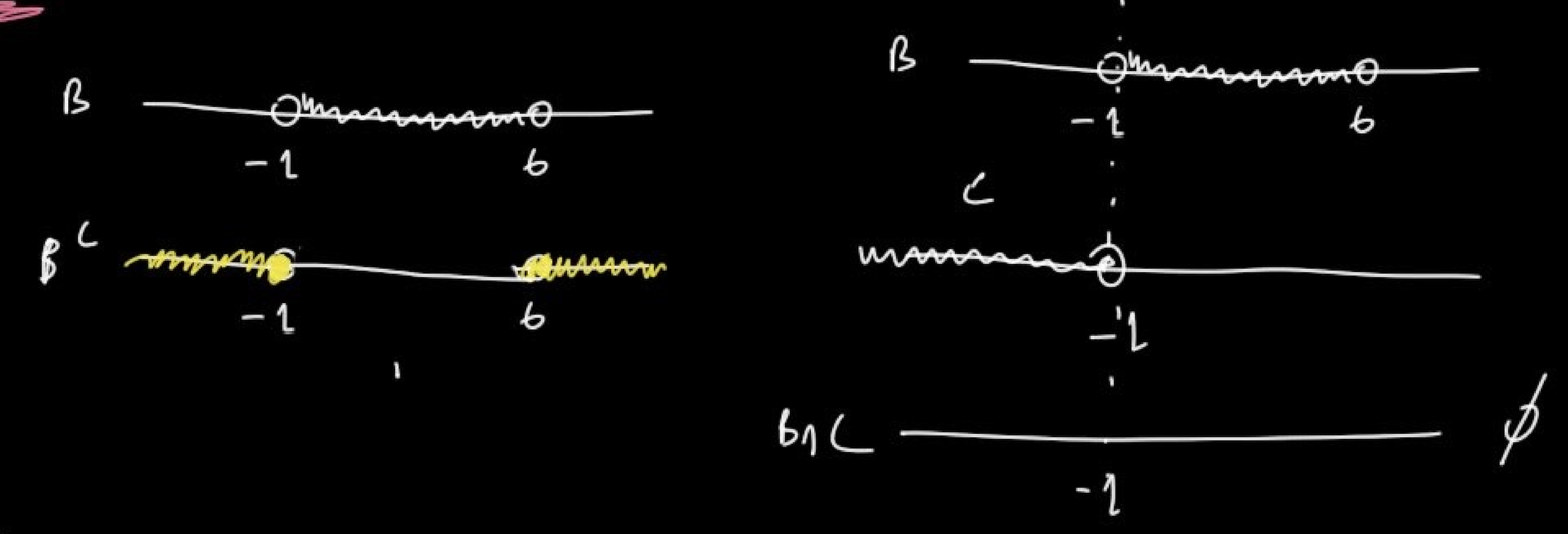
LISTA 01.

1. Dados os intervalos $A = [-3, 2]$, $B = (-1, 6)$ e $C = (-\infty, -1)$, obtenha o resultado de cada operação abaixo:

(a) $A \cup B$ (b) $A \setminus B$ (c) $A \cap C$ (d) $A \setminus C$ (e) $B \setminus A$ (f) $B^{\complement} \setminus (B \cap C)$

(f) $B^c \setminus (B \cap C)$.



Logo, temos:

$$B^c \setminus (B \cap C) = B^c \setminus \emptyset = B^c = (-\infty, -1] \cup [6, +\infty).$$

2. Encontre os valores de $x \in \mathbb{R}$ que verificam cada desigualdade abaixo.

(a) $5x + 2 > x - 6$ (b) $3x - 5 < \frac{3}{4}x + \frac{1-x}{3}$ (c) $2 \leq 5 - 3x \leq 11$

(d) $\frac{2}{1-x} \leq 1$ (e) $\frac{1}{3x-7} \geq \frac{4}{3-2x}$ (f) $|2x - 4| \leq 2$

