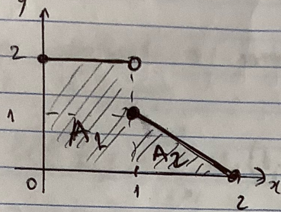
GABARITO DA PROVA 1 DE CÁLCULO 2. 28/11/23.

01)

(a)



$f \geq 0$ em $[0,2]$. Então,

$\int_0^2 f$ = área abaixo do gráfico.

No caso, $\int_0^2 f = A_1 + A_2 = 1 \cdot 2 + \frac{1 \cdot 1}{2}$

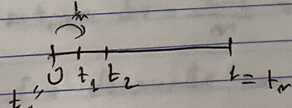
$= 2 + \frac{1}{2} = \frac{5}{2}$ //

(b) $\int_0^2 f = \int_0^1 f + \int_1^2 f$.

1º: $\int_0^1 f = \int_0^1 2 \cdot dx$.

Seja P_n partição regular que divide $[0,1]$ em n subintervalos de comprimento

$t_i - t_{i-1} = \Delta x = \frac{1-0}{n} = \frac{1}{n}$.



$t_i = 0 + \frac{i}{n} = \frac{i}{n}$; $i \in \{1, 2, \ldots, n\}$.

Como $f \equiv 2$ em $[0,1]$, temos que

$m_i = \inf_{x\in[t_{i-1}, t_i)} f(x) = 2$ e $M_i = \sup_{x\in[t_{i-1}, t_i]} f(x) = 2.$

Assim, montando a soma superior de f em rel. à partição P_n, obtemos:

$$S(f, P_n) = \sum_{i=1}^{n} M_i \underbrace{(t_i - t_{i-1})}_{=\frac{1}{n}} = \sum_{i=1}^{n} 2 \cdot \frac{1}{n} =$$

$$= \frac{2}{n} \cdot \sum_{i=1}^{n} 1 = \frac{2}{\cancel{n}} \cdot \cancel{n} = 2.$$

Assim, $\int_0^1 f = \lim_{n\to\infty} S(f, P_n) =$

$$= \lim_{n\to\infty} 2 = 2.$$

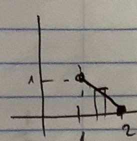
Do mesmo modo se mostra que $\underline{\int_0^1} f = 2.$

$\underline{\underline{2^{\underline{o}}}}$: $\int_1^2 f = \int_1^2 (2-x)\,dx.$

Seja P_n partição regular que divide [1, 2] em n subintervalos de comprimento

$$t_i - t_{i-1} = \Delta x = \frac{2-1}{n} = \frac{1}{n}.$$

Como f é decrescente em [1, 2] então



$m_i = \inf_{x\in[t_{i-1}, t_i]} f(x) = f(t_i)$ e

$$M_i = \sup_{x \in [t_{i-1}, t_i]} f(x) = f(t_{i-1}) \quad , \quad \text{onde:}$$

$$t_i = 1 + i \cdot \frac{1}{n} \; , \quad \forall n \in \{1, 2, 3, \dots, n\} .$$

Assim, montando $S(f; P_n)$, obtemos:

$$S(f; P_n) = \sum_{i=1}^{n} M_i \cdot \underbrace{(t_i - t_{i-1})}_{\frac{1}{n}} = \sum_{i=1}^{n} f(t_{i-1}) \cdot \frac{1}{n} =$$

$$= \frac{1}{n} \cdot \sum_{i=1}^{n} (2 - t_{i-1}) = \frac{1}{n} \cdot \sum_{i=1}^{n} 2 - \frac{1}{n} \cdot \sum_{i=1}^{n} t_{i-1} =$$

$$= \frac{2}{n} \cdot \underbrace{\sum_{i=1}^{n} 1}_{n} - \frac{1}{n} \cdot \sum_{i=1}^{n} \left(1 + \frac{i-1}{n}\right) = \frac{2}{\cancel{n}} \cdot \cancel{n} - \frac{1}{n} \underbrace{\sum_{i=1}^{n} 1}_{n} - \frac{1}{n^2} \cdot \sum_{i=1}^{n} (i-1)$$

