GABARITO PROVA 1.

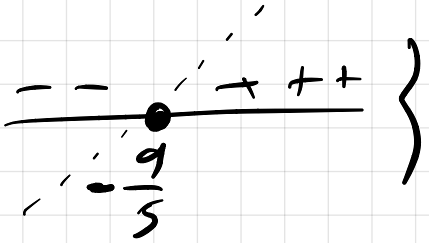
01) $$-\frac{2}{3} \le \frac{3x+1}{2x-3} \le \frac{2}{3}$$

(I): $\frac{3x+1}{2x-3} \le \frac{2}{3}$ (II): $-\frac{2}{3} \le \frac{3x+1}{2x-3}$

(I): $$\frac{3x+1}{2x-3} - \frac{2}{3} \le 0 \iff \frac{3(3x+1) - 2(2x-3)}{3(2x-3)} \le 0$$

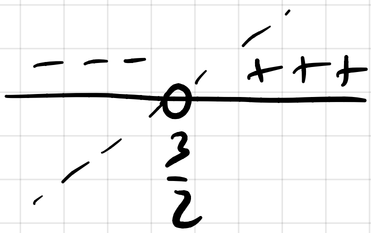
$$\iff \frac{9x+3-4x+6}{6x-9} \le 0 \iff \frac{5x+9}{6x-9} \le 0$$

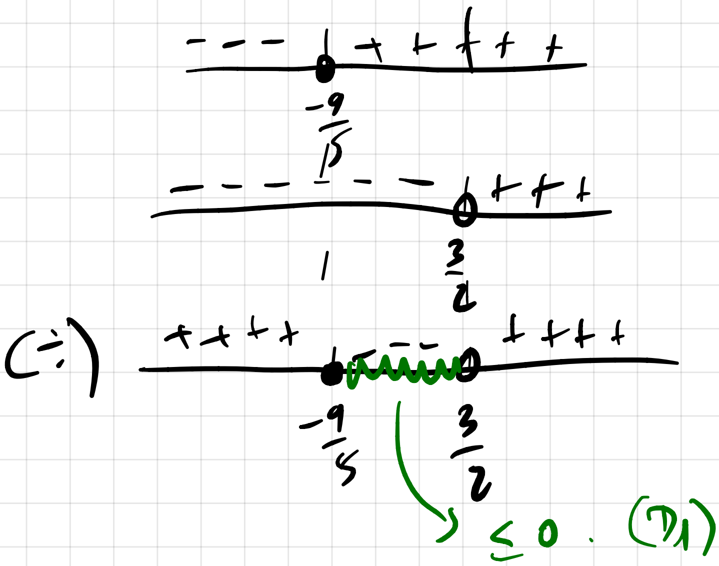
ZEROS DO NUMERADOR: $5x+9=0 \iff x = -\frac{9}{5}$



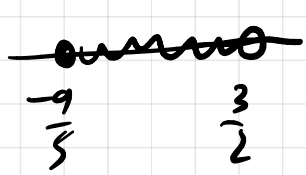
ZEROS DO DENOMINADOR ($\neq 0$):

$6x-9=0 \iff x = \frac{3}{2}$





$\rightarrow$ D_1 =

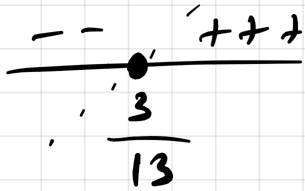


$$(\text{II}):\ \frac{3x+1}{2x-3} \geq -\frac{2}{3} \iff \frac{3x+1}{2x-3} + \frac{2}{3} \geq 0 \iff$$

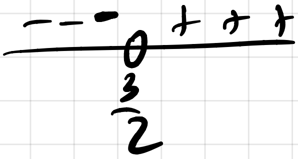
$$\iff \frac{3(3x+1)+2.(2x-3)}{3(2x-3)} \geq 0 \iff \frac{9x+3+4x-6}{6x-9} \geq 0$$