(g) $\left|\frac{2-x}{3x-1}\right| \leq \frac{2}{3}$ (h) $\left|\frac{1}{x-2} - \frac{x}{3}\right| \leq 1$ (i) $\left|\frac{2x-4}{2-3x}\right| \leq \frac{1}{4}$

(b) $3x - 5 < \frac{3}{4}x + \frac{1-x}{3}$

$3x - 5 < \frac{9x + 4 \cdot (1 - x)}{12}$ $(\times 12 > 0)$

$12 \cdot (3x - 5) < 9x + 4 - 4x$

$36x - 60 < 9x + 4$

$36x - 5x < 4 + 60$

$31x < 64$ $(\div 31 > 0)$

$\boxed{x < \frac{64}{31}}$

$S = \{x \in \mathbb{R} : x < \frac{64}{31}\}$.

h) $\left|\frac{1}{x-2} - \frac{x}{3}\right| \leq 1 \iff \left|\frac{3 - x(x-2)}{3(x-2)}\right| \leq 1$

$\iff \left|\frac{3 - x^2 + 2x}{3(x-2)}\right| \leq 1$

$\iff \left|\frac{-x^2 + 2x + 3}{3(x-2)}\right| \leq 1$

$\iff \left|\frac{-1(x^2 - 2x - 3)}{3(x-2)}\right| \leq 1 = \underbrace{|-1|}_{= 1} \cdot \left|\frac{x^2 - 2x - 3}{3(x-2)}\right| \leq 1 \iff \left|\frac{x^2 - 2x - 3}{3(x-2)}\right| \leq 1$

$|\alpha| \leq 1 \iff -1 \leq \alpha \leq 1.$

Disso, temos que:

$$\left|\frac{x^2-2x-3}{3(x-2)}\right| \le 1 \iff \underbrace{-1 \le \overbrace{\frac{x^2-2x-3}{3(x-2)} \le 1}^{(II)}}_{(I)}$$

$$(I):\quad \frac{x^2-2x-3}{3(x-2)} \ge -1 \iff \frac{x^2-2x-3}{3(x-2)} + \frac{1}{1} \ge 0$$

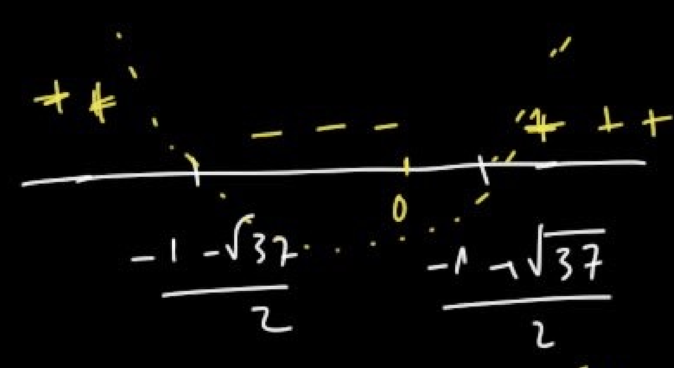
$$\frac{x^2-2x-3+3(x-2)}{3(x-2)} \ge 0 \iff \frac{x^2-2x-3+3x-6}{3(x-2)} \ge 0$$

$$\iff \frac{x^2+x-9}{3(x-2)} \ge 0$$

- zeros do numerador (isto para fazer o estudo do sinal)

$$x^2+x-9=0$$

$$\iff x = \frac{-1 \pm \sqrt{1+36}}{2\cdot 1} \quad \begin{cases} x = \dfrac{-1+\sqrt{37}}{2} \\[2mm] x = \dfrac{-1-\sqrt{37}}{2} \end{cases}$$



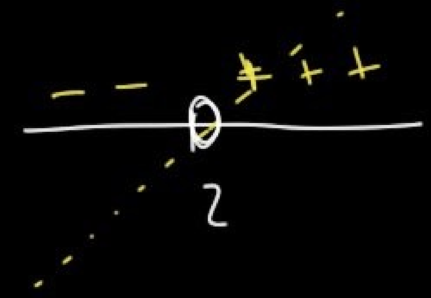
PENSE COMO FUNÇÃO QUADRÁTICA:

$y = x^2+x-9$

$a = 1 > 0$ c.p.c.

