RESOLUÇÃO DE EXERCÍCIOS DAS LISTAS.

LISTA 05.

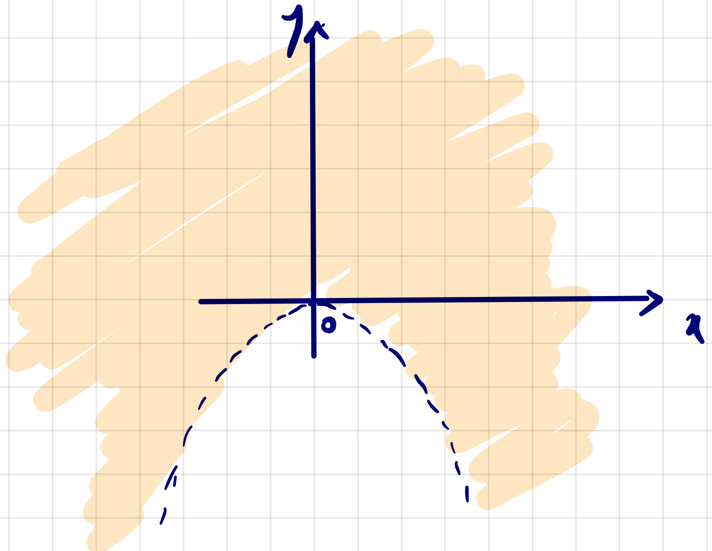
01) - e) $f(x,y) = \ln(x^2+y)$ $\qquad f: \Omega \subset \mathbb{R}^2 \to \mathbb{R}$

$\Omega = D(f) = ?$

$$x^2+y > 0$$

$$y > -x^2$$

$$\Omega = D(f) = \{(x,y) \in \mathbb{R}^2 : y > -x^2\}$$



01 - f) $f(x,y) = \text{arcsen}(x+y)$

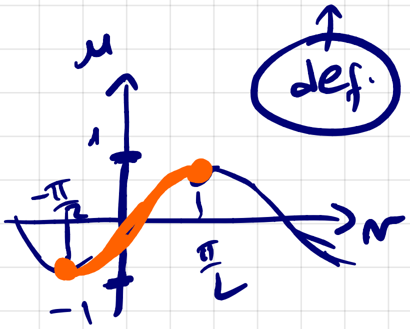
Obs: $u = \text{sen}\, v \iff v = \text{arcsen}\, u$

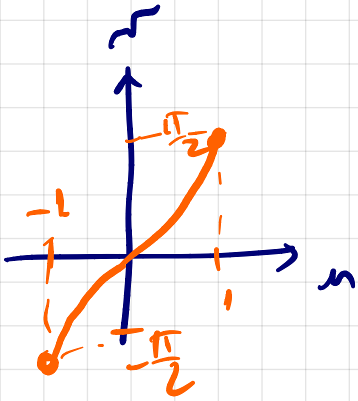
def.