$$= 2 - 1 - \frac{1}{n^2} \cdot (1 + 2 + \dots + (n-1)) = 1 - \frac{1}{n^2} \cdot \frac{(\cancel{1} + (n - \cancel{1})) \cdot (n-1)}{2}$$

$$= 1 - \frac{1}{n^{\cancel{2}}} \cdot \frac{\cancel{n} \cdot (n-1)}{2} = 1 - \frac{1}{2} \cdot \left(1 - \frac{1}{n}\right)$$

Então: $\displaystyle\int_1^2 f = \lim_{n \to \infty} S(f; P_n) =$

$$= \lim_{n \to \infty} 1 - \frac{1}{2}\left(1 - \frac{1}{n}\right) = 1 - \frac{1}{2} = \frac{1}{2}$$

Do mesmo modo se mostra que $\int_1^2 f = \frac{1}{2}$.

Portanto, $\int_1^2 f = \frac{1}{2}$.

Por fim, obtemos:

$$\int_0^2 f = \int_0^1 f + \int_1^2 f = 2 + \frac{1}{2} = \frac{5}{2}$$

(c) Pelo T.F.C:

$$\int_0^2 f = \int_0^1 f + \int_1^2 f$$

$$= \int_0^1 2\,dx + \int_1^2 (2-x)\,dx =$$

$$= 2x\Big|_0^1 + \left(2x - \frac{x^2}{2}\right)\Big|_1^2 =$$

$$= 2 - 0 + \left(4 - 2 - \left(2 - \frac{1}{2}\right)\right)$$

$$= 2 + \cancel{2} - \cancel{2} + \frac{1}{2} = \frac{5}{2}$$

02)

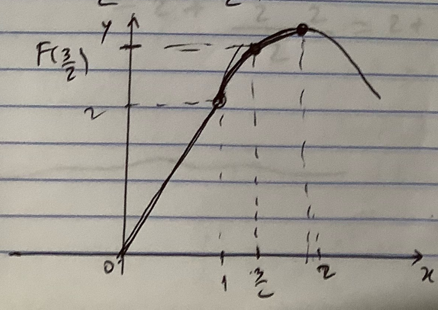
$$F(x) = \begin{cases} \int_0^x 2\,dt \; ; \text{ se } 0 \leq x \leq 1 \\ \int_0^1 2\,dt + \int_1^x (2-t)\,dt, \text{ se } 1 \leq x \leq 2 \end{cases}$$

$$F(x) = \begin{cases} 2t\Big|_0^x, \text{ se } 0 \leq x < 1 \\ 2t\Big|_0^1 + \left(2t - \frac{t^2}{2}\right)\Big|_1^x, \text{ se } 1 \leq x \leq 2 \end{cases}$$

$$F(x) = \begin{cases} 2x, \text{ se } 0 \leq x < 1 \\ 2 + 2x - \frac{x^2}{2} - \left(2 - \frac{1}{2}\right), \text{ se } 1 \leq x \leq 2 \end{cases}$$

$$F(x) = \begin{cases} 2x, \text{ se } 0 \leq x < 1 \\ \cancel{2} + 2x - \frac{x^2}{2} \cancel{-2} + \frac{1}{2}, \text{ se } 1 \leq x \leq 2 \end{cases}$$

$$F(x) = \begin{cases} 2x, \text{ se } 0 \leq x < 1 \\ -\frac{x^2}{2} + 2x + \frac{1}{2}, \text{ se } 1 \leq x \leq 2. \end{cases}$$



$$x_v = -\frac{b}{2a} = \frac{-(2)}{2\cdot(-\frac{1}{2})} = 2$$

05

$$F\left(\frac{3}{2}\right) = \frac{-\left(\frac{3}{2}\right)^2}{2} + 2\cdot\left(\frac{3}{2}\right) + \frac{1}{2}$$