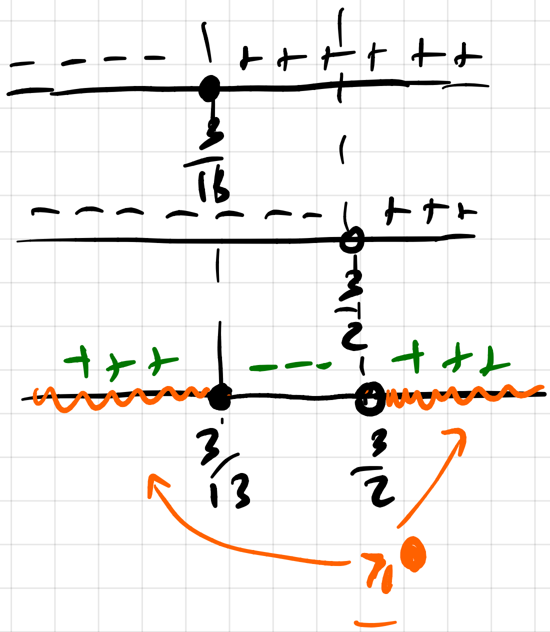
$$\iff \frac{13x-3}{6x-9} \geq 0$$

ZEROS DO NUMERADOR: $13x-3=0 \iff x = \frac{3}{13}$



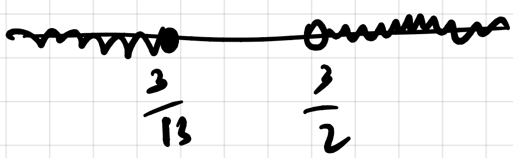
ZEROS DO DENOM.: ($\neq 0$) IGUAL AO CASO ACIMA:

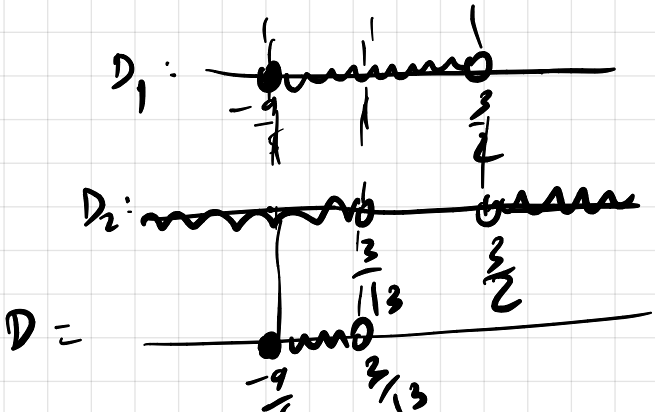




$\frac{3}{2}\times\frac{13}{13}$ $\quad \frac{3}{13}\times\frac{2}{2}$

$\Rightarrow D_2 =$





$S = \left[-\frac{9}{5}, \frac{3}{13}\right)$

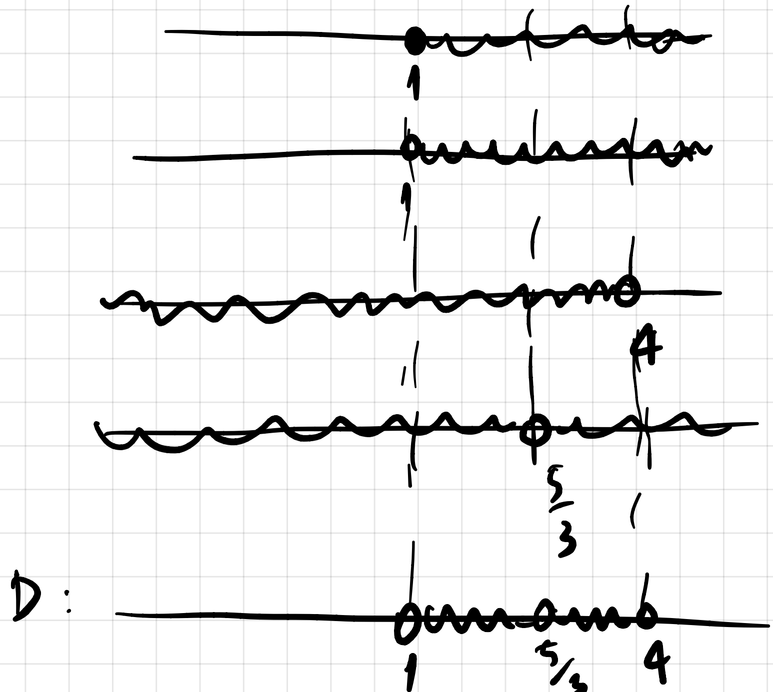
02) Note que podemos escrever:

$$f(x) = \frac{1}{\sqrt{x-1}} - 3(x-1)\cdot\left[\ln(x-1) - \ln(4-x)\right] - \frac{4}{3x-5}$$

condições de existência:

- $x-1 \geq 0 \iff x \geq 1$ $\quad (D_1)$
- $x-1 \in \mathbb{R}$.
- $x-1 > 0 \iff x > 1$. $\quad (D_2)$
- $4-x > 0 \iff x < 4$ $\quad (D_3)$
- $3x-5 \neq 0 \iff x \neq \frac{5}{3}$ $\quad (D_4)$

$D = D_1 \cap D_2 \cap D_3 \cap D_4$:



$D = (1, \frac{5}{3}) \cup (\frac{5}{3}, 4)$.

$f \circ g : \mathbb{R} \to \mathbb{R}$.

03) $(f \circ g)(x) = f(g(x)) = f(x-3) =$

$$= \begin{cases} (x-3)^2 + 2\cdot(x-3) + 4, & \text{se } x-3 \geq 1 \\ 3(x-3)+4, & \text{se } x-3 < 1 \end{cases}$$

$$= \begin{cases} x^2 - 6x + 9 + 2x - 6 + 4, & \text{se } x \geq 4 \\ 3x - 9 + 4, & \text{se } x < 4 \end{cases}$$

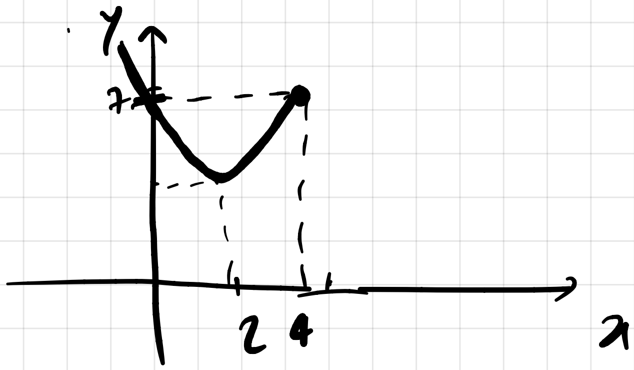
$$\Rightarrow (f \circ g)(x) = \begin{cases} x^2 - 4x + 7, & \text{se } x \geq 4 \\ 3x - 5, & \text{se } x < 4. \end{cases}$$

$y = x^2 - 4x + 7 = 0$ (zeros)

$\Leftrightarrow x = \dfrac{4 \pm \sqrt{16-28}}{2}$ $\nexists$ zeros.

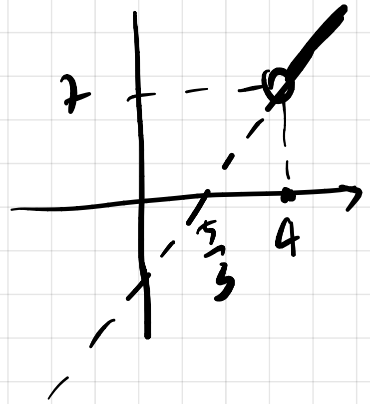
$f(0) = 7 > 0$. f está acima, com c.p.c. do eixo x.