ou

$-1 \leq u \leq 1$

$-\frac{\pi}{2} \leq v \leq \frac{\pi}{2}$

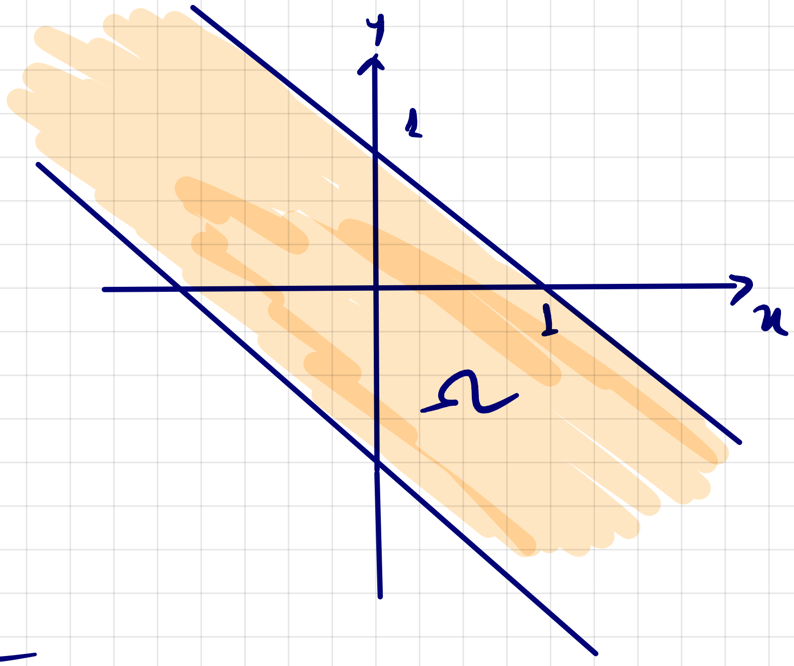




condição de existência: $-1 \leq x+y \leq 1$

$$-1-x \leq y \leq 1-x$$

$D(f) = \Omega = \{(x,y) \in \mathbb{R}^2 : -1 \leq x + y \leq 1\}$



02 – d) $\quad f(x,y) = \sqrt{x+y}$.

$\Omega = D(f) = ?$

$x + y \geq 0$

$y \geq -x$

$D(f) = \{(x,y) \in \mathbb{R}^2 : y \geq -x\}$

gráfico do domínio:

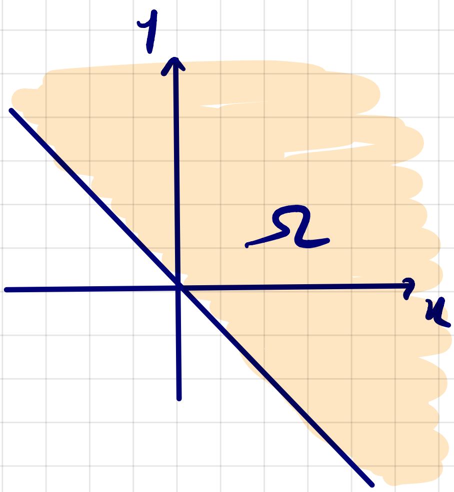


gráfico de f : ?

$f : \Omega \subset \mathbb{R}^2 \to \mathbb{R}$.

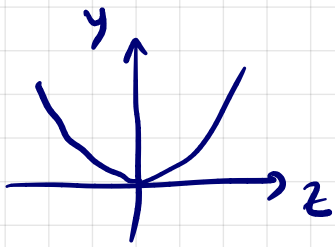
$z = f(x,y) \Rightarrow z = \sqrt{x+y}$

Elevando ao quadrado, obtemos:

$z^2 = x + y$ $\qquad z \geq 0$

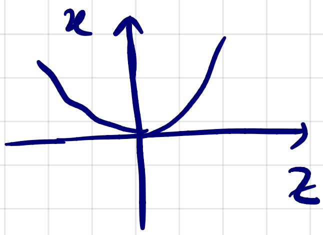
traços • $x = 0$: (plano zy):

$z^2 = y$ (parábola)



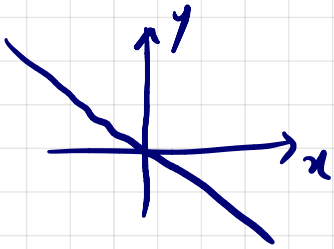
• $y = 0$: (plano xz)