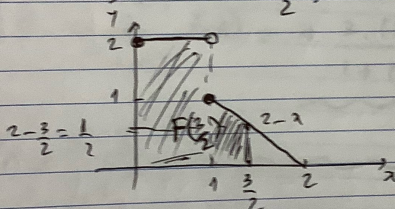
$$= -\frac{9}{8} + \frac{6}{2} + \frac{1}{2} = -\frac{9}{8} + \frac{7}{2} = -\frac{9}{8} + \frac{14}{4} =$$

$$= -\frac{9}{8} + \frac{28}{8} = \frac{19}{8} \,//$$

No gráfico de f, temos que

$$F\left(\frac{3}{2}\right) = \int_0^{\frac{3}{2}} f \quad = \text{área da figura}$$

entre 0 e $\frac{3}{2}$:



$$F\left(\frac{3}{2}\right) = 1\cdot 2 + \frac{\left(\frac{3}{2}-1\right)\cdot\left(1+\frac{1}{2}\right)}{2}$$

$$= 2 + \frac{\frac{1}{2}\cdot\frac{3}{2}}{2} = 2 + \frac{3}{8} = \frac{19}{8} \,//$$

03)

$$F(x) = \int_{1-3x}^{1} \frac{t^3}{1+t^2}\,dt = -\int_{1}^{1-3x} \frac{t^3}{1+t^2}\,dt$$

$$\frac{dF}{dx} = -\frac{d}{dx}\left(\int_{1}^{1-3x} \frac{t^3}{1+t^2}\,dt\right) =$$

$\hookrightarrow u = 1-3x$

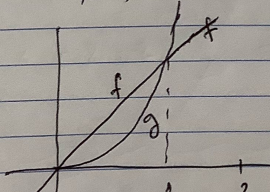
$u' = -3$

$$-\frac{d}{du}\left(\int_{1}^{u} \frac{t^3}{1+t^2}\,dt\right)\cdot\frac{du}{dx} =$$

$$= -\frac{u^3}{1+u^2}\cdot(-3) = +\frac{3\cdot(1-3x)^3}{1+(1-3x)^2}$$

04) Escreva $f(x) = x$ e $g(x) = x^2$.

Note que, temos os gráficos:



Então,

$f \geq g$ em $[0,1]$,

donde segue que

$\int_0^1 f \geq \int_0^1 g$, ou

seja, $\int_0^1 x\,dx \geq \int_0^1 x^2\,dx$.

Do mesmo modo, como $f \leq g$ em $[1,2]$, segue que

$$\int_1^2 f \leq \int_1^2 g \text{, i.e.;}$$

$$\int_1^2 x \, dx \leq \int_1^2 x^2 dx$$

05)

a) $\int x^{-\frac{1}{3}} dx + \int \frac{2x\,dx}{6-5x^2} =$

$v = 6-5x^2$

$dv = -10x\,dx$

$\Rightarrow 2x\,dx = -\frac{1}{5} dv$

$$= \frac{x^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + \int \frac{-\frac{1}{5}\,dv}{v} =$$

$$= \frac{x^{\frac{2}{3}}}{\frac{2}{3}} - \frac{1}{5} \ln|v| + C =$$

$$\frac{3}{2} x^{\frac{2}{3}} - \frac{1}{5} \ln|6-5x^2| + C.$$

b) $\int (4-\sec 2\varphi)^{-\frac{5}{2}} (\cancel{4}\sec 2\varphi \tan 2\varphi)\, d\varphi = \int r^{k}\, dr$

$r = 4-\sec 2\varphi \Rightarrow dr = -\sec 2\varphi \tan 2\varphi . 2d\varphi$

$\Rightarrow -\frac{1}{2} dr = \sec 2\varphi . \tan 2\varphi\, d\varphi$

Logo:

$\int (4-\sec 2\varphi)^{-\frac{5}{2}} . \underbrace{(\sec 2\varphi \tan 2\varphi\, d\varphi)}_{-\frac{1}{2} dr} = \int r^{-\frac{5}{2}} . \left(-\frac{1}{2}\right) dr =$

