Lista 04)

01 - d (d) $\int \arccos 2x \, dx$

$$\int u\,dv = u \cdot v - \int v\,du$$

$$\begin{cases} u = \arccos 2x \Rightarrow du = -\dfrac{2}{\sqrt{1-(2x)^2}}\,dx = -\dfrac{2\,dx}{\sqrt{1-4x^2}} \\ dv = dx \Rightarrow v = x \end{cases}$$

$u = \arccos v \Leftrightarrow v = \cos u$

$\sin^2 u + \cos^2 u = 1$

$\sin^2 u = 1 - \cos^2 u$

$\sin u = \sqrt{1-\cos^2 u}$

$v' = -\sin u \cdot u' \longrightarrow u' = \dfrac{v'}{-\sin u}$

$$u' = -\frac{v'}{\sqrt{1-\cos^2 u}} = -\frac{v'}{\sqrt{1-v^2}}$$

Assim, temos:

$$\int \arccos 2x\,dx = \int u\,dv = u \cdot v - \int v\,du$$

$$= x \cdot \arccos 2x - \int x \cdot \left(\frac{-2\,dx}{\sqrt{1-4x^2}}\right)$$

$$= x \cdot \arccos 2x + \int \frac{2x\,dx}{\sqrt{1-4x^2}} = x \cdot \arccos 2x + \int (1-4x^2)^{-\frac{1}{2}} \cdot (2x\,dx)$$

$\int v^k\,dv$

$v = 1-4x^2 \Rightarrow dv = -8x\,dx$

$\dfrac{dv}{-4} = 2x\,dx$

$$= x \cdot \arccos 2x + \int v^{-\frac{1}{2}} \cdot \left(-\frac{1}{4}\,dv\right) =$$

$$= x \cdot \arccos 2x - \frac{1}{4}\int v^{-\frac{1}{2}}\,dv$$

$$= x \cdot \arccos 2x - \frac{1}{4} \cdot \frac{v^{+\frac{1}{2}}}{\frac{1}{2}} + C = x \cdot \arccos 2x - \frac{1}{2} \cdot \sqrt{1-4x^2} + C$$

07 - c) (c) $\int_0^1 x^2 e^{-x} dx$

$$\int x^2 e^{-x}\,dx = \int u\,dv = u \cdot v - \int v\,du$$

$$\begin{cases} u = x^2 \longrightarrow du = 2x\,dx \\ dv = e^{-x}\,dx \longrightarrow v = -e^{-x} \end{cases}$$

$$\int x^2 e^{-x}\,dx = x^2 \cdot (-e^{-x}) - \int e^{-x} \cdot 2x\,dx$$

$$= -x^2 e^{-x} - 2\int e^{-x} \cdot x\,dx$$

$$\begin{cases} u = x \Rightarrow du = dx \\ dv = e^{-x}\,dx \Rightarrow v = -e^{-x} \end{cases}$$

$$= -x^2 \cdot e^{-x} - 2 \cdot \left(x \cdot (-e^{-x}) - \int -e^{-x}\,dx\right)$$

$$= -x^2 e^{-x} - 2x e^{-x} + 2\int \underbrace{e^{-x}}_{e^w} \cdot \underbrace{(-dx)}_{dw} =$$

$$= -x^2 e^{-x} - 2x e^{-x} + 2 \cdot e^{-x} + C = (2 - 2x - x^2) \cdot e^{-x} + C$$

Por fim, faz-se:

$$\int_0^1 x^2 e^{-x}\,dx = (2 - 2x - x^2)\,e^{-x}\Big|_0^1$$

$$= (2 - 2(1) - (1)^2) \cdot e^{-1} - (2 - 0 - 0) \cdot e^{0}$$

$$= -1 \cdot e^{-1} - 2 = -\frac{1}{e} - 2$$

08-e) (e) $\displaystyle\int \frac{dx}{\sqrt{1+2x+3x^2}}$

$$3x^2+2x+1 = 3\cdot\left[(x+a)^2+b\right] = 3\cdot\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]$$

