LISTA 03

06) (c) $\int \frac{dx}{x\ln^2 x} = \int (\underbrace{\ln x}_{r})^{-2} \cdot \underbrace{\frac{dx}{x}}_{dr} = \int r^{-2}\,dr = \frac{r^{-2+1}}{-2+1} + C$

$r = \ln x$

$\Rightarrow dr = \frac{1}{x}dx$

ok!

$= \frac{r^{-1}}{-1} + C = -\frac{1}{r} + C = -\frac{1}{\ln x} + C$

LISTA 02 – 03

(c) $F(x) = \int_1^{\sqrt{x}} \frac{z^2}{z^4+1}\,dz$

Obs.! Lembre que:

$F(x) = \int_a^x f(t)\,dt$

$\frac{d}{dx}F(x) = \frac{d}{dx}\int_a^x f(t)\,dt$

$= f(t)$

no entanto, neste item não temos apenas x como limite superior.

temos $u = \sqrt{x}$.

Então, vamos ter que usar a regra da cadeia

$F(u) = \int_1^u \frac{z^2}{z^4+1}\cdot dz$; onde $u = x^{\frac{1}{2}}$.

Assim, obtemos:

$$\frac{d}{dx}\Big(\underbrace{\int_1^u \frac{z^2}{z^4+1}\,dz}_{F(u)}\Big) = F'(u)\cdot u' = f(u)\cdot u' = \frac{u^2}{u^4-1}\cdot\frac{1}{2}\cdot x^{-\frac{1}{2}} = \frac{(x^{\frac{1}{2}})^2}{(x^{\frac{1}{2}})^4-1}\cdot\frac{1}{2}x^{\frac{1}{2}}$$

