19/09/23

LISTA 08

02) $\vec{f}(t) = t\vec{i} - \vec{j} + 0\vec{k} = (t, -1, 0)$

$\vec{g}(t) = \vec{i} + t\vec{j} + 0\vec{k} = (1, t, 0)$

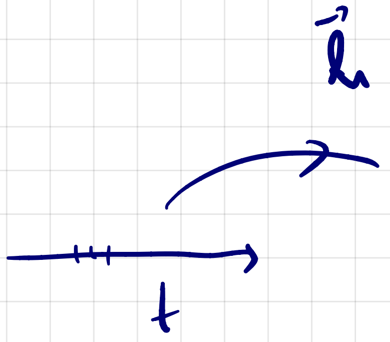
Defina $\vec{h}(t) = \vec{f}(t) \times \vec{g}(t)$; ou seja:

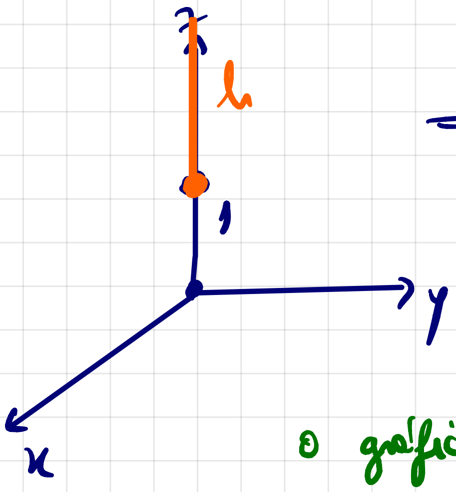
$$\vec{h}(t) = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ t & -1 & 0 & t & -1 \\ 1 & t & 0 & 1 & t \end{array}\right|$$

$$= 0\vec{i} + 0\vec{j} + t^2\vec{k} + \vec{k} - 0\vec{i} - 0\vec{j}$$

$$\vec{h}(t) = 0\vec{i} + 0\vec{j} + (t^2+1)\vec{k}$$

$$\vec{h}(t) = (0, 0, t^2+1), \ \forall t \in \mathbb{R}.$$





$t^2 \geq 0, \ \forall t,$

$\Rightarrow t^2 + 1 \geq 1, \ \forall t,$

O gráfico de $\vec{h} = \vec{f} \times \vec{g}$ será uma semi-reta com origem em $(0,0,1)$, sobre o eixo z, com orientação ascendente.

LISTA 08

03) $\vec{r}(t) = \left(t, \frac{1}{t-2}, 1\right)$

(a) $t = 0$; $\vec{r}(t) = ?$

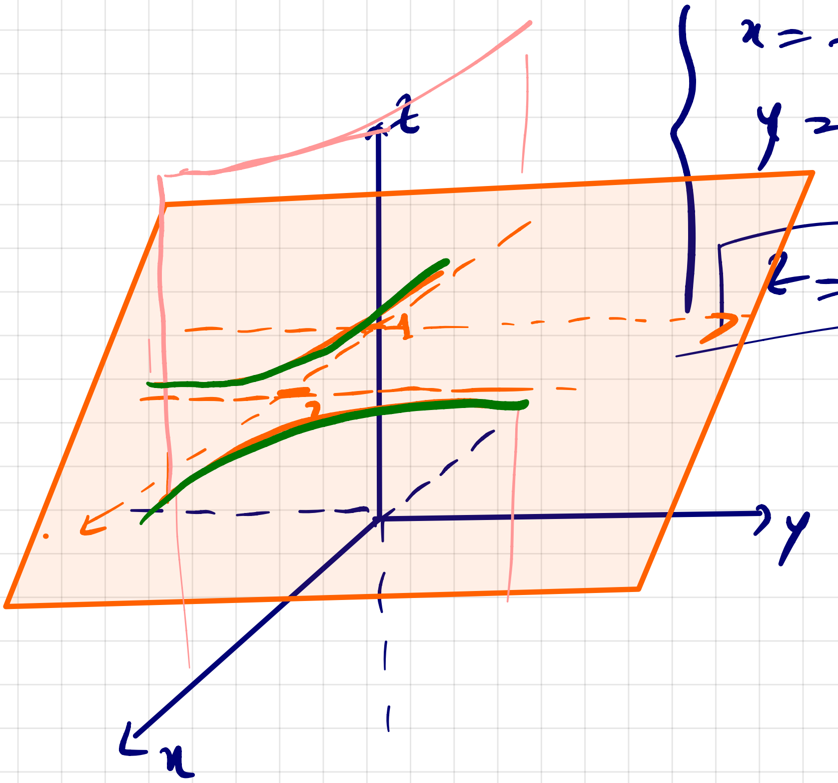
$t = 1$; $\vec{r}(t) = ?$

- $t=0$: $\vec{r}(0) = \left(0, \frac{1}{0-2}, 1\right) = \left(0, -\frac{1}{2}, 1\right)$

