CÁLCULO 1 – Resolução de algumas questões da Lista 04.

01) (f) $\lim_{x\to+\infty} x.(\sqrt{x^2+1}-x) = \infty.(\infty-\infty)$ INDETERMINAÇÃO

Vamos racionalizar para transformar a indeterminação $\infty-\infty$ em outra forma de indeterminação que possa ser removida por algum tipo de simplificação:

$$\lim_{x\to+\infty} x.(\sqrt{x^2+1}-x) = \lim_{x\to+\infty} x.(\sqrt{x^2+1}-x).\frac{(\sqrt{x^2+1}+x)}{\sqrt{x^2+1}+x} =$$

$$= \lim_{x\to+\infty} \frac{x.[(\sqrt{x^2+1})^2 - x^2]}{\sqrt{x^2+1}+x} = \lim_{x\to+\infty}\frac{x.(\cancel{x^2}+1-\cancel{x^2})}{\sqrt{x^2+1}+x}$$

$$= \lim_{x\to+\infty}\frac{x}{\sqrt{x^2+1}+x} \overset{\frac{\infty}{\infty}}{=} \lim_{x\to+\infty}\frac{\frac{x}{x}}{\frac{\sqrt{x^2+1}+x}{x}} =$$

TRANSFORMOU-SE EM INDETERMINAÇÃO DA FORMA $\frac{\infty}{\infty}$.

VAMOS AGORA DIVIDIR NUMERADOR E DENOMINADOR POR x:

$$= \lim_{x\to+\infty}\frac{1}{\frac{\sqrt{x^2+1}}{x}+1} = \lim_{x\to+\infty}\frac{1}{\sqrt{\frac{x^2+1}{x^2}}+1} = \lim_{x\to+\infty}\frac{1}{\sqrt{1+\frac{1}{x^2}}+1}$$

$\frac{1}{x^2} \to 0$

COMO $x\to+\infty$, então $x=\sqrt{x^2}$ (introduzimos x no radical)

$$= \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

01) (g) $\lim\limits_{x \to -1} \dfrac{2}{x+1} = \dfrac{2}{0}$

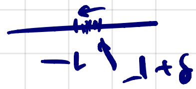
PRECISAREMOS VER OS LIMITES LATERAIS PARA DECIDIR SE EXISTE OU NÃO (POIS RESULTOU EM DIVISÃO POR ZERO)

- $\lim\limits_{x \to -1^+} \dfrac{2}{x+1} = ?$

Tome $\delta > 0$ e considere $x = -1+\delta$

Então, $x \to -1^+ \iff \delta \to 0^+$ $(\delta > 0)$

Assim:

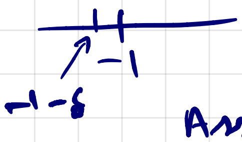


$$\lim_{x \to -1^+} \frac{2}{x+1} = \lim_{\delta \to 0^+} \frac{2}{-1+\delta+1} = \lim_{\delta \to 0^+} \frac{2}{\delta} = \frac{2}{0^+} = +\infty$$

Logo, $\boxed{\lim\limits_{x \to -1^+} \dfrac{2}{x+1} = +\infty}$ $(*)$

- $\lim\limits_{x \to -1^-} \dfrac{2}{x+1} = ?$

Tome $\delta > 0$ e considere $x = -1-\delta$.



Então: $x \to -1^- \iff \delta \to 0^+$ $(\delta > 0)$.

Assim:

$$\lim_{x \to -1^-} \frac{2}{x+1} = \lim_{\delta \to 0^+} \frac{2}{-1-\delta+1} = \lim_{\delta \to 0^+} \frac{2}{-\delta} = -\infty.$$

Logo, $\boxed{\lim\limits_{x \to -1^-} \dfrac{2}{x+1} = -\infty}$ $(**)$

De $(*)$ e $(**)$ segue que $\nexists \lim\limits_{x \to -1} \dfrac{2}{x+1}$.