$, u = u(x)$

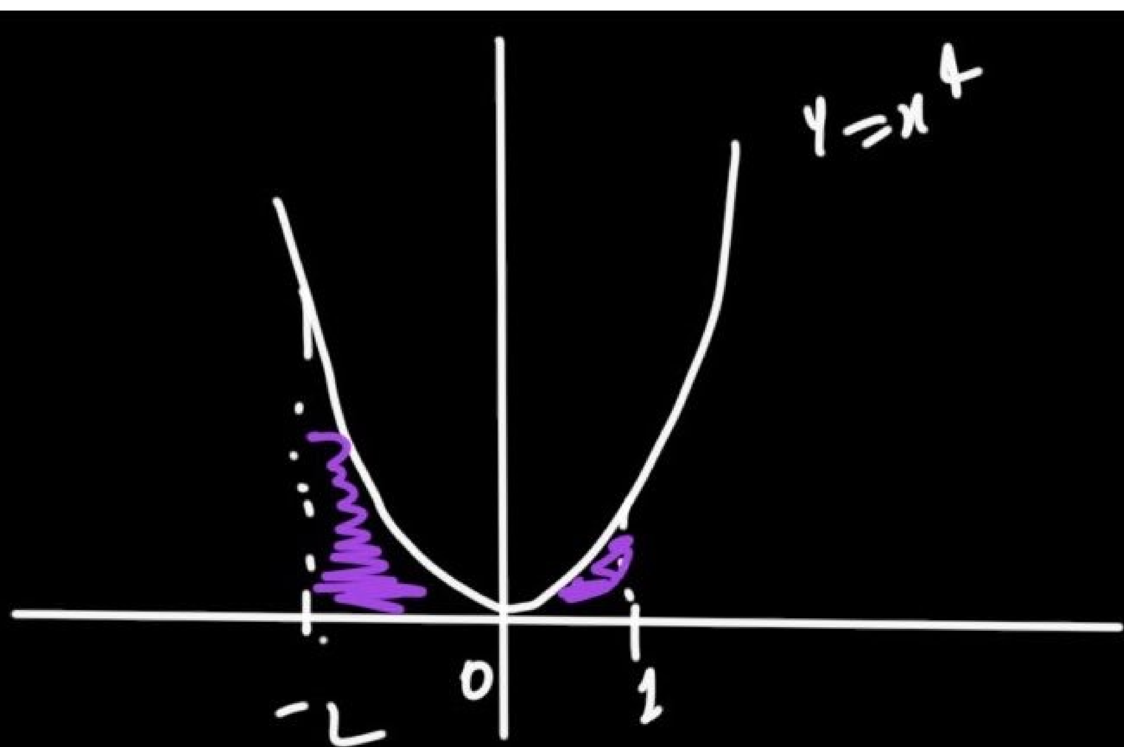
$$\boxed{f(z) = \frac{z^2}{z^4+1}}$$

$$u = x^{\frac{1}{2}} = \frac{1}{2}x^{-\frac{1}{2}}$$

$$= \frac{x}{x^2-1}\cdot\frac{1}{2}\sqrt{x} = \frac{x\sqrt{x}}{2(x^2-1)}$$

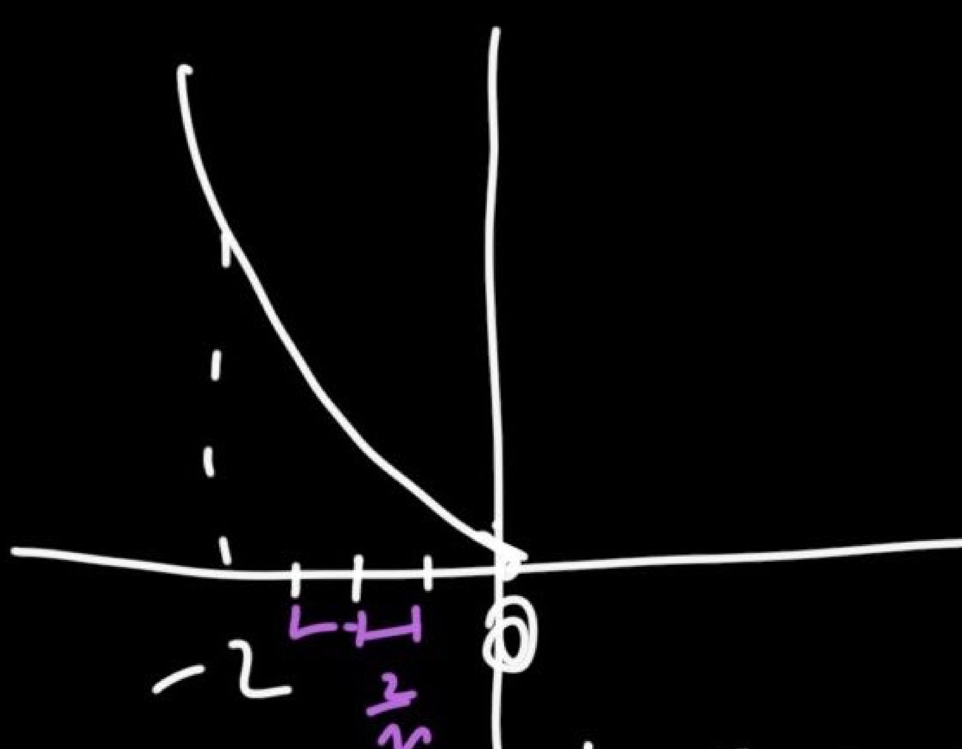
LISTA 01.

8. Calcule $\int_{-2}^{1} x^4 dx$, usando uma partição regular.



$$\int_{-2}^{1} f = \underbrace{\int_{-2}^{0} f} + \int_{0}^{1} f$$

- $\displaystyle\int_{-2}^{0} x^4\,dx$:



Seja P_n partição regular de $[-2, 0]$ que divide este intervalo em n subintervalos de mesmo comprimento:

$$\Delta x = \frac{0-(-2)}{n} = \frac{2}{n} \quad (\ldots)$$

etc.

PROVA ANTIGA.