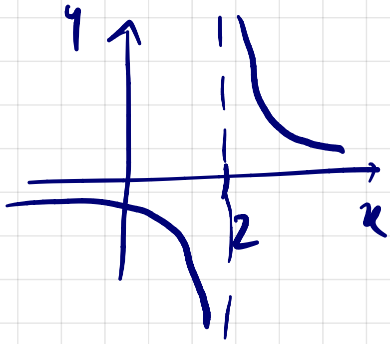
- $t=1$: $\vec{r}(1) = \left(1, \frac{1}{1-2}, 1\right) = (1, -1, 1)$

(b)

$$\vec{r}(t) = \Big(\underbrace{t}_{x(t)}, \underbrace{\frac{1}{t-2}}_{y(t)}, \underbrace{1}_{z(t)}\Big)$$



$$\begin{cases} x = t \\ y = \dfrac{1}{t-2} \\ \boxed{z = 1} \end{cases} \;\Big> \; y = \frac{1}{x-2}$$



(c)

$$\lim_{t\to 2} \vec{f}(t) = \lim_{t\to 2} \left(t, \frac{1}{t-2}, 1\right) =$$

$$= \left(\lim_{t\to 2} t, \lim_{t\to 2} \frac{1}{t-2}, \lim_{t\to 2} 1\right)$$

$$(2, \nexists, 1)$$

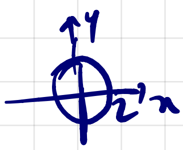
- $\lim_{t\to 2^-} \frac{1}{t-2} = -\infty$
- $\lim_{t\to 2^+} \frac{1}{t-2} = +\infty$

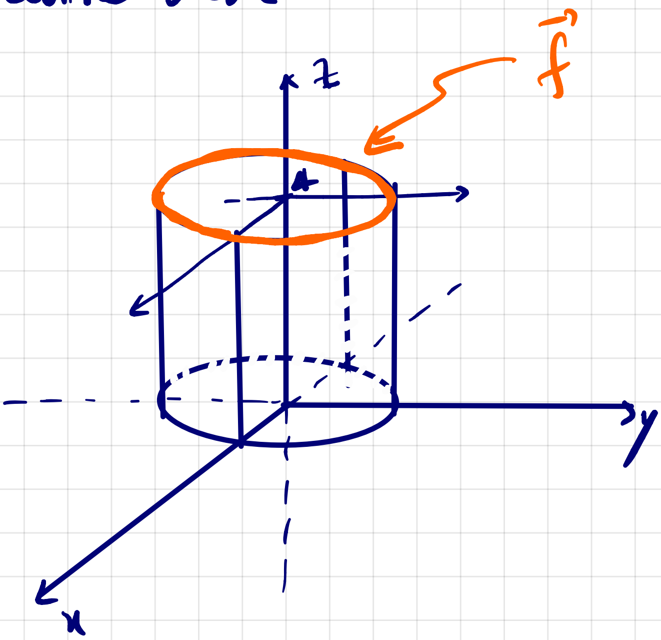
O deslocamento é descontínuo, pois "antes" de $t=2$, próximo, a partícula ocupa um ramo da trajetória, e, ligeiramente depois de 2, a mesma dá um "salto" para o outro ramo.

---

04) eq. vetorial para a curva dada pela interseção entre:

$$\left\{\begin{array}{l} x^2+y^2=4 \\ \text{e} \\ z=4 \end{array}\right.$$





$$\frac{x^2}{4} + \frac{y^2}{4} = 1$$

$$\text{sen}\,t = \frac{y}{2} \;;\; \cos t = \frac{x}{2}$$

$$\Rightarrow \quad x = 2\cos t \qquad \vec{f}(t) = (x, y, z)$$

$$y = 2\operatorname{sen} t$$

$$z = 4 \qquad = (2\cos t, 2\operatorname{sen} t, 4)$$

---

05) $x = \ln(z^2+2)$

$y = x z^3$.

$\Big\}$ eq. vetorial para curva definida por estas duas equações

Escreva $z = t$.