- zeros do denominador $(\neq 0)$

$$3(x-2)=0 \iff x=2$$

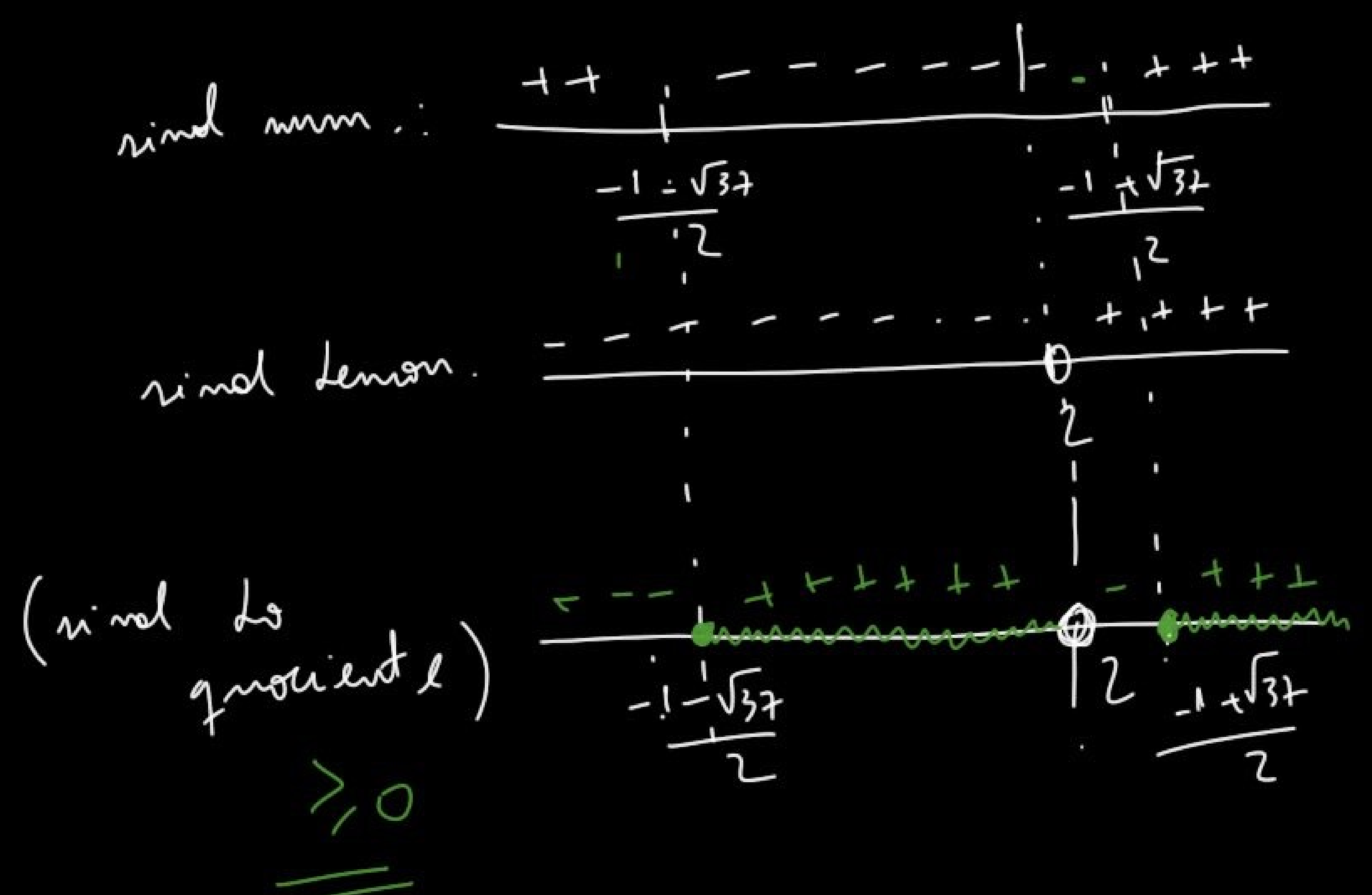


PENSE COMO FUNÇÃO AFIM

$y = 3x-6$

$a = 3 > 0$

f é cresc.