$= -\frac{1}{2} \frac{r^{-\frac{5}{2}+1}}{-\frac{5}{2}+1} + c = -\frac{1}{2} . \frac{r^{-\frac{3}{2}}}{-\frac{3}{2}} + c$

$= -\frac{1}{\cancel{2}} . \left(-\frac{\cancel{2}}{3}\right) . r^{-\frac{3}{2}} + c = +\frac{1}{3} (4-\sec 2\varphi)^{-\frac{3}{2}} + c$

(c) $\int \underbrace{(\cos x)^{-\frac{1}{3}} . \operatorname{sen} x\, dx}_{-dr} = -\int r^{-\frac{1}{3}} dr =$

$r = \cos x \Rightarrow dr = -\operatorname{sen} x\, dx .$

$= -\frac{r^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + c = -\frac{r^{\frac{2}{3}}}{\frac{2}{3}} + c =$

$= -\frac{3}{2} (\cos x)^{\frac{2}{3}} + c$

(d) $\int \frac{dx}{(x-2)^2+9} = \frac{1}{3} \cdot \operatorname{artan}\left(\frac{x-2}{3}\right) + C.$

$r = x-2 \Rightarrow dr = dx$ ok!

$a^2 = 9 \Rightarrow a = 3$

(e) $\int \frac{e^{2x}\,dx}{e^{2x}+3} = \int \frac{\frac{dr}{2}}{r} =$

$r = e^{2x}+3 \Rightarrow dr = 2 \cdot e^{2x}\,dx$

$\Rightarrow e^{2x}\,dx = \frac{dr}{2}$

$= \frac{1}{2}\int \frac{dr}{r} = \frac{1}{2}\ln|r| + C = \frac{1}{2}\ln|e^{2x}+3| + C$

(f) $\int \frac{\sec^2 e^{\sqrt{x}} \cdot e^{\sqrt{x}}\,dx}{\sqrt{x}} = \int \sec^2 r \cdot (2\,dr) =$

$r = e^{x^{\frac{1}{2}}} \Rightarrow dr = e^{x^{\frac{1}{2}}} \cdot \frac{1}{2} x^{-\frac{1}{2}}\,dx$

$dr = \frac{e^{\sqrt{x}}\,dx}{2\sqrt{x}} \Rightarrow 2\,dr = \frac{e^{\sqrt{x}}\,dx}{\sqrt{x}}$

$= \tan r + C = \tan e^{\sqrt{x}} + C$

06) $\displaystyle\int \frac{dx}{x^2+4x+3} = \int \frac{dx}{(x+2)^2-1} =$

$u = x+2 \Rightarrow du = dx$ ok!

$$= \frac{1}{1} \cdot \ln \left| \frac{x+2-1}{\sqrt{(x+2)^2-1}} \right| + C =$$

$$= \ln \left| \frac{x+1}{\sqrt{x^2+4x+3}} \right| + C.$$

$$\boxed{\int \frac{du}{u^2-a^2} = \frac{1}{a} \ln \left| \frac{u-a}{\sqrt{u^2-a^2}} \right| + c}$$

Assim;

$$\int_0^1 \frac{dx}{x^2+4x+3} = \ln \left| \frac{x+1}{\sqrt{x^2+4x+3}} \right| \Bigg|_0^1 =$$

$$\ln \left| \frac{1+1}{\sqrt{1+4+3}} \right| - \ln \left| \frac{1}{\sqrt{3}} \right| = \ln \frac{2}{\sqrt{8}} - \ln \frac{1}{\sqrt{3}}$$

$$= \ln \frac{\cancel{2}}{\cancel{2}\sqrt{2}} - \ln \frac{1}{\sqrt{3}} = \underbrace{\ln 1}_{0} - \ln \sqrt{2} - (\underbrace{\ln 1}_{0} - \ln \sqrt{3})$$

$$= \ln \sqrt{3} - \ln \sqrt{2} = \frac{1}{2} \ln \frac{3}{2}$$