Disso; teremos:

$$\boxed{x = \ln(t^2+2)}$$

$$y = x \cdot z^3 = \underline{\ln(t^2+2) \cdot t^3}$$

$$\vec{f}(t) = (x, y, z) =$$

$$\Rightarrow \vec{f}(t) = (\ln(t^2+2), t^3 \cdot \ln(t^2+2), t)$$

LISTA 08

01) (e) $\vec{f}(t) = \big((8 - 4\,\text{sen}\,t),\ 2\cos t,\ 4\,\text{sen}\,t\big)$

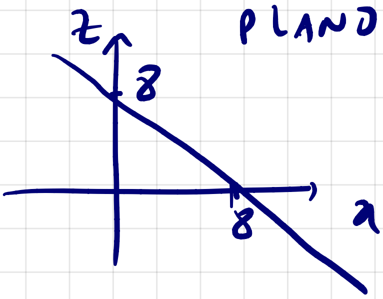
$x = 8 - 4\,\text{sen}\,t$

$y = 2\cos t$

$z = 4\,\text{sen}\,t$

$\boxed{x = 8 - z}$

PLANO

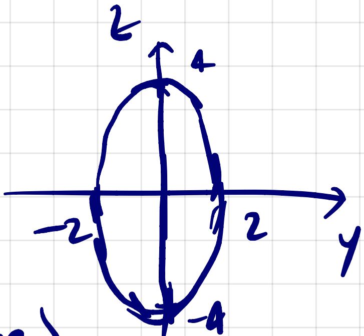


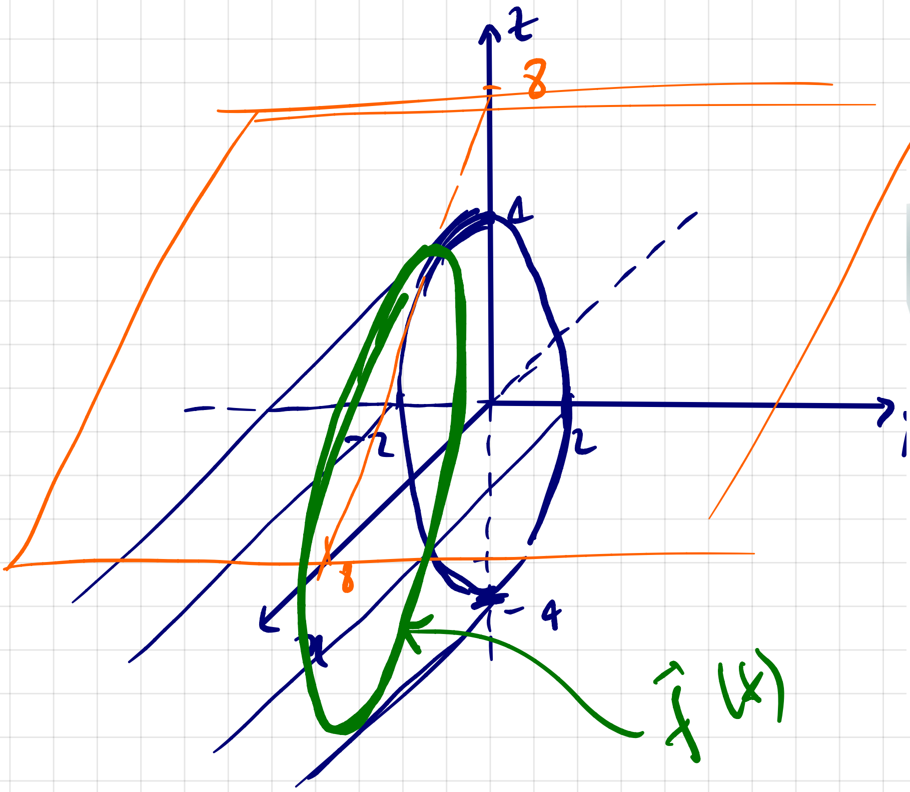
$\dfrac{y}{2} = \cos t \;;\; \dfrac{z}{4} = \text{sen}\,t$

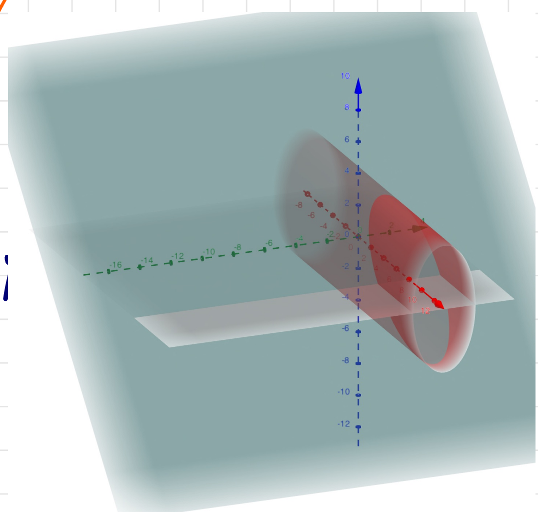
$\cos^2 t + \text{sen}^2 t = 1$

$\left(\dfrac{y}{2}\right)^2 + \left(\dfrac{z}{4}\right)^2 = 1$

$\dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$ (elipse)