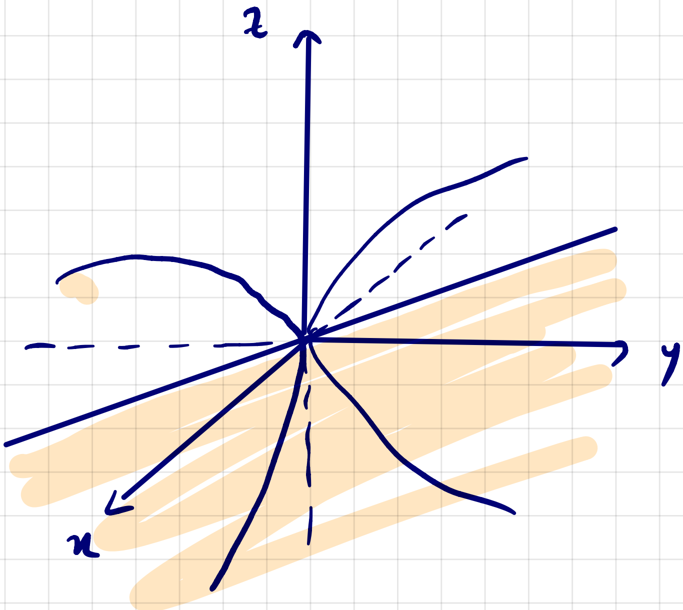
$z^2 = x$ (parábola)



• $z = 0$: $x + y = 0$

$y = -x$ (reta).





?

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02 – c) $f(x,y) = 4x^2 + 9y^2$

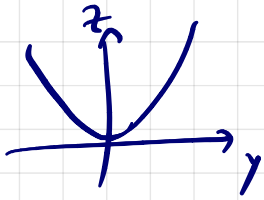
$\Omega = D(f) = \mathbb{R}^2$ [não há restrição para (x,y).]

$z = 4x^2 + 9y^2$

traços:

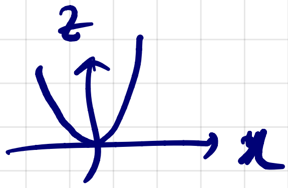
• $x = 0$ (plano yz)

$z = 9y^2$ (parábola)



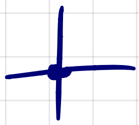
• $y = 0$: (plano xz)

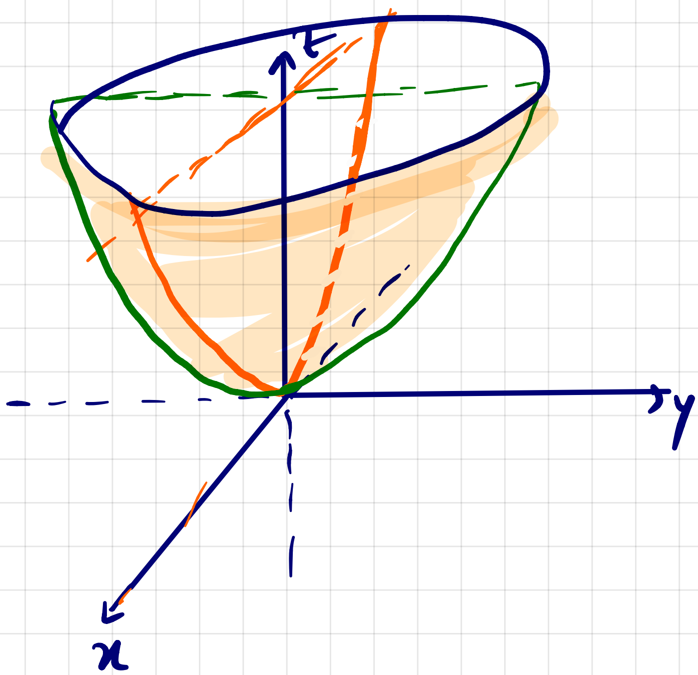
$z = 4x^2$ (parábola



• $z = 0$: (plano xy)

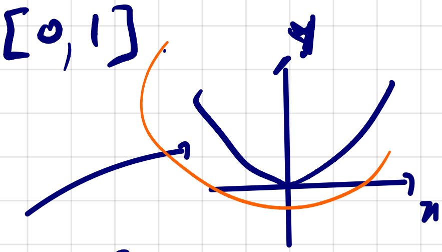
$4x^2 + 9y^2 = 0 \iff (x,y) = (0,0)$ (origem)

