05/09/23

PARABOLÓIDE HIPERBÓLICO: são equações do tipo:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = c\cdot z \quad \text{ou}$$

$$\frac{y^2}{b^2} - \frac{z^2}{c^2} = a x \quad \text{ou}$$

$$\frac{x^2}{a^2} - \frac{z^2}{c^2} = b y .$$

Olhando a primeira eq, por exemplo, sendo $z =$ constante, teremos, no plano $z = \text{CONST}$, teremos a elipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = cz := k$.

Ex.1 $$\frac{x^2}{4} - \frac{y^2}{16} = z$$

traços: • no plano xy: $(z=0)$

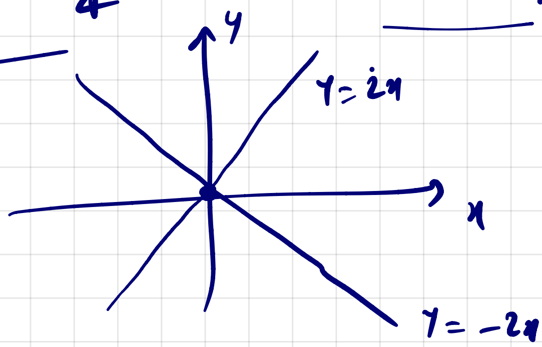
$$\frac{x^2}{4} - \frac{y^2}{16} = 0$$

$$\left(\frac{x}{2} - \frac{y}{4}\right)\cdot\left(\frac{x}{2} + \frac{y}{4}\right) = 0$$

$$\frac{x}{2} = \frac{y}{4} \quad \text{ou} \quad \frac{x}{2} = -\frac{y}{4} \quad \text{(retas)}$$

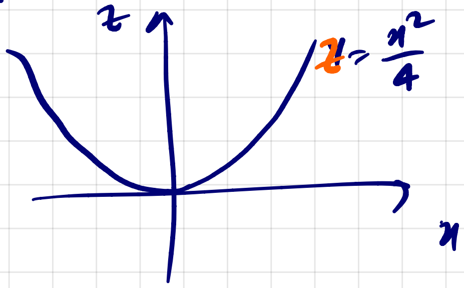
$x = \frac{y}{2}$

$x = -\frac{y}{2}$



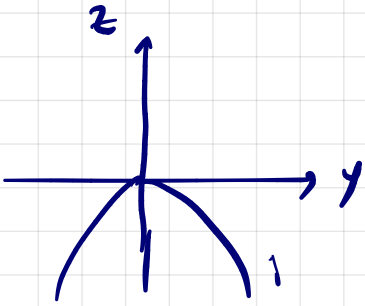
• no plano xz : $(y=0)$

$$\frac{x^2}{4} = z \qquad \text{(parábola)}$$

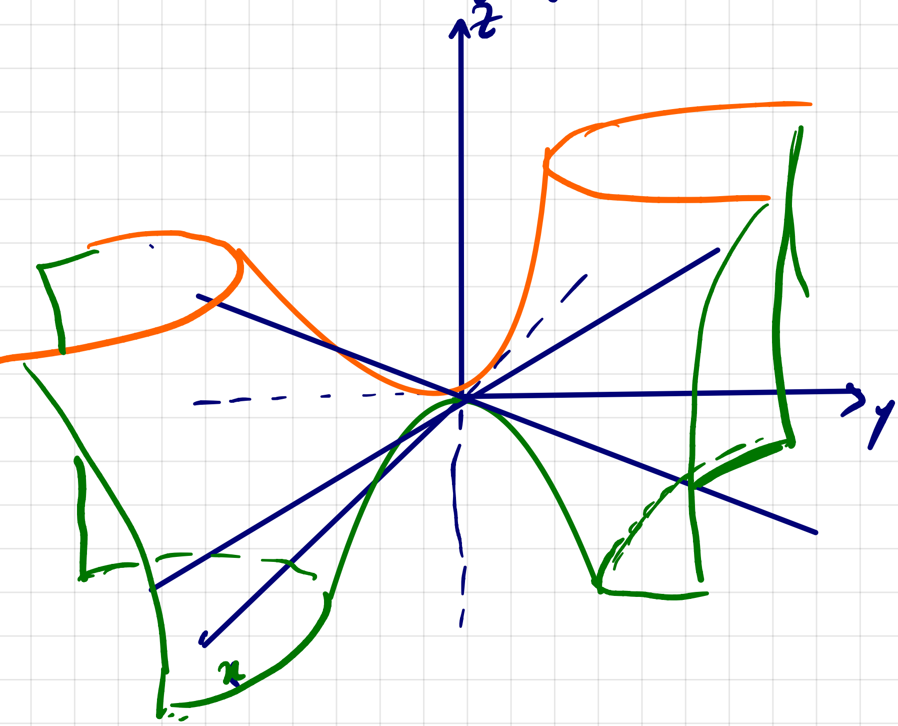


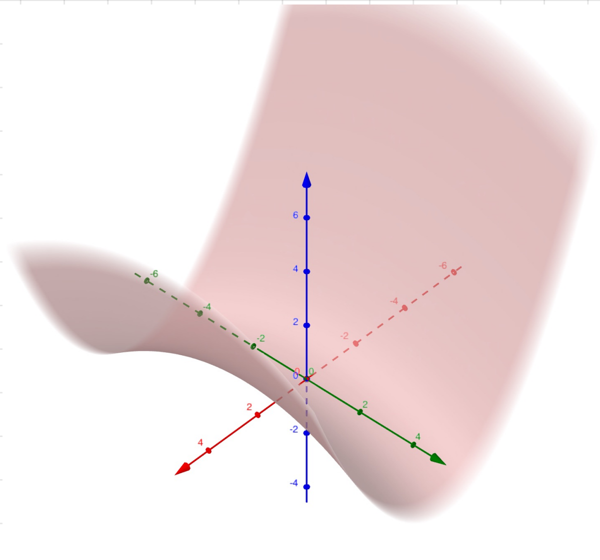
• no plano yz $(x=0)$:

$$z = -\frac{y^2}{16} \qquad \text{(parábola)}$$



esboço gráfico:

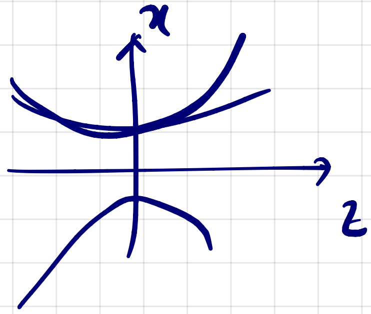




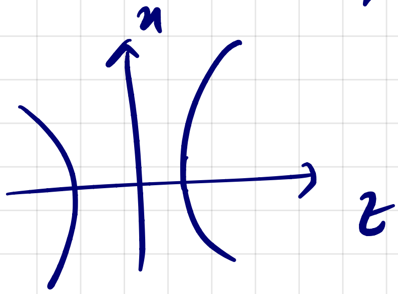
02) $\dfrac{x^2}{81} - \dfrac{z^2}{