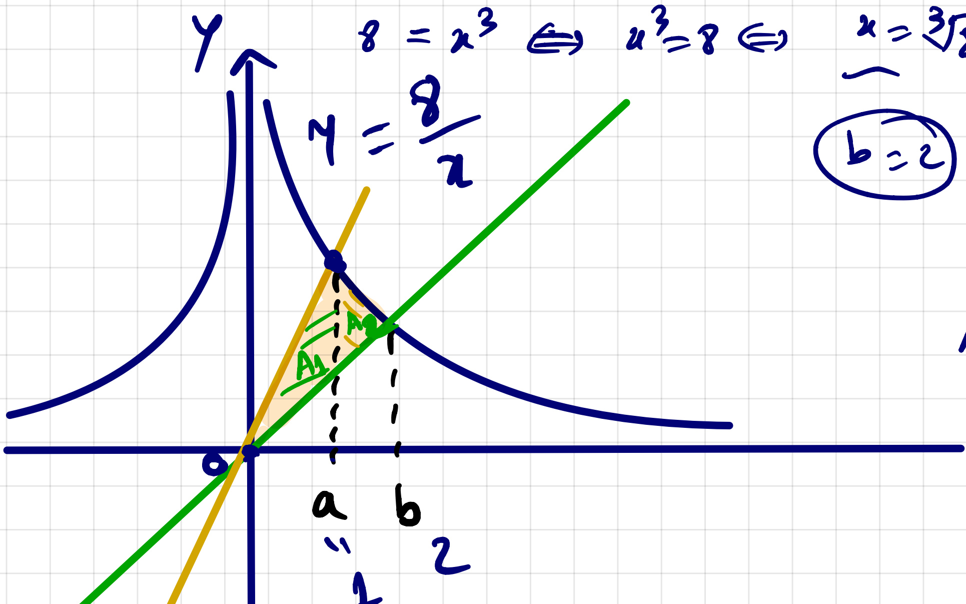
$a = 1$

para achar "b":

$\frac{8}{x^2} = x$

$8 = x^3 \iff x^3 = 8 \iff x = \sqrt[3]{8} = \sqrt[3]{2^3} = 2$

$b = 2$



$A = A_1 + A_2$

$$A_1 = \int_0^1 (8x - x)\,dx = \int_0^1 7x\,dx = 7\frac{x^2}{2}\Big|_0^1 = \frac{7}{2}.$$

$$A_2 = \int_1^2 \frac{8}{x^2}\,dx - \int_1^2 x\,dx = 8 \cdot \int_1^2 x^{-2}\,dx - \int_1^2 x\,dx$$

$$= \left(8\,\frac{x^{-1}}{-1} - \frac{x^2}{2}\right)\Big|_1^2 = \left(-\frac{8}{x} - \frac{x^2}{2}\right)\Big|_1^2 =$$

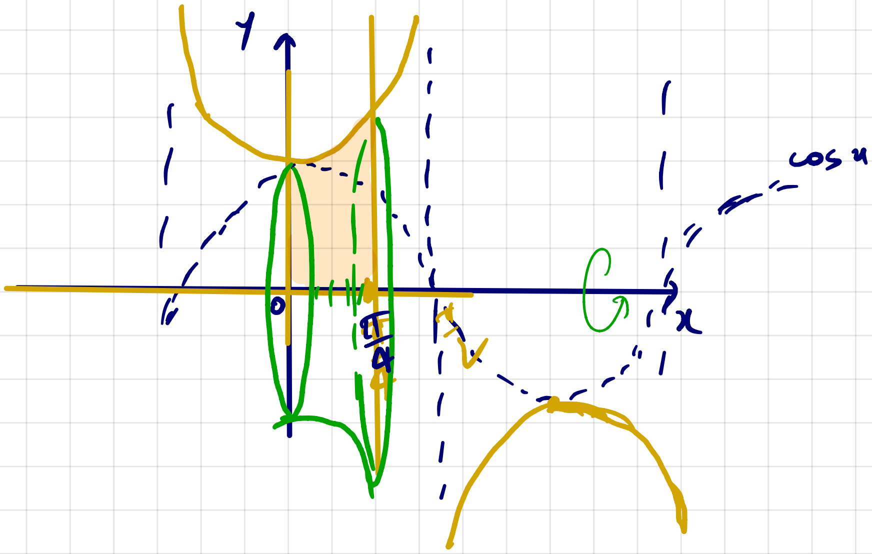
$$= -\frac{8}{2} - \frac{4}{2} - \left(-\frac{8}{1} - \frac{1}{2}\right) =$$

$$= -4 - 2 + 8 + \frac{1}{2} = 2 + \frac{1}{2} = \frac{5}{2}$$

Portanto, a medida da área procurada será:

$$A = A_1 + A_2 = \frac{7}{2} + \frac{5}{2} = \frac{12}{2} = 6 \text{ unidades de área}$$