$x_v = -\dfrac{b}{2a} = -\dfrac{(-4)}{2\cdot 1} = 2$



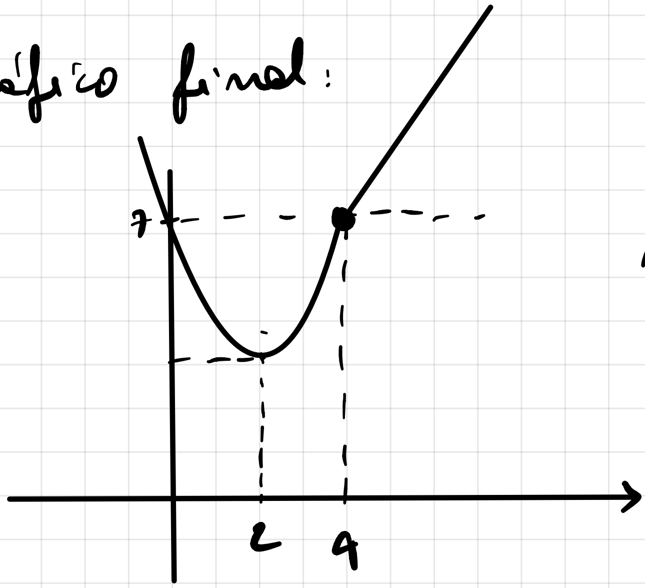
$f(4) = 7$

$y = 3x - 5$ (reta) zero: $3x - 5 = 0 \Leftrightarrow x = \frac{5}{3}$



$f(4) = 3 \cdot (4) - 5$
$= 7$

Esboço gráfico final:



$h = f \circ g$.

h não é bijetiva pois não é sobrejetiva, visto que $Im(h) \neq \mathbb{R}$.

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04) $t = 0 \Rightarrow m = m_0$

$t = 5750 \Rightarrow m = m_0 - \frac{1}{2} m_0 = \frac{1}{2} m_0 = m_0 \left(\frac{1}{2}\right)^1$.

$t = 2 \times 5750 \rightarrow m = \frac{1}{2} m_0 - \frac{1}{2}\left(\frac{1}{2} m_0\right) = \frac{1}{4} m_0$.

$= m_0 \cdot \left(\frac{1}{2}\right)^2$

$t = 3 \cdot 5750 \Rightarrow m = m_0 \cdot \left(\frac{1}{2}\right)^2 - \frac{1}{2} m_0 \cdot \left(\frac{1}{2}\right)^2 =$

$= m_0 \cdot \left(\frac{1}{2}\right)^2 \left[1 - \frac{1}{2}\right] = m_0 \left(\frac{1}{2}\right)^3$

⋮

$$t = k \cdot 5750 \Rightarrow m = m_0 \cdot \left(\frac{1}{2}\right)^k.$$

$$k = \frac{t}{5750}$$

$$m(t) = m_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5750}}$$

$m = \frac{23}{100} m_0$; então , temos:

$$\frac{23}{100} m_0 = m_0 \cdot \left(\frac{1}{2}\right)^{\frac{t}{5750}}$$

$$\frac{23}{100} = \left(\frac{1}{2}\right)^{\frac{t}{5750}}$$

$$\log 23 - \underbrace{\log 100}_{2} = \frac{t}{5750}\left[\underbrace{\log(1)}_{=0} - \log 2\right]$$

$$t = 5750 \cdot \left[\frac{\log 23 - 2}{-\log 2}\right]$$

$$t \cong 5750 \cdot \left[\frac{1,3617278360175 93 - 2}{0,301029995663981}\right]$$

$\cong 12.191,69$ anos.

05) (a) $f(x) = x \cdot |1-x|$;

$$|1-x| = \begin{cases} 1-x, & \text{se } 1-x \geq 0 \\ -(1-x), & \text{se } 1-x < 0 \end{cases}$$

$$= \begin{cases} 1-x, & \text{se } x \leq 1 \\ -1+x, & \text{se } x > 1 \end{cases}$$

Logo:

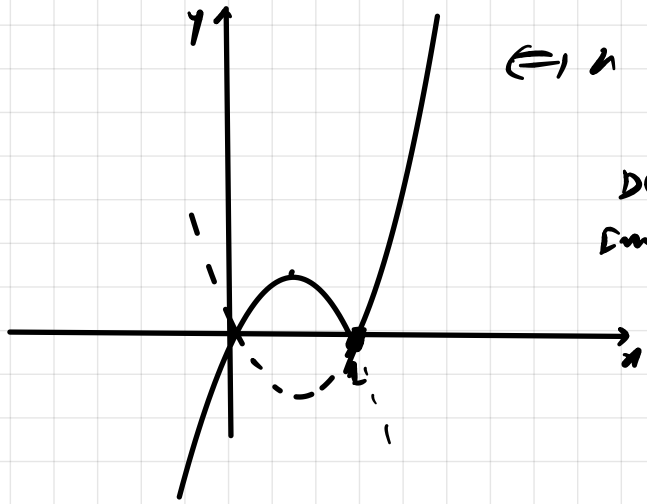
$$f(x) = x \cdot |1-x| = \begin{cases} x \cdot (1-x), & \text{se } x \leq 1 \\ x \cdot (-1+x), & \text{se } x > 1 \end{cases}$$