01) (i) $\lim\limits_{x\to 0} \dfrac{1}{x^2} = \dfrac{1}{0}$

PRECISAREMOS VER OS LIMITES LATERAIS PARA DECIDIR SE EXISTE OU NÃO (POIS RESULTOU EM DIVISÃO POR ZERO)

- $\lim\limits_{x\to 0^+} \dfrac{1}{x^2} = \dfrac{1}{(0^+)^2} = +\infty$
- $\lim\limits_{x\to 0^-} \dfrac{1}{x^2} = \dfrac{1}{(0^-)^2} = \dfrac{1}{0^+} = +\infty$

logo, $\lim\limits_{x\to 0} \dfrac{1}{x^2} = +\infty$.

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01) (k) $\lim\limits_{x\to 0} \dfrac{\operatorname{sen} 5x}{\operatorname{sen} 7x} = \dfrac{0}{0}$ (INDETERM.)

COMO ENVOLVE NÚMEROS TRIGONOMÉTRICOS, PRECISAMOS CHEGAR AO 1º LIMITE NOTÁVEL:

$$\lim_{x\to 0} \frac{\operatorname{sen} 5x}{\operatorname{sen} 7x} = \lim_{x\to 0} \frac{5x \cdot \underbrace{\frac{\operatorname{sen} 5x}{5x}}_{\to 1}}{7x \cdot \underbrace{\frac{\operatorname{sen} 7x}{7x}}_{\to 1}} = \frac{5}{7}.$$

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01) (l) $\lim\limits_{x\to \frac{1}{2}} \dfrac{\operatorname{sen}(2x-1)}{2-4x} = \dfrac{\operatorname{sen} 0}{0} = \dfrac{0}{0}$ (INDET.)

$$\lim_{x\to \frac{1}{2}} \frac{\operatorname{sen}(2x-1)}{2-4x} = \lim_{x\to \frac{1}{2}} \underbrace{\frac{\operatorname{sen}(2x-1)}{-2(2x-1)}}_{\to 1} = -\frac{1}{2}$$

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01) (p) $\lim\limits_{x\to 0} \dfrac{\operatorname{arctan} 2x}{\operatorname{sen} 3x} = \dfrac{\operatorname{arctan} 0}{\operatorname{sen} 0} = \dfrac{0}{0}$ (INDETERM.)

$$= \lim_{x\to 0} \frac{\cancel{2x} \cdot \underbrace{\frac{\operatorname{arctan} 2x}{2x}}_{\to 1}}{\cancel{3x} \cdot \underbrace{\frac{\operatorname{sen} 3x}{3x}}_{\to 1}} = \frac{2}{3}$$

01) (1) $\lim\limits_{x\to 0} \dfrac{1-\cos 2x}{x\cdot\tan 3x} = \lim\limits_{x\to 0} \dfrac{\frac{\operatorname{sen}^2 2x}{1+\cos 2x}}{x\cdot \tan 3x} =$ $\quad \frac{0}{0}$ (INDETERM.)

NOTE QUE: $\operatorname{sen}^2\alpha + \cos^2\alpha = 1$

$\Rightarrow 1-\cos^2\alpha = \operatorname{sen}^2\alpha$

$(1-\cos\alpha)\cdot(1+\cos\alpha) = \operatorname{sen}^2\alpha$

$\Rightarrow 1-\cos\alpha = \dfrac{\operatorname{sen}^2\alpha}{1+\cos\alpha}$

$$= \lim_{x\to 0} \frac{\operatorname{sen}^2 2x}{1+\cos 2x}\cdot\frac{1}{x\cdot\tan 3x} = \lim_{x\to 0} \frac{\operatorname{sen} 2x\cdot\operatorname{sen} 2x}{1+\cos 2x} \cdot \frac{1\cdot 2 \cdot 2x}{2x \cdot \tan 3x \cdot 2x}$$

$$= \lim_{x\to 0} \frac{4x\cdot 3}{(1+\cos 2x)\,\tan 3x \cdot 3} = \frac{4}{(1+\cos 0)\cdot 3} = \frac{4}{2\cdot 3} = \frac{2}{3}$$

01) (2) $\lim\limits_{x\to 0} \left(\dfrac{x^2+3x+1}{x^3+1}\right)^{\frac{1}{x}} = \quad 1^{\infty}$ (INDETERM.)