$$D_1 = \left[\frac{-1-\sqrt{37}}{2},\, 2\right) \cup \left[\frac{-1+\sqrt{37}}{2},\, +\infty\right)$$

$$(II):\quad \frac{x^2-2x-3}{3(x-2)} \le 1 \iff \frac{x^2-2x-3}{3(x-2)} - 1 \le 0$$

... et cetera ...

↳ vamos obter solução D_2.

A solução final será

$$D = D_1 \cap D_2$$ → POIS PRECISA SATISFAZER AS DUAS DESIGUALDADES

4. Sejam $f : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{-\frac{1}{2}\}$, $g : \mathbb{R} \setminus \{\frac{5}{2}\} \to \mathbb{R} \setminus \{0\}$ e $h : \mathbb{R} \setminus \{0\} \to \mathbb{R} \setminus \{0\}$, dadas respectivamente por

$$f(x) = -\frac{1+x}{2}; \quad g(x) = 2x - 5 \quad \text{e} \quad h(x) = \frac{1}{x}.$$

Justifique por que é possível montar a composição $u = f \circ h \circ g$. Em seguida, determine a função $u : A \to B$ exibindo quem é o domínio A e quem é o contradomínio B.

$$f(h(g(x)))$$

$$\begin{tikzcd}
\mathbb{R}\setminus\{\frac{5}{2}\} \arrow[r, "g"] \arrow[rrr, bend right] & \mathbb{R}\setminus\{0\} \arrow[r, "h"] & \mathbb{R}\setminus\{0\} \arrow[r, "f"] & \mathbb{R}\setminus\{-\frac{1}{2}\}
\end{tikzcd}$$

$$u =: f \circ h \circ g$$

Logo, é possível montar a composição acima, devido aos domínios e contra-domínios dados

Disso, temos:

$$u : \underbrace{\mathbb{R}\setminus\{\tfrac{5}{2}\}}_{A} \longrightarrow \underbrace{\mathbb{R}\setminus\{-\tfrac{1}{2}\}}_{B}.$$

$\forall x \in \mathbb{R}\setminus\{\frac{5}{2}\}$, temos:

$$u(x) = (f \circ h \circ g)(x) = f(h(g(x))) = f(h(2x-5))$$

$$= f\left(\frac{1}{2x-5}\right) = -\frac{\frac{1}{1} - \frac{1}{2x-5}}{2} = -\frac{\frac{2x-5-1}{2x-5}}{2}$$

$$= \frac{2x-6}{2x-5} \cdot \left(-\frac{1}{2}\right) = \frac{\cancel{2}(x-3)}{(2x-5)(-\cancel{2})}$$

$$= \frac{3-x}{2x-5}$$

conclusão: obtemos $u : \mathbb{R}\setminus\{\frac{5}{2}\} \to \mathbb{R}\setminus\{-\frac{1}{2}\}$

$$u(x) = \frac{3-x}{2x-5}$$

5. Construa o gráfico de cada função a seguir, indicando domínio e imagem.

(a) $f(x) = \begin{cases} x^2 - 4x + 3, & \text{se } x \geq 1 \\ x - 1, & \text{se } x < 1 \end{cases}$

(b) $f(x) = |2x - 2| + |x + 3|$

(c) $f(x) = x^2 - 4|x| + 3$

(d) $f(x) = (x^2 - 1) + |x^2 - 1| + 1$

(b) $y = |2x-2| + |x+3|$

Note que:

• $|2x-2| = \begin{cases} 2x-2, & \text{se } 2x-2 \geq 0 \\ -(2x-2), & \text{se } 2x-2 < 0 \end{cases} = \begin{cases} 2x-2, & \text{se } x \geq 1 \\ -2x+2, & \text{se } x < 1 \end{cases}$