Questão 02. Seja $f : [0,2] \to \mathbb{R}$ dada por

$$f(x) = \begin{cases} x+1, & \text{se } x \in [0,1) \\ 1, & \text{se } x \in [1,2] \end{cases}$$

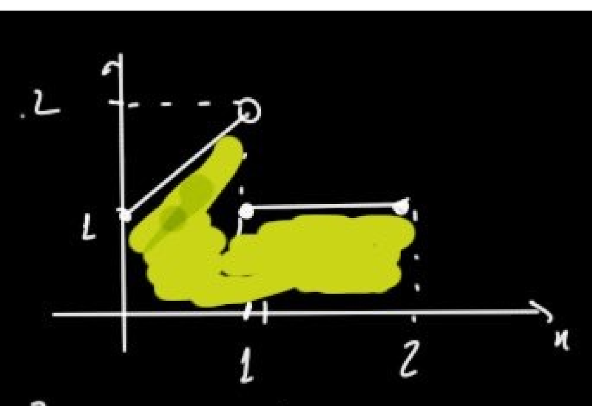
(a) Esboce o gráfico de f e calcule $\int_0^2 f$ das seguintes formas:

(i) através do esboço gráfico de f;

(ii) usando a definição de integral definida;

(iii) usando o Teorema Fundamental do Cálculo.

(i)



$$\int_0^2 f = \int_0^1 f + \int_1^2 f$$

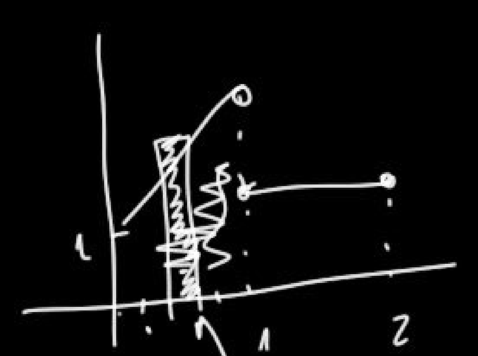
- $\int_0^1 f = \int_0^1 (x+1)\,dx = \dfrac{(B+b)\cdot h}{2} = \dfrac{(2+1)\cdot 1}{2} = \dfrac{3}{2}$

ÁREA DE UM TRAPÉZIO

- $\int_1^2 f = \int_1^2 1\,dx = b \times h = 1 \times 1 = 1$

Logo, $\int_0^2 f = \dfrac{3}{2} + 1 = \dfrac{5}{2}$

(ii)



$$\int_0^1 f = \int_0^1 (x+1)\,dx.$$

Seja P_n partição regular do intervalo $[0,1]$, que divide em n subintervalos de comprimento

$$t_i - t_{i-1} = \Delta x = \frac{1-0}{n} = \frac{1}{n}.$$

neste caso, $t_i = 0 + i \cdot \dfrac{1}{n} = \dfrac{i}{n}$, $i \in \{1, 2, \dots, n\}$

Como f é crescente $[0,1]$, então

$$M_i = \sup_{x \in [t_{i-1}, t_i]} f(x) = f(t_i) = t_i + 1 = \frac{i}{n} + 1.$$

Assim, montando a soma superior, temos:

$$S(f, P_n) = \sum_{i=1}^{n} M_i \cdot \underbrace{(t_i - t_{i-1})}_{\Delta t_i = \Delta x} = \sum_{i=1}^{n} \left(\frac{i}{n} + 1\right) \cdot \frac{1}{n} =$$

$$= \sum_{i=1}^{n} \left(\frac{i}{n^2} + \frac{1}{n}\right) = \sum_{i=1}^{n} \frac{i}{n^2} + \sum_{i=1}^{n} \frac{1}{n} =$$

$$= \frac{1}{n^2} \underbrace{\sum_{i=1}^{n} i}_{\frac{(1+n)n}{2}} + \frac{1}{n} \cdot \underbrace{\sum_{i=1}^{n} 1}_{n} = \frac{1}{n^{\cancel{2}}} \cdot \frac{(1+n)\cdot \cancel{n}}{2} + \frac{1}{\cancel{n}} \cdot \cancel{n} = \frac{1+n}{2n} + 1$$