ESTE SÍMBOLO DE INDETERM. SUGERE O USO DO 2º LIMITE NOTÁVEL.

$$\lim_{x\to 0}\left(\frac{x^2+3x+1}{x^3+1}\right)^{\frac{1}{x}} = \lim_{x\to 0}\left(1+\frac{x^2+3x+1}{x^3+1}-1\right)^{\frac{1}{x}} =$$

$$= \lim_{x\to 0}\left(1+\frac{x^2+3x+\cancel{1}-x^3-\cancel{1}}{x^3+1}\right)^{\frac{1}{x}} = \lim_{x\to 0}\left(1+\frac{x^2+3x-x^3}{x^3+1}\right)^{\frac{1}{x}} =$$

$$\lim_{x\to 0}\left(1+\text{expr.}\right)^{\frac{1}{\text{expr.}}} = e$$

$$= \left[\underbrace{\lim_{x\to 0}\left(1+\frac{x^2+3x-x^3}{x^3+1}\right)^{\frac{x^3+1}{x^2+3x-x^3}}}_{\to e}\right]^{\frac{x^2+3x-x^3}{x^3+1}\cdot\frac{1}{x}} =$$

$$= e^{\lim\limits_{x\to 0}\frac{x^2+3x-x^3}{x(x^3+1)}} = e^{\lim\limits_{x\to 0}\frac{x(x+3-x^2)}{x(x^3+1)}} = e^{\lim\limits_{x\to 0}\frac{x+3-x^2}{x^3+1}} = e^3$$

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01) (β) $\lim\limits_{x\to 0} \dfrac{2x+\operatorname{sen}3x}{x^3}\cdot \ln \sec x = \underbrace{\dfrac{0}{0}}_{\text{INDET.}} \cdot \underbrace{\ln 1}_{=0}$

lembre que:
$a.\ln b = \ln b^a$.

$$= \lim_{x\to 0} \ln(\sec x)^{\frac{2x+\operatorname{sen}3x}{x^3}} = \ln \lim_{x\to 0} (\sec x)^{\frac{2x+\operatorname{sen}3x}{x^3}}$$

$$= \ln \lim_{x\to 0}(1+\underbrace{\sec x-1}_{\text{expr.}})^{\frac{2x-\operatorname{sen}3x}{x^3}} = \ln\left[\underbrace{\lim_{x\to 0}(1+\sec x-1)^{\frac{1}{\sec x-1}}}_{\to e}\right]^{\frac{\sec x-1}{1}\cdot\frac{2x+\operatorname{sen}3x}{x^3}}$$

$\lim(1+\text{expr})^{\frac{1}{\text{expr.}}}$

$$= \ln e^{\lim\limits_{x\to 0}(\sec x-1)\cdot\frac{(2x+\operatorname{sen}3x)}{x^3}} =$$

$$= \lim_{x\to 0}(\sec x-1)\cdot\left(\frac{2x+\operatorname{sen}3x}{x^3}\right)\cdot\underbrace{\ln e}_{=1}$$

$1+\tan^2 x = \sec^2 x$

$\tan^2 x = \sec^2 x - 1$

$\tan^2 x = (\sec x+1)(\sec x-1)$

$$\Rightarrow \quad \sec x - 1 = \frac{\tan^2 x}{\sec x + 1}$$

$$\Leftrightarrow \lim_{x\to 0} \frac{\tan^2 x}{\sec x + 1} \cdot \left(\frac{2x + \operatorname{sen} 3x}{x^3} \right) =$$

$$= \lim_{x\to 0} \underbrace{\frac{\tan x}{x}}_{1} \cdot \underbrace{\frac{\tan x}{x}}_{1} \cdot \left(\frac{2x + \operatorname{sen} 3x}{x} \right) \cdot \frac{1}{\sec x + 1}$$

$$= \lim_{x\to 0} 1 \cdot 1 \cdot \left(\frac{2\cancel{x}}{\cancel{x}} + \underbrace{\frac{3 \cdot \operatorname{sen} 3x}{3 \cdot x}}_{1} \right) \cdot \frac{1}{\sec x + 1} =$$

$$= (2+3) \cdot \left(\frac{1}{\sec 0 + 1} \right) = 5 \cdot \frac{1}{1+1} = \frac{5}{2} \;//$$

