LISTA 02

3. Use o Teorema Fundamental do Cálculo, primeiro formato, para obter a derivada de cada função abaixo:

(a) $F(x) = \int_1^x \frac{1}{t^3+1} dt$ (b) $F(x) = \int_1^{e^x} \ln t dt$

(c) $F(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1} dz$ (d) $F(x) = \int_0^{x^4} \cos t dt$

$$\boxed{\frac{d}{dx} \int_a^x f(t) . dt = f(x)}$$

a) $f(t) = \frac{1}{t^3+1}$. $\frac{d}{dx} F(x) = \frac{d}{dx} \int_1^x \frac{1}{t^3+1} dt = \frac{1}{x^3+1}$.

b) $\frac{dF}{dx} = \frac{d}{dt} \int_1^{e^x} \ln t \, dt = \frac{d}{dt} \int_1^{u} \ln t \, dt$

Escreva $u = e^x \Rightarrow u' = e^x$.

$= \ln u . u' = \ln e^x . e^x$

$= \log_e e^x . e^x$

$= x . e^x$

4. De acordo com o Teorema Fundamental do Cálculo, calcule cada integral definida a seguir[1]:

(a) $\int_0^2 x^2 dx$ (b) $\int_{-2}^2 (x^3+1) dx$ (c) $\int_1^4 (x^2+4x+5) dx$

QUAL A FUNÇÃO NA QUAL x^2+4x+5 É SUA DERIVADA?

c) $\int_1^4 (x^2+4x+5) dx = \int_1^4 x^2 dx + 4\int_1^4 x \, dx + 5\int_1^4 1 . dx$

$= \left(\frac{x^3}{3} + 2x^2 + 5x \right) \Big|_1^4$

$= \frac{(4)^3}{3} - 2.(4)^2 + 5.4 - \left(\frac{1}{3} - 2.(1)^2 + 5 \right)$

$= \frac{64}{3} - 32 + 20 - \frac{1}{3} + 2 - 5$

$= \frac{63}{3} - 12 - 3 = 21 - 15 = 6$

(f) $\int_{-3}^3 \sqrt{3+|x|} dx$

Note que $|x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$

$\sqrt{3+|x|} = \begin{cases} \sqrt{3+x}, & \text{se } x \geq 0 \\ \sqrt{3-x}, & \text{se } x < 0 \end{cases}$

$\int_{-3}^3 \sqrt{3+|x|} \, dx = \int_{-3}^0 \sqrt{3-x} \, dx + \int_0^3 \sqrt{3+x} \, dx$

$= \int_{-3}^0 (3-x)^{\frac{1}{2}} dx + \int_0^3 (3+x)^{\frac{1}{2}} dx =$

$\int u^k du$. $u = 3-x$ $du = -1dx$ $dx = -du$

$\int u^k du$ $u = 3+x$ $du = 1dx$

$$= \int_{-3}^{0} (3-x)^{\frac{1}{2}}\,dx + \int_{0}^{3} (3+x)^{\frac{1}{2}}\,dx \;\; = $$

Note que: • $\int (3-x)^{\frac{1}{2}}dx = \int v^{k}\cdot(-dv) = -\int v^{k}dv = -\frac{v^{k+1}}{k+1} + C = -\frac{(3-x)^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$

$$= -\frac{2}{3}(3-x)^{\frac{3}{2}} + C$$

• $\int (3+x)^{\frac{1}{2}}dx = \int v^{k}dv = \frac{v^{k+1}}{k+1} + C = \frac{2}{3}(3+x)^{\frac{3}{2}} + C$

Assim.

$$= \; -\frac{2}{3}(3-x)^{\frac{3}{2}}\Big|_{-3}^{0} + \frac{2}{3}(3+x)^{\frac{3}{2}}\Big|_{0}^{3} = \dots$$

7. Calcule cada integral indefinida a seguir usando as regras estudadas em aula.

(a) $\int\left(\frac{1}{x^2} + \sin 3x\right) dx$ (b) $\int \frac{4x^2 - 2\sqrt{x}}{x}\, dx$ (c) $\int \frac{(2x+3)dx}{\sqrt{x^2+3x}}$ (d) $\int \frac{\cos ax\, dx}{\sqrt{b + \sin ax}}$

(e) $\int \frac{dy}{\sqrt{a - by}}$ (f) $\int \left(\frac{\sec x}{1+\tan x}\right)^2 dx$ (g) $\int \frac{\sec 2\theta \tan 2\theta d\theta}{3\sec 2\theta - 2}$ (h) $\int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}} dx$

a) $\int \frac{dx}{x^2} + \int \operatorname{sen} 3x\,dx = \underbrace{\int x^{-2}dx}_{\int v^k dv} + \underbrace{\int \operatorname{sen} 3x\,dx}_{\int \operatorname{sen} w\,dw}$

$w = 3x \Rightarrow dw = 3dx$

$dx = \frac{dw}{3}$

$$= \frac{x^{-1}}{-1} + \int \operatorname{sen} w\cdot\left(\frac{dw}{3}\right)$$

$$= -\frac{1}{x} + \frac{1}{3}\int \operatorname{sen} w\,dw = -\frac{1}{x} - \frac{1}{3}\cdot\cos w + C$$

$$= -\frac{1}{x} - \frac{1}{3}\cdot\cos 3x + C$$

b) $\int \frac{4x^2 - 2\sqrt{x}}{x}dx = \int\left(\frac{4x^2}{x} - \frac{2\sqrt{x}}{x}\right)dx =$

$$= \int 4x\,dx - 2\int x^{\frac{1}{2}}\cdot x^{-1}\,dx$$

$$= 4\int \underbrace{x^1}_{v^k}dx - 2\int \underbrace{x^{-\frac{1}{2}}}_{v^k}dx$$