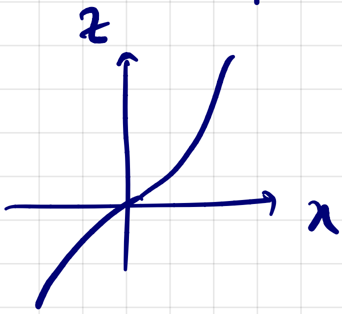


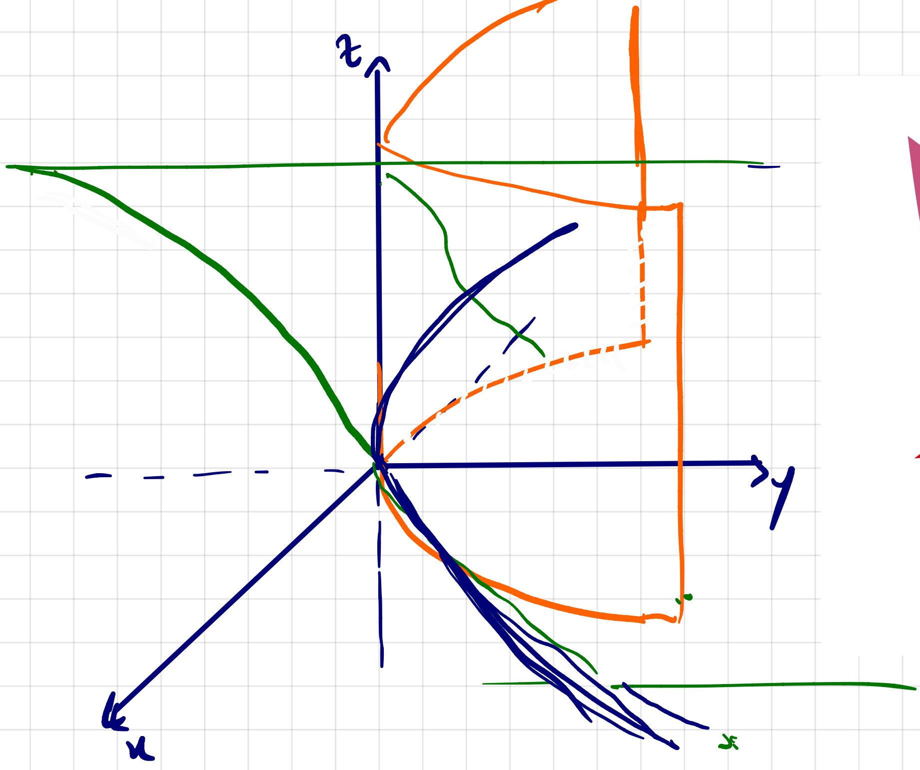


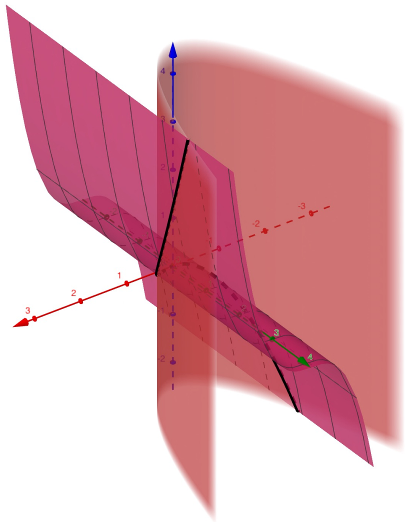
03) (a) $f(t) = (t, t^2, t^3)$; $t \in [0,1]$

$$\begin{cases} x = t \\ y = t^2 \\ z = t^3 \end{cases} \quad \begin{matrix} y = x^2 \\ z = x^3 \end{matrix}$$









(PELO GEOGEBRA)

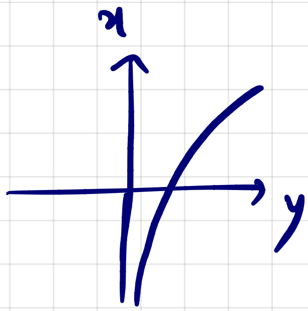
c) $f(t) = (\ln t,\ t,\ 1)$
$\qquad\qquad\quad x \qquad y \quad z$