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04) (b) $f(x) = \dfrac{3-2x}{9-x^2}$

$$\text{zero: } f(x) = 0 \Leftrightarrow \frac{3-2x}{9-x^2} = 0 \Leftrightarrow$$

$$\Leftrightarrow 3 - 2x = 0 \Leftrightarrow \boxed{x = \frac{3}{2}}$$

DOMÍNIO:

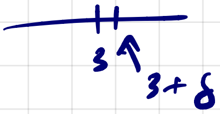
$$9 - x^2 \neq 0 \Leftrightarrow x \neq \pm 3$$

- $\displaystyle \lim_{x\to +\infty} f(x) = \lim_{x\to +\infty} \frac{3-2x}{9-x^2} = \lim_{x\to +\infty} \frac{-2x}{-x^2} = \lim_{x\to +\infty} \frac{1}{x} = 0$

- $\displaystyle \lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{3-2x}{9-x^2} = \lim_{x\to -\infty} \frac{-2x}{-x^2} = \lim_{x\to -\infty} \frac{1}{x} = 0.$

$\Rightarrow$ ASSÍNTOTA HORIZONTAL: $y = 0$ (eixo $0x$).

- $\displaystyle\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} \frac{3-2x}{9-x^2} =$

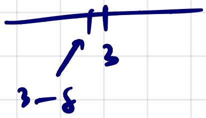


Tome $\delta > 0$ e escreva $x = 3+\delta$, Assim: $x \to 3^+ \Leftrightarrow \delta \to 0^+$

$$= \lim_{\delta\to 0^+} \frac{3-2.(3+\delta)}{9-(3+\delta)^2} = \lim_{\delta\to 0^+} \frac{3-6-2\delta}{\cancel{9}-\cancel{9}-6\delta-\delta^2} =$$

$$= \lim_{\delta\to 0^+} \frac{-3-2\delta}{-6\delta-\delta^2} = \lim_{\delta\to 0^+} \frac{3+2\delta}{6\delta+\delta^2} = \frac{3}{0^+} = +\infty$$

- $\displaystyle\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} \frac{3-2x}{9-x^2} =$



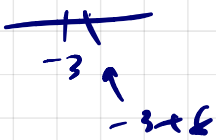
Tome $\delta > 0$ e escreva $x = 3-\delta$. Assim, $x \to 3^- \Leftrightarrow \delta \to 0^+$, e daí:

$$= \lim_{\delta\to 0^+} \frac{3-2(3-\delta)}{9-(3-\delta)^2} = \lim_{\delta\to 0^+} \frac{3-6+2\delta}{\cancel{9}-\cancel{9}+6\delta-\delta^2} =$$

$$= \lim_{\delta\to 0^+} \frac{-3+2\delta}{6\delta-\delta^2} = \lim_{\delta\to 0^+} \frac{-3+2\delta}{\delta(6-\delta)} = \frac{-3}{0^+} = -\infty$$

Assim, temos uma assíntota vertical: $x = 3$

- $\displaystyle\lim_{x\to -3^+} f(x) = \lim_{x\to -3^+} \frac{3-2x}{9-x^2} =$



Tome $\delta > 0$ e considere $x = -3+\delta$.

Logo, $x \to -3^+ \Leftrightarrow \delta \to 0^+$

$$\Leftrightarrow \lim_{\delta\to 0^+} \frac{3-2\cdot(-3+\delta)}{9-(-3+\delta)^2} = \lim_{\delta\to 0^+} \frac{3+6-2\delta}{\cancel{9}-\cancel{9}+6\delta-\delta^2} =$$

$$= \lim_{\delta\to 0^+} \frac{9-2\delta}{\delta(6-\delta)} = \frac{9}{0^+} = +\infty.$$

- $\lim_{x\to -3^-} f(x) = \dots = -\infty$ Logo, $x=-3$ é assíntota vertical.

exercício.

Então, o gráfico de f é tal que, tem zero em $x = \frac{3}{2}$, possui o eixo x $(y=0)$ como assíntota horizontal, não está definida em $+3$ e em -3, e $x=-3$ e $x=3$ são assíntotas verticais, pois que

$$\lim_{x\to 3^+} f(x) = +\infty \;;\; \lim_{x\to 3^-} f(x) = -\infty;$$

$$\lim_{x\to -3^+} f(x) = +\infty \;;\; \lim_{x\to -3^-} f(x) = -\infty.$$