$$= \begin{cases} -x^2+x, & \text{se } x \leq 1 \\ x^2-x, & \text{se } x > 1. \end{cases}$$

$y = -x^2 + x$ zeros: $-x^2+x = 0$

$\Leftrightarrow x(-x+1) = 0$

$\Leftrightarrow x = 0$; $x = 1$.



$D(f) = \mathbb{R}$

$Im(f) = \mathbb{R}$.

(b) $y = 1 - 2^{1-3x} \Leftrightarrow y - 1 = -2^{1-3x}$

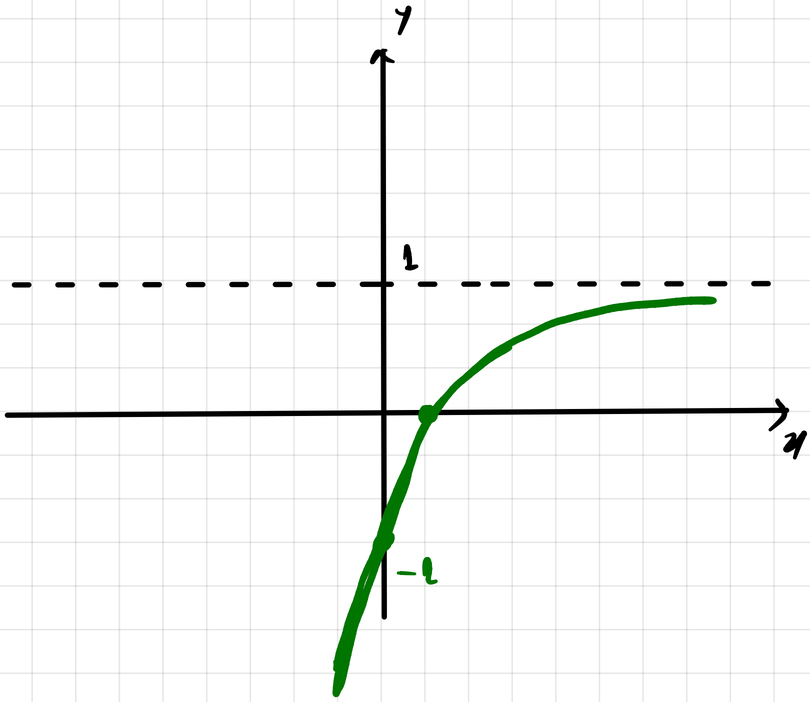
$\Leftrightarrow 1 - y = 2^{1-3x} > 0. \Leftrightarrow 1 - y > 0$

$\Leftrightarrow y < 1.$

$Im(f) = (-\infty, 1).$

ASSÍNTOTA HORIZONTAL: $y = 1$.

x	$y = 1 - 2^{1-3x}$
0	$1 - 2 = -1$
$\frac{1}{3}$	$1 - 2^0 = 0$



$D(f) = \mathbb{R}$

$Im(f) = (-\infty, 1).$

(c) $y = \left| 2 - 2\log_{\frac{1}{2}}(2x-1) \right|$

1.º: $y = 2 - 2\log_{\frac{1}{2}}(2x-1) \Leftrightarrow$

$\frac{2-y}{2} = \log_{\frac{1}{2}}(2x-1) \Leftrightarrow \left(\frac{1}{2}\right)^{\frac{2-y}{2}} = 2x - 1$

> 0

$$\Leftrightarrow \quad x = \frac{1 + \left(\frac{1}{2}\right)^{\frac{2-y}{2}}}{2}$$