13. A região limitada pela curva $y = \sec x$, pelo eixo x, pelo eixo y e pela reta $x = \frac{\pi}{4}$ gira em torno do eixo x. Determine o volume do sólido gerado.



$$\sec x = \frac{1}{\cos x}$$

$$V = \pi \cdot \int_0^{\frac{\pi}{4}} [f(x)]^2 dx = \pi \cdot \int_0^{\frac{\pi}{4}} (\sec x)^2 dx =$$

$$\pi \int_0^{\frac{\pi}{4}} \sec^2 x \, dx = \pi \cdot (\tan x)\Big|_0^{\frac{\pi}{4}} = \pi \cdot \left(\tan\frac{\pi}{4} - \tan 0\right)$$

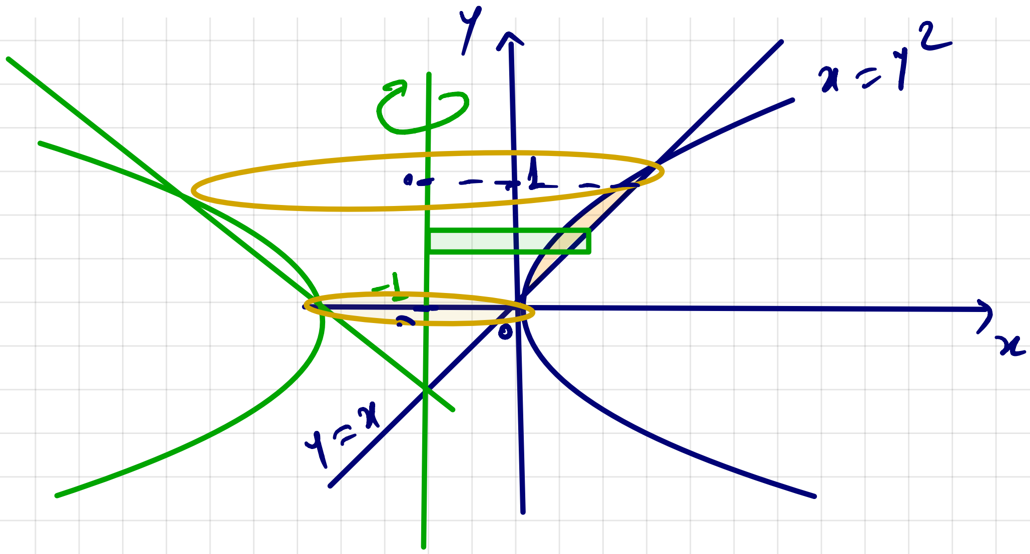
$$(\tan u)' = \sec^2 u \cdot u'$$

$$\int \sec^2 u \, du = \tan u + C$$

$$= \pi \cdot (1 - 0)$$

$= \pi$ unidades de volume.

15. Ache o volume do sólido que resulta quando a região limitada por $x = y^2$ e $x = y$ é feita girar em torno da reta $y = -1$.



interceptos: $y^2 = y$ $(x = x)$

$$y^2 - y = 0$$

$$y(y-1) = 0$$

$$\begin{cases} y = 0 \\ y = 1 \end{cases}$$

$V = V_{MAIOR} - V_{MENOR}$; onde:

- V_{MAIOR} é dado pela reta
- V_{MENOR} é dado pela parábola.

$$V_{MAIOR} = \pi \cdot \int_0^1 [\underbrace{f(y)}_{y}+1]^2\,dy = \pi \int_0^1 [y+1]^2\,dy =$$

$$\left[\begin{array}{l} u = y+1 \\ du = dy \end{array}\right. \quad \int u^k\,du$$

$$= \pi \left.\frac{(y+1)^3}{3}\right|_0^1 = \pi \cdot \frac{(1+1)^3}{3} - \pi \cdot \frac{(0+1)^3}{3}$$

$$= \frac{8\pi}{3} - \frac{\pi}{3} = \frac{7\pi}{3} \text{ u.v.}$$

$$V_{MENOR} = \pi \cdot \int_0^1 [g(y)+1]^2 dy = \pi \int_0^1 \cdot [y^2+1]^2 \cdot dy$$

$$= \pi \cdot \int_0^1 (y^4 + 2y^2 + 1)\, dy = \pi \cdot \left(\frac{y^5}{5} + 2\frac{y^3}{3} + y \right) \Big|_0^1 =$$

$$= \pi \cdot \left[\frac{1}{5} + \frac{2}{3} + 1 - 0 \right] = \pi \cdot \left(\frac{3+10+15}{15} \right) = \frac{28\pi}{15}$$

Portanto, a medida ~~do~~ volume V será:

$$V = V_{MAIOR} - V_{MENOR} = \frac{7\pi}{3} - \frac{28\pi}{15} = \frac{35\pi - 28\pi}{15}$$

$$= \frac{7\pi}{15} \text{ unidades de volume.}$$