• $|x+3| = \begin{cases} x+3, & \text{se } x+3 \geq 0 \\ -(x+3), & \text{se } x+3 < 0 \end{cases} = \begin{cases} x+3, & \text{se } x \geq -3 \\ -x-3, & \text{se } x < -3 \end{cases}$

Ou seja, obtemos:

$$|2x-2| = \begin{cases} 2x-2, & \text{se } x \geq 1 \\ -2x+2, & \text{se } x < 1 \end{cases}$$

$$|x+3| = \begin{cases} x+3, & \text{se } x \geq -3 \\ -x-3, & \text{se } x < -3 \end{cases}$$

$y = |2x-2| + |x+3|$

$|2x-2| = -2x+2$ | $|2x-2| = -2x+2$ | $|2x-2| = 2x-2$

-3 1

$|x+3| = -x-3$ | $|x+3| = x+3$ | $|x+3| = x+3$

-3 1

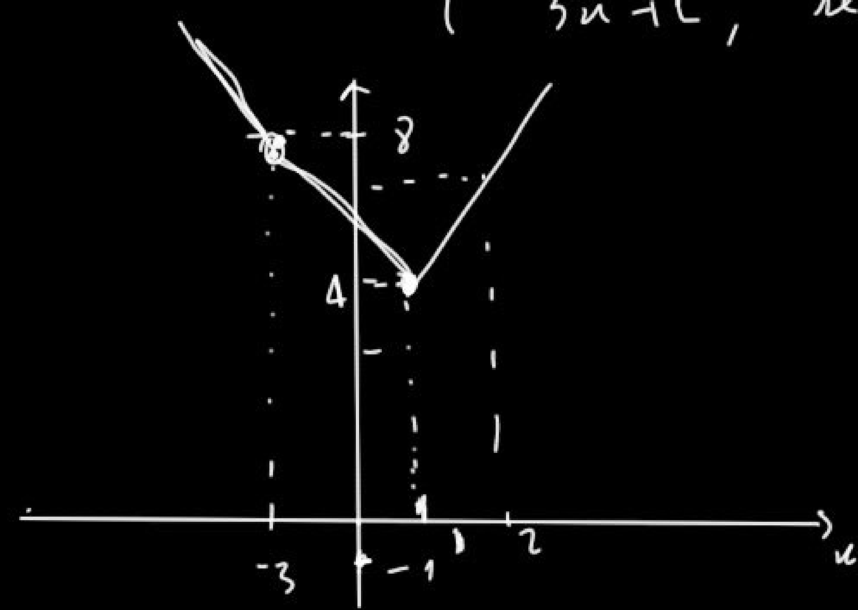
$|2x-2| + |x+3| = -2x+2-x-3 = -3x-1$

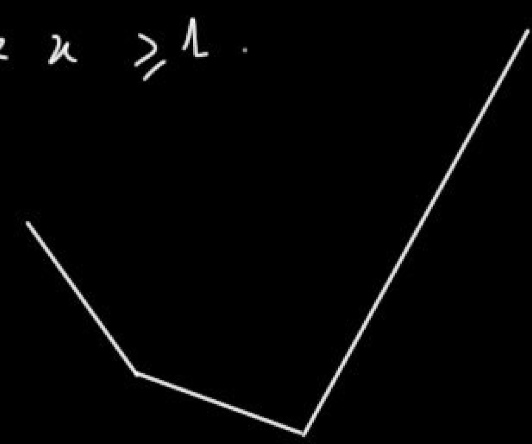
$|2x-2| + |x+3| = -2x+2+x+3 = -x+5$

$|2x-2| + |x+3| = 2x-2+x+3 = 3x+1$.

Ou seja, obtemos

$$f(x) = |2x-2| + |x+3| = \begin{cases} -3x-1, & \text{se } x < -3 \\ -x+5, & \text{se } -3 \leq x < 1 \\ 3x+1, & \text{se } x \geq 1. \end{cases}$$





$D(f) = \mathbb{R}$. $\text{Im}(f) = [4, +\infty)$.