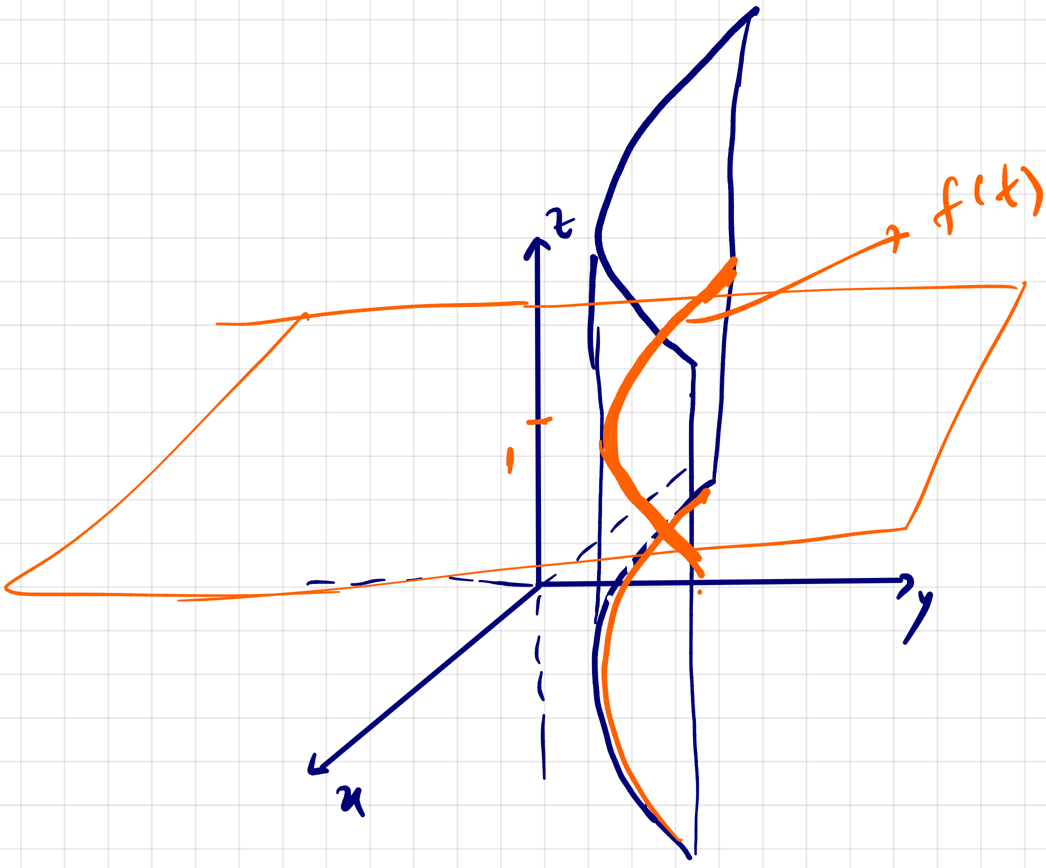
$x = x(t)$
$y = y(t)$
$z = z(t)$

$$\left\{\begin{array}{l} x = \ln t \\ y = t \\ z = 1. \end{array}\right.$$

$x = \ln y$

$z = 1$





01) a) $\vec{f}(t) = \left(\underbrace{\sqrt{2t+6}}_{D_1}, \underbrace{\sqrt{\frac{1-t}{2-t}}}_{D_2}, \underbrace{\ln(2-t)}_{D_3} \right)$

CORRIGIR.

CORRIGIR NA LISTA.