$\int v^k dv = \frac{v^{k+1}}{k+1} + C$

$$= \frac{4x^2}{2} - 2\cdot\frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= 2x^2 - 2\cdot x^{\frac{1}{2}}\cdot\frac{2}{1} + C = 4x^2 - 4\sqrt{x} + C$$

(f) $\int \left(\frac{\sec x}{1+\tan x}\right)^2 dx$

Note que $(\tan x)' = \sec^2 x$

Disso, escrevemos:

$$\int \left(\frac{\sec x}{1+\tan x}\right)^2 dx = \int \frac{\sec^2 x}{(1+\tan x)^2} dx =$$

$$= \int \underbrace{(1+\tan x)}_{v}^{-2} \cdot \underbrace{\sec^2 x\, dx}_{dv} = \int v^k \cdot dv = \frac{v^{k+1}}{k+1} + C$$

$v = 1 + \tan x \Rightarrow dv = \sec^2 x \cdot dx$

$$= \frac{(1+\tan x)^{-2+1}}{-2+1} + C = -\frac{1}{1+\tan x} + C$$

(g) $\int \frac{\sec 2\theta \tan 2\theta d\theta}{3 \sec 2\theta - 2}$

$$\int \frac{dv}{v} = \ln|v| + C$$

$v = 3 \cdot \sec 2\theta - 2 \Rightarrow dv = 3 \cdot \sec 2\theta \cdot \tan 2\theta \cdot 2 \cdot d\theta$

$\sec 2\theta \cdot \tan 2\theta \cdot d\theta = \frac{1}{6} dv$

$$\int \frac{\overbrace{\sec 2\theta \cdot \tan 2\theta \, d\theta}^{\frac{1}{6} dv}}{\underbrace{3 \cdot \sec 2\theta - 2}_{v}} = \frac{1}{6} \int \frac{dv}{v} =$$

$$= \frac{1}{6} \cdot \ln|v| + C = \frac{1}{6} \cdot \ln|3 \cdot \sec 2\theta - 2| + C.$$

(h) $\int \frac{e^{\sqrt{x}} - 3}{\sqrt{x}} dx$

$$= \underbrace{\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx}_{\int e^v dv} - 3 \underbrace{\int \frac{dx}{\sqrt{x}}}_{\int v^k dv} =$$

$v = \sqrt{x} = x^{\frac{1}{2}}$

$\Rightarrow dv = \frac{1}{2} x^{-\frac{1}{2}} \cdot dx = \frac{dx}{2\sqrt{x}}$

$\Rightarrow dv = \frac{dx}{2\sqrt{x}} \Rightarrow 2dv = \frac{dx}{\sqrt{x}}$

$$= \int e^v \cdot (2dv) - 3 \int x^{-\frac{1}{2}} \cdot dx$$

$$= 2 \cdot \int e^v dv - 3 \cdot \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C = 2 \cdot e^v - 3 \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$= 2 e^{\sqrt{x}} - 6\sqrt{x} + C$$

LISTA 03

06) (r) $\int \frac{4dx}{x^2+9}$

$$= 4 \int \frac{dx}{x^2+9}$$

$$\int \frac{dv}{v^2+a^2} = \frac{1}{a} \cdot \arctan\left(\frac{v}{a}\right) + C$$

$v = x \Rightarrow dv = dx$

$a^2 = 9 \Rightarrow a = 3$

$$= 4 \cdot \left(\frac{1}{3} \cdot \arctan\left(\frac{x}{3}\right) + C\right)$$

$$= \frac{4}{3} \arctan\left(\frac{x}{3}\right) + C$$

LISTA 03 –

06) (v) $\int \frac{dx}{\sqrt{x^2+2x+5}}$ $= \int \frac{dx}{\sqrt{(x+1)^2+2^2}} = \int \frac{du}{\sqrt{u^2+a^2}} = \ln\left|u+\sqrt{u^2+a^2}\right| + C$

Note que $x^2+2x+5 = \underbrace{x^2+2x+1}_{(x+1)^2}+4 = (x+1)^2+4$

$u = x+1 \Rightarrow du = dx$ ok!

$= \ln\left|x+1+\sqrt{(x+1)^2+4}\right| + C$

$= \ln\left|x+1+\sqrt{x^2+2x+5}\right| + C$

L3

1. Podemos integrar $\int \sec^2 x \tan x\, dx$ de duas maneiras como segue:

(a) $\int \sec^2 x \tan x\, dx = \int \tan x(\sec^2 x\, dx) = \frac{1}{2}\tan^2 x + c$,

(b) $\int \sec^2 x \tan x\, dx = \int \sec x(\sec x \tan x\, dx) = \frac{1}{2}\sec^2 x + c$.

Explique a diferença aparente entre as duas respostas.

Basta notar que; sendo

$F(x) = \frac{1}{2}\tan^2 x + C_1$ e $G(x) = \frac{1}{2}\sec^2 x + C_2$,

então.

$F(x) = \frac{1}{2}\tan^2 x + C_1 = \frac{1}{2}\left(\sec^2 x - 1\right) + C_1 =$

$1 + \tan^2 x = \sec^2 x$

$\frac{1}{2}\sec^2 x \underbrace{- \frac{1}{2} + C_1}_{= C_2} = G(x)$

Ou seja, ambas as soluções diferem entre si por apenas uma constante. A saber, pela notação feita:

$C_2 = \frac{1}{2} + C_1$