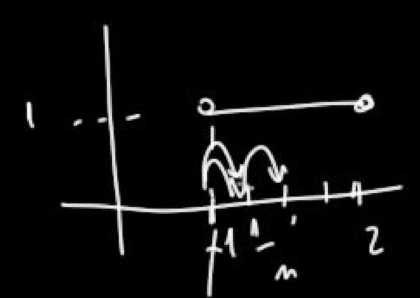
$S(f;P_n) = \frac{1+n}{2n} + 1$

$$\overline{\int_0^1} f = \lim_{n\to\infty} S(f;P_n) = \lim_{n\to\infty}\left(\frac{1+n}{2n}+1\right)$$

$$= \frac{1}{2} + 1 = \frac{3}{2}$$

Do mesmo modo tem-se que

$\underline{\int_0^1} f = \frac{3}{2}$ Logo, $\int_0^1 f = \frac{3}{2}$



Seja P_n partição regular que divide $[1,2]$ em n subintervalos de comprimento $\Delta x = \frac{2-1}{n} = \frac{1}{n}$.

$t_0 = 1$

$t_1 = 1 + \frac{1}{n}$ .... $t_j = 1 + j\cdot\frac{1}{n}$ ....

$t_2 = 1 + 2\cdot\frac{1}{n}$

f é constante então, $M_i = \sup_{(t_{i-1},t_i)} f(x) = 1$

Assim, $S(f,P_n) = \sum_{i=1}^{n} M_i \cdot \underbrace{(t_i - t_{i-1})}_{\frac{1}{n}} = \sum_{i=1}^{n} 1\cdot\frac{1}{n} = \sum_{i=1}^{n} \frac{1}{n} =$

$$= \frac{1}{n}\cdot\underbrace{\sum_{i=1}^{n} 1}_{n} = \frac{1}{\cancel{n}}\cdot\cancel{n} = 1.$$

$\Rightarrow S(f,P_n) = 1, \quad \forall P_n.$

$$\overline{\int_1^2} f = \lim_{n\to\infty} S(f,P_n) = 1$$

Do mesmo modo, $\underline{\int_1^2} f = 1$

Então, $\int_1^2 f = 1$

Enfim, obtemos:

$$\int_0^2 f = \int_0^1 f + \int_1^2 f = \frac{3}{2} + 1 = \frac{5}{2}$$

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vi.) pelo T.F.C.:

$$\int_0^2 f = \int_0^1 (x+1)\,dx + \int_1^2 1\,dx$$

$$\left(\frac{x^2}{2} + x\right)\Big|_0^1 + x\Big|_1^2 = \frac{1}{2} + 1 - 0 + (2-1) = \frac{5}{2}$$

EXTRA:

(b) $\int \frac{xe^{\sqrt{3x^2+2}}dx}{\sqrt{3x^2+2}}$

$\int e^u \, du = e^u + c$

$u = \sqrt{3x^2+2} = (3x^2+2)^{\frac{1}{2}}$

$du = \frac{1}{2}(3x^2+2)^{\frac{1}{2}-1} \cdot 6x \, dx$

$du = \frac{3x \, dx}{\sqrt{3x^2+2}}$

$\frac{du}{3} = \frac{x \, dx}{\sqrt{3x^2+2}}$

$\int \frac{x e^{\overbrace{\sqrt{3x^2+2}}^{u}} dx}{\sqrt{3x^2+2}} = \int e^u \frac{du}{3} = \frac{1}{3} \int e^u \, du =$

$\frac{du}{3}$

$= \frac{1}{3} e^u + c = \frac{1}{3} \cdot e^{\sqrt{3x^2+2}} + c$