$D = D_1 \cap D_2 \cap D_3$

D_1: $2t+6 \geq 0 \iff 2t \geq -6 \iff t \geq -3$

-3

D_2: $\frac{1-t}{2-t} \geq 0$ zeros num. e sinal: $1-t=0$, $t=1$

$++ \quad --$

1

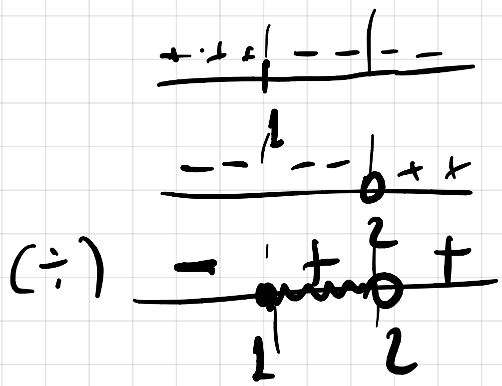
$\begin{cases} t>1 \Rightarrow 1-t<0 \\ t<1 \Rightarrow 1-t>0 \end{cases}$

zeros e sinal do denom.: $2-t=0 \iff t=2$

$-- \quad ++$

2

$\begin{cases} t<2 \Rightarrow 2-t>0 \\ t>2 \Rightarrow 2-t<0 \end{cases}$



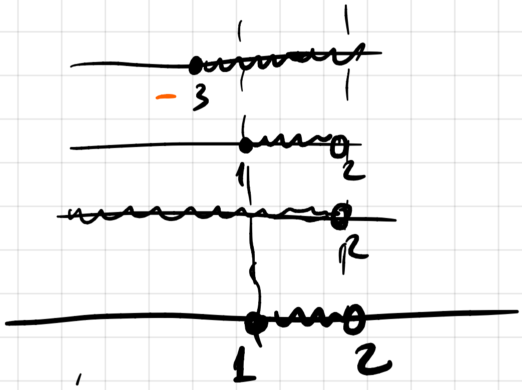
$D_2 = [1, 2)$

$1 \quad 2$

D_3: $2-t>0 \iff t<2$

2

$D_1 \cap D_2 \cap D_3$:

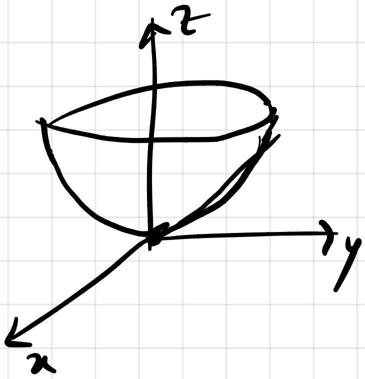


$D(f) = [1, 2)$

05) $f(x,y) = \dfrac{1}{x^2+y^2}$

$z = x^2+y^2$

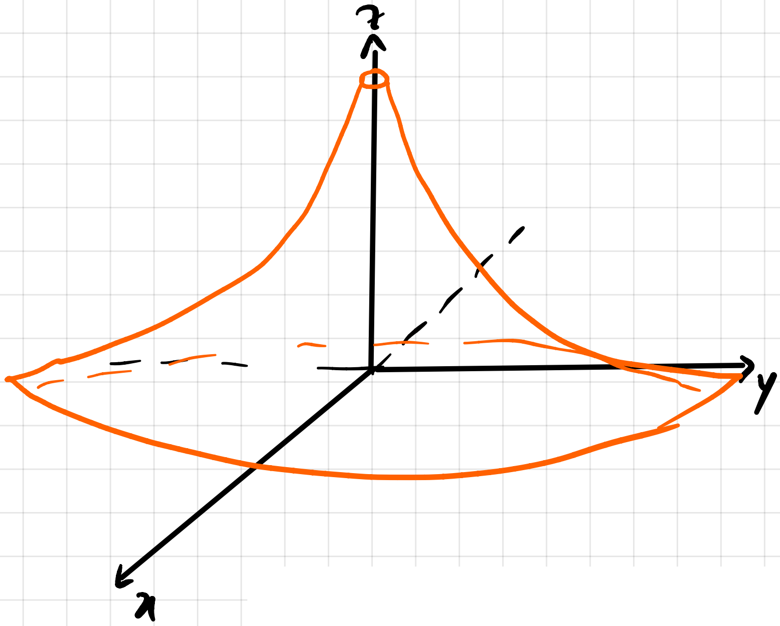
PARABOLÓIDE



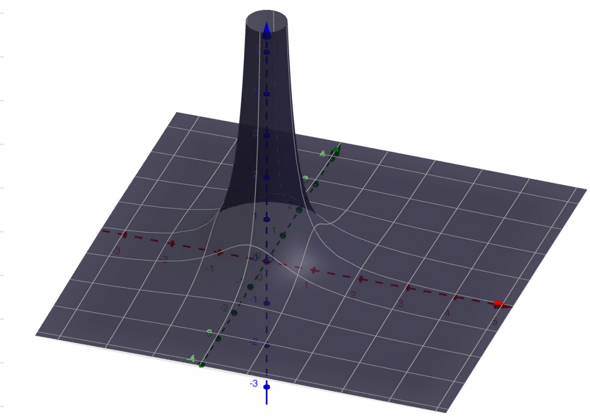
o gráfico de f será "o inverso" deste.

- o inverso de "ir para zero" é "ir para o infinito";

e vice-versa.



GEOGEBRA



07)(a)

$$f(u,v) = (x,y,z) \; ;$$

$$f : \underset{\mathbb{R}^2}{\underset{\parallel}{\Omega}} \subset \mathbb{R}^2 \to \mathbb{R}^3$$

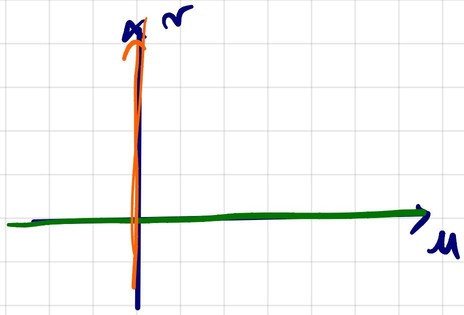
pois $u, v \in \mathbb{R}$

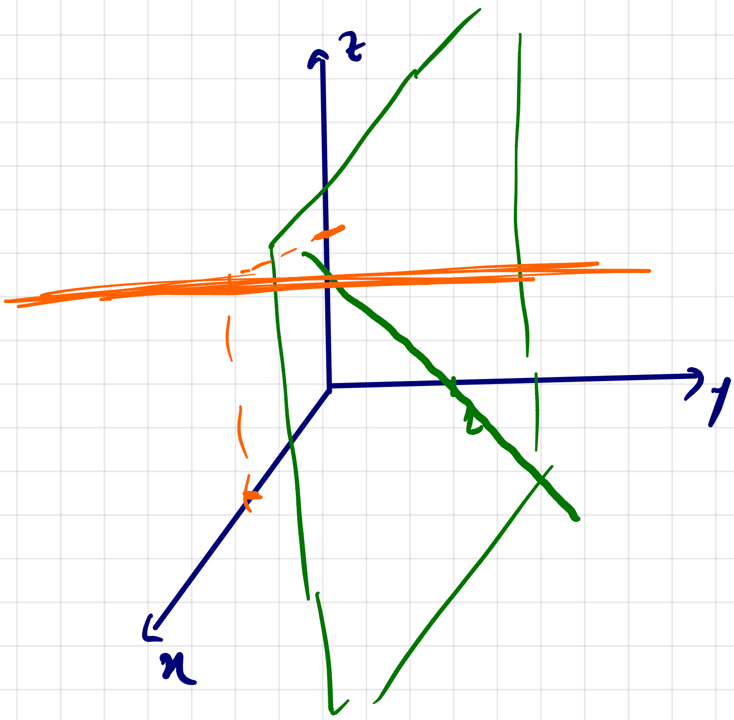
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} u \\ v \\ u \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{cases} x = u+1 \\ y = v+1 \\ z = u+1 \end{cases} \qquad x = z$$

$$y = v+1 \in \mathbb{R}.$$





eixo u: $v = 0$

$$f(u,0) = (\overset{x}{\underbrace{u+1}}, 1, \overset{z}{\underbrace{u+1}})$$

$$f(u,v) = (u+1, v+1, u+1)$$

eixo v: $u = 0$

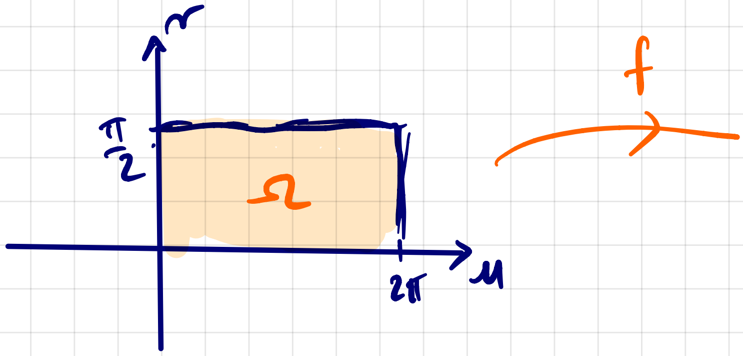
$$f(0,v) = (\underbrace{1}_{x},\ v+1,\ \underbrace{1}_{z})$$

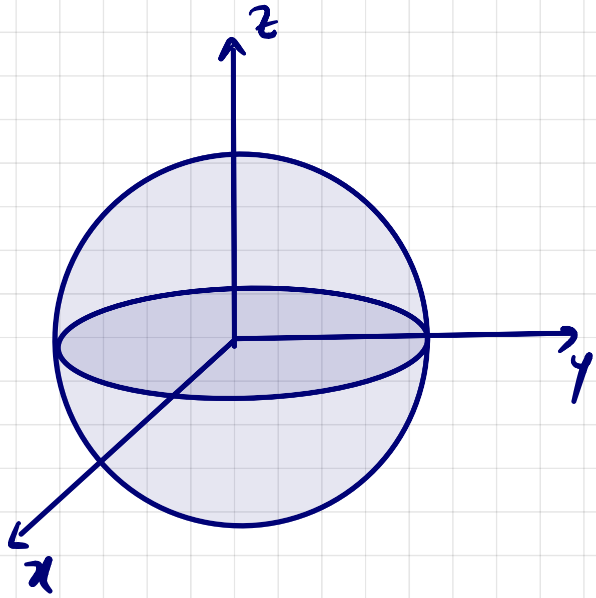
este exemplo $\mathbb{R}^2 \to \mathbb{R}^3$ tem apenas como ver algumas curvas do $\mathbb{R}^2$ sendo transformadas em outras curvas no $\mathbb{R}^3$, como fizemos com os dois eixos.

(b) $f(u,v) = (\overbrace{\cos u \,\text{sen}\, v}^{x},\ \overbrace{\text{sen}\, u \cdot \text{sen}\, v}^{y},\ \overbrace{\cos v}^{z})$

$$0 \leq u \leq 2\pi$$

$$0 \leq v \leq \frac{\pi}{2}$$





Note que

$$x^2 + y^2 + z^2 = (\cos u \,\text{sen}\, v)^2 + (\text{sen}\, u \cdot \text{sen}\, v)^2 + (\cos v)^2$$

$$= \cos^2 u \cdot \text{sen}^2 v + \text{sen}^2 u \cdot \text{sen}^2 v + \cos^2 v$$

$$= \text{sen}^2 v \underbrace{\left[\cos^2 u + \text{sen}^2 u\right]}_{=1} + \cos^2 v$$

$= \operatorname{sen}^2 v + \cos^2 v = 1.$

$\Longrightarrow \boxed{x^2+y^2+z^2=1} \rightarrow$ esfera no $\mathbb{R}^3$.