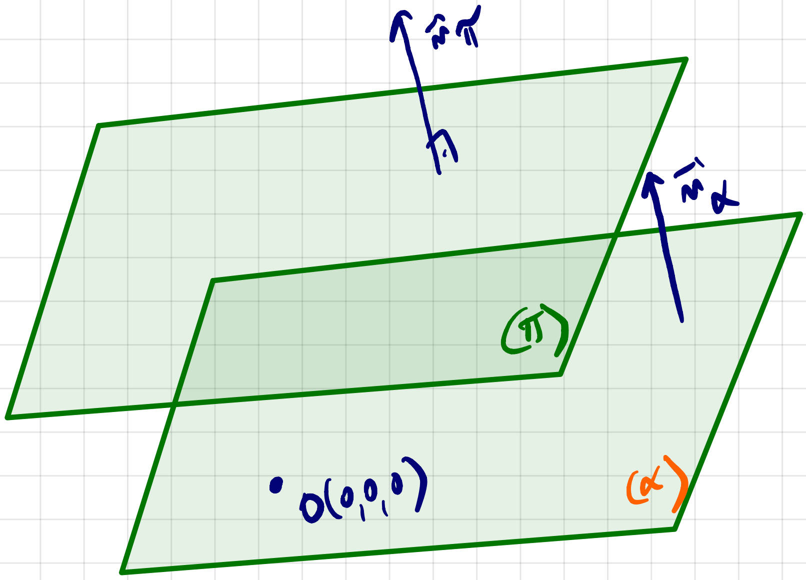






LISTA 06

11) plano passa pela origem $O(0,0,0)$;
e paralelo ao plano $(\pi): 4x - 2y + 7z + 12 = 0$



$\vec{n}_\pi = (4, 2, 7)$ CORRIGIR NA LISTA.

$(\alpha) = ?$

$\vec{n}_\pi = \vec{n}_\alpha$ (pois (π) e (α) são paralelos)

$\Rightarrow \vec{n}_\alpha = (4, -2, 7)$

$(\alpha): 4x - 2y + 7z + d = 0$. $O(0,0,0) \in (\alpha)$:

$4 \cdot 0 - 2 \cdot 0 + 7 \cdot 0 + d = 0$

$d = 0$

$\Rightarrow (\alpha): 4x - 2y + 7z + 0 = 0$

$$\boxed{(\alpha): 4x - 2y + 7z = 0}$$

LISTA 05

13) $\vec{u} = (1, 2, -3)$; $\vec{v} = (2, 0, -1)$; $\vec{w} = (3, 1, 0)$

obter $\vec{s} = (x, y, z)$ tal que

$$\begin{cases} \vec{s} \cdot \vec{u} = -16 \\ \vec{s} \cdot \vec{v} = 0 \\ \vec{s} \cdot \vec{w} = 3 \end{cases}$$

- $\vec{s} \cdot \vec{u} = (x, y, z) \cdot (1, 2, -3) = -16$

$$\boxed{x + 2y - 3z = -16}$$

- $\vec{s} \cdot \vec{v} = 0$:

$(x, y, z) \cdot (2, 0, -1) = 0$

$$\boxed{2x + 0y - z = 0}$$

- $\vec{s} \cdot \vec{w} = 3$

$(x, y, z) \cdot (3, 1, 0) = 3$

$$\boxed{3x + y + 0z = 3}$$

Ou seja, obtemos o seguinte sistema linear:

$$\begin{cases} x + 2y - 3z = -16 \\ 2x - z = 0 \\ 3x + y = 3 \end{cases}$$

$\rightsquigarrow z = 2x$

$\rightsquigarrow y = 3 - 3x$

$x + 2y - 3z = -16$

$x + 2.(3-3x) - 3.(2x) = -16$

$x + 6 - 6x - 6x = -16$

$-11x = -22$

$\boxed{x = 2}$

$y = 3 - 3x$

$y = 3 - 3.(2)$

$\boxed{y = -3}$

$z = 2x$

$z = 2.2 \Rightarrow \boxed{z = 4}$

Resposta: $\vec{s} = (x, y, z)$

$\boxed{\vec{s} = (2, -3, 4)}$

LISTA 05

15) $m = ?$

$\vec{u} = (m, 3, -4)$

$\vec{v} = (2, 1-2m, 3)$

$\vec{u} \perp \vec{v} \Leftrightarrow \vec{u} \cdot \vec{v} = 0$

$\Leftrightarrow (m, 3, -4).(2, 1-2m, 3) = 0$

$2m + 3.(1-2m) - 4.3 = 0$

$2m + 3 - 6m - 12 = 0$

$-4m = 9 \Rightarrow \boxed{m = -\frac{9}{4}}$