$$= 3\left[x^2+2ax+a^2+b\right]$$

$$= 3x^2+6ax+3a^2+3b$$

$$6a=2 \Rightarrow \boxed{a=\frac{1}{3}}$$

$$3a^2+3b=1$$

$$3\left(\frac{1}{3}\right)^2+3b=1$$

$$\frac{1}{3}+3b=1$$

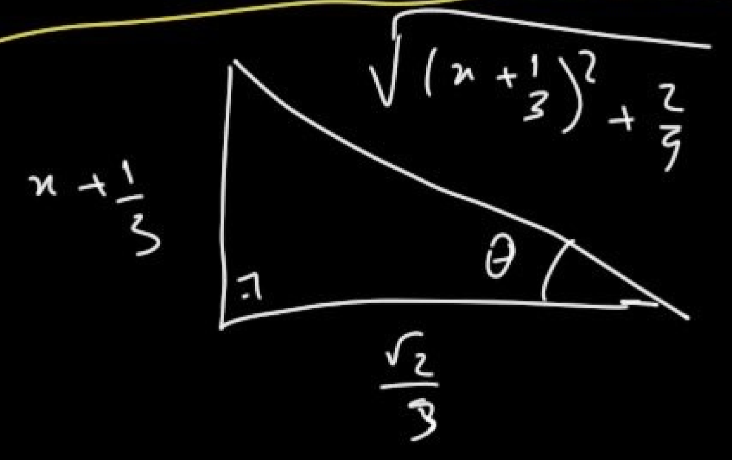
$$3b=\frac{3}{3}-\frac{1}{3}$$

$$3b=\frac{2}{3}$$

$$\boxed{b=\frac{2}{9}}$$

Assim, teremos:

$$\int \frac{dx}{\sqrt{3x^2+2x+1}} = \int \frac{dx}{\sqrt{3\cdot\left[\left(x+\frac{1}{3}\right)^2+\frac{2}{9}\right]}} = \frac{1}{\sqrt3}\int \frac{dx}{\sqrt{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}} = \frac{1}{\sqrt3}\cdot\int \frac{\cancel{\frac{\sqrt2}{3}}\sec^2\theta\, d\theta}{\cancel{\frac{\sqrt2}{3}}\cdot\cos\theta} \;(=)$$



$$\tan\theta = \frac{x+\frac{1}{3}}{\frac{\sqrt2}{3}}$$

$$\cos\theta = \frac{\frac{\sqrt2}{3}}{\sqrt{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}}}$$

$$x = \frac{\sqrt2}{3}\cdot\tan\theta - \frac{1}{3} \Rightarrow dx = \frac{\sqrt2}{3}\cdot\sec^2\theta\cdot d\theta$$

$$\Rightarrow \sqrt{\left(x+\frac{1}{3}\right)^2+\frac{2}{9}} = \frac{\sqrt2}{3}\cdot\cos\theta$$

$$(=)\;\frac{1}{\sqrt3}\cdot\int \sec^2\theta\cdot\sec\theta\, d\theta = \frac{1}{\sqrt3}\cdot\int (1+\tan^2\theta)\cdot\sec\theta\, d\theta = \frac{1}{\sqrt3}\int \sec\theta\, d\theta + \frac{1}{\sqrt3}\underbrace{\int \tan^2\theta\cdot\sec\theta\, d\theta}_{\int u\, dv}$$

$$1+\tan^2\theta = \sec^2\theta$$

$$\int \sec\theta\, d\theta = \ln|\sec\theta+\tan\theta| + C$$

$$u=\sec\theta \Rightarrow du = \sec\theta\tan\theta\, d\theta$$

$$dv = \tan^2\theta\, d\theta \Rightarrow$$

$$v = \int \tan^2\theta\, d\theta = \tan\theta - \theta$$

$$= \frac{1}{\sqrt3}\ln|\sec\theta+\tan\theta| + \frac{1}{\sqrt3}\cdot\left[\sec\theta\cdot(\tan\theta-\theta) - \int(\tan\theta-\theta)\cdot\sec\theta\cdot\tan\theta\, d\theta\right]$$

(...)