$\Downarrow$

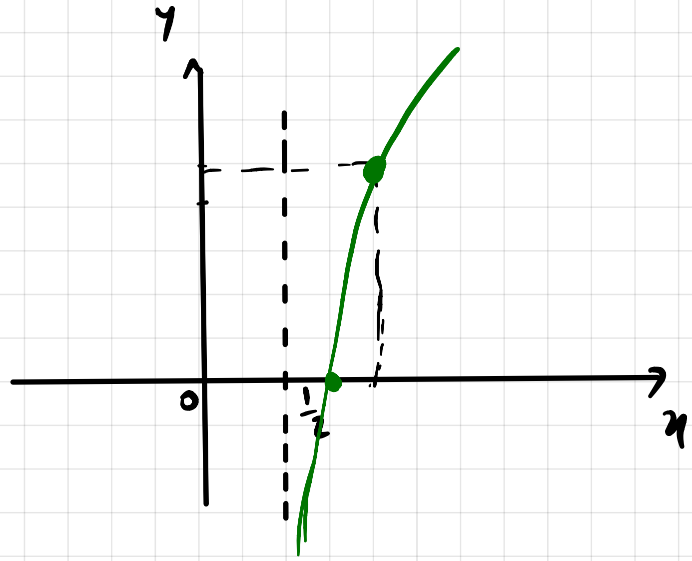
$2x - 1 > 0$

$\Updownarrow$

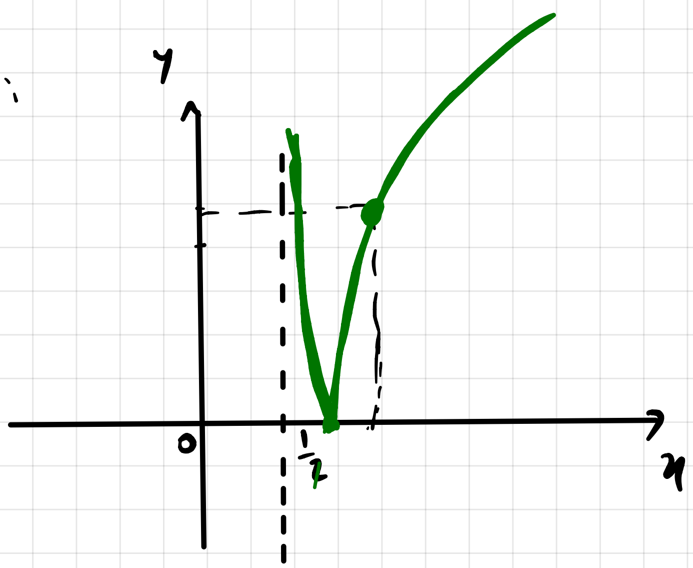
$x > \frac{1}{2}$

(ASSINTOTA VERT.)

$x = \frac{1 + \left(\frac{1}{2}\right)^{\frac{2-y}{2}}}{2}$	y
$\frac{1+1}{2} = 1$	2
$\frac{3}{4}$	0



2° $f(x) = |y|$;



$D(f) = \left(\frac{1}{2}, +\infty\right)$

$Im(f) = [0, +\infty)$.

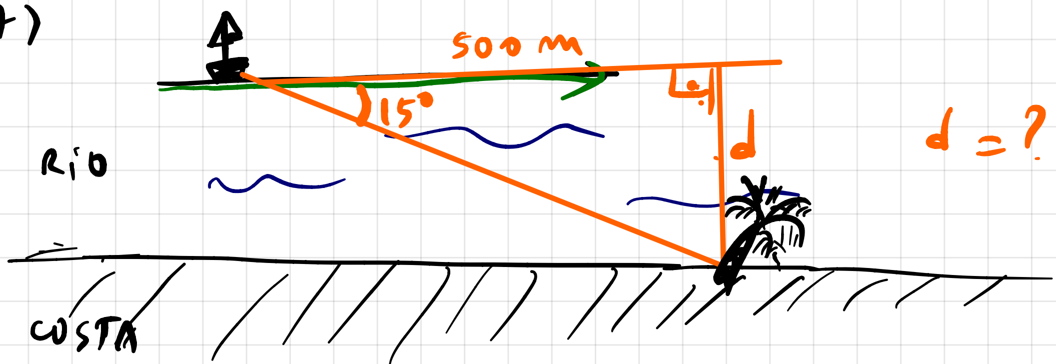
06)

$$\log_\beta \frac{A\cdot B^2}{\sqrt[3]{C}} = \log_\beta A\cdot B^2 - \log_\beta \sqrt[3]{C} =$$

$$= \log_\beta A + 2.\log_\beta B - \frac{1}{3}\log_\beta C$$

$$= 2 + 2.(-1) - \frac{1}{3}.\left(\frac{1}{2}\right) = -\frac{1}{6}$$

07)



$$\tan 15^\circ = \frac{d}{500m} \Rightarrow d = 500.\tan 15^\circ;$$

onde:

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) = \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ.\tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + 1.\frac{\sqrt{3}}{3}} = \frac{3-\sqrt{3}}{3+\sqrt{3}}.$$

$$\Rightarrow d = 500.\left(\frac{3-\sqrt{3}}{3+\sqrt{3}}\right).\frac{3-\sqrt{3}}{3-\sqrt{3}} = \frac{500.(3-\sqrt{3})^2}{9-3}$$

$$d = \frac{500 \cdot (9 - 6\sqrt{3} + 3)}{6} = \frac{500 \cdot (12 - 6\sqrt{3})}{6}$$

$$d = 500(2 - \sqrt{3}) \text{ m.}$$

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08) $x \in 1^{\circ}q$; $y \in 3^{\circ}q$.

$\cos y = ?$

$x + y = 300^{\circ} \Rightarrow y = 300^{\circ} - x$

$\cos y = \cos(300^{\circ} - x) = \cos 300^{\circ} \cdot \cos x + \operatorname{sen} 300^{\circ} \cdot \operatorname{sen} x$

$\cos 300^{\circ} = \cos 60^{\circ} = \frac{1}{2}$; $\operatorname{sen} 300^{\circ} = -\operatorname{sen} 60^{\circ} = -\frac{\sqrt{3}}{2}$

$$= -\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\operatorname{sen} x;$$

Sendo $\cot x = \frac{2}{3} \Rightarrow \tan x = \frac{3}{2}$.

$1 + \tan^2 x = \sec^2 x$

$1 + \frac{9}{4} = \sec^2 x$

$\Rightarrow \sec x = +\sqrt{\frac{13}{4}} = +\frac{\sqrt{13}}{2}$ ($x \in 1^{\circ}q$)

$\Rightarrow \cos x = \frac{2}{\sqrt{13}}$.

$\operatorname{sen} x = \sqrt{1 - \cos^2 x} = \sqrt{1 - \frac{4}{13}} = \frac{3}{\sqrt{13}}$ ($1^{\circ}q$)

Assim obtemos:

$$\cos\gamma = -\frac{1}{2}\cos x - \frac{\sqrt{3}}{2}\,\text{sen}\,x = -\frac{1}{2}\cdot\frac{2}{\sqrt{13}} - \frac{\sqrt{3}}{2}\cdot\frac{3}{\sqrt{13}}$$

$$= -\frac{2+3\sqrt{3}}{2\sqrt{13}}$$

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09)

$$\text{sen}(\alpha+\beta) = \text{sen}\,\alpha\cdot\cos\beta + \text{sen}\,\beta\cos\alpha \quad ; \ \alpha,\beta \in 1^{\circ}Q.$$

$$= \frac{2}{3}\cdot\frac{3}{4} + \text{sen}\,\beta\cdot\cos\alpha$$

- $\text{sen}\,\beta = \sqrt{1-\cos^2\beta} = \sqrt{1-\frac{9}{16}} = \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{4}$
- $\cos\alpha = +\sqrt{1-\text{sen}^2\alpha} = \sqrt{1-\frac{4}{9}} = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

Logo:

$$\text{sen}(\alpha+\beta) = \frac{1}{2} + \text{sen}\,\beta\cos\alpha = \frac{1}{2} + \frac{\sqrt{7}}{4}\cdot\frac{\sqrt{5}}{3} =$$

$$= \frac{6+\sqrt{35}}{12}$$