Universidade Federal de Pelotas
Disciplina de Cálculo 2 - Turma T2
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Lista 05 de Exercícios - Integrais de funções racionais e irracionais. Decomposição em frações parciais

1. Calcule as integrais a seguir:

(a) $\displaystyle\int \frac{(2x+3)dx}{x^2-4x+1}$ (b) $\displaystyle\int \frac{(7x-4)dx}{3x^2-4x+5}$ (c) $\displaystyle\int \frac{(2x-4)dx}{\sqrt{x^2-12x+4}}$

(d) $\displaystyle\int \frac{(3x-1)dx}{\sqrt{x^2+x+1}}$ (e) $\displaystyle\int \frac{(1-3x)dx}{4x^2+12x-5}$ (f) $\displaystyle\int \frac{(2x+3)dx}{\sqrt{2-3x^2}}$

2. Calcule cada integral indefinida a seguir, comprovando a resposta indicada ao lado.

(a) $\displaystyle\int \frac{(4x-2)dx}{x^3-x^2-2x} = \ln\frac{|x^2-2x|}{(x+1)^2} + c$

(b) $\displaystyle\int \frac{(x+1)dx}{x(x-2)(x+3)} = -\frac{2}{15}\ln|x+3| - \frac{1}{6}\ln|x| + \frac{3}{10}\ln|x-2| + c$

(c) $\displaystyle\int \frac{4dx}{x^3-x^2-2x} = \frac{4}{3}\ln|x+1| - \ln x^2 + \frac{2}{3}\ln|x-2| + c$

(d) $\displaystyle\int \frac{(x^4+2x+1)dx}{x^3-x^2-2x} = \frac{x^2}{2} + x - \frac{1}{2}\ln|x| + \frac{21}{6}\ln|x-2| + c$

(e) $\displaystyle\int \frac{(x^2-3x+4)dx}{(x-1)(x+2)(x+3)} = \frac{11}{2}\ln|x+3| - \frac{14}{3}\ln|x+2| + \frac{1}{6}\ln|x+1| + c$

(f) $\displaystyle\int \frac{dx}{x^3-x} = \frac{1}{2}\ln|x-1| + \frac{1}{2}\ln|x+1| - \ln|x| + c$

(g) $\displaystyle\int \frac{(2x-1)dx}{x^3-2x^2-5x+6} = -\frac{1}{6}\ln|x-1| + \frac{1}{2}\ln|x-3| - \frac{1}{3}\ln|x+2| + c$

(h) $\displaystyle\int \frac{5x^2-3}{x^3-x}\,dx = 2\ln|2x+3| + \ln|x| - \frac{1}{2}\ln|2x+1| + c$

(i) $\displaystyle\int \frac{(4x+3)dx}{4x^3+8x^2+3x} = -\frac{1}{2}\ln|2x+3| + \ln|x| - \frac{1}{2}\ln|2x+1| + c$

(j) $\displaystyle\int \frac{(4x^2+6x)dx}{(x-1)^2(x+1)} = -\frac{5}{x-1} + \frac{9}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + c$

(k) $\displaystyle\int \frac{(x^2+x)dx}{(x-1)(x^2+1)} = \ln|x-1| + \arctan(x) + c$

(ℓ) $\displaystyle\int \frac{x^2dx}{(x^2-1)^3} = \frac{1}{16}\left[-\frac{1}{x-1} - \ln|x-1| + \ln|x+1| + \frac{1}{(x+1)^2} - \frac{1}{(x+1)} - \frac{1}{(x-1)^2}\right] + c$

(m) $\displaystyle\int \frac{(4x^2+2x+8)dx}{x(x^2+2)^2} = -\ln(x^2+2) + \frac{x}{2(x^2+2)} + \frac{\sqrt{2}}{4}\arctan(\frac{x\sqrt{2}}{4}) + 2\ln|x| + c$

(n) $\displaystyle\int \frac{dx}{x^3+x^2+x} = -\frac{1}{2}\ln|x^2+x+1| - \frac{\sqrt{3}}{3}\arctan\left(\frac{\sqrt{3}}{3}(2x+1)\right) + \ln|x| + c$

(o) $\displaystyle\int \frac{(1-x^3)dx}{(x^2+1)(x-1)^2} = -\frac{3}{2}\ln|x-1| + \frac{1}{4}\ln(x^2+1) - \frac{1}{2}\arctan(x) + c$

(p) $\displaystyle\int \frac{(1-x^3)dx}{(x^2+1)^2(x-1)^2} = -\frac{3}{4}\ln|x-1| - \frac{x+1}{4x^2+4} + \frac{1}{2}\arctan(x) + \frac{3}{8}\ln(x^2+1) + c$

(q) $\displaystyle\int \frac{(x+2)dx}{x(x^4-1)} = \frac{1}{16}[\frac{1}{x+1} - \frac{5}{x-1} - 11\ln|x-1| + \frac{2x+4}{x^2+1} + 6\arctan(x) - 8\ln(x^2+1) +$

$+ 32\ln|x| - 5\ln|x+1|] + c$

(r) $\displaystyle\int \frac{dx}{(x^3-1)^2} = \frac{1}{9}\left[-\frac{1}{x-1} - 2\ln|x-1| + \ln|x-1| + \ln|x^2+x+1| + \right.$

$$\left. + 2\sqrt{3}\arctan\left(\frac{\sqrt{3}}{3}(2x+1)\right) + \frac{x-1}{x^2+x+1}\right] + c$$

3. Mostre que:

   (a) $\displaystyle\int_2^4 \frac{x^3-2}{x^3-x^2}dx = \frac{5}{2} + \ln\frac{4}{3}$

   (b) $\displaystyle\int_0^5 \frac{(x^2-3)dx}{(x+2)(x+1)^2} = \ln\frac{7}{2} - \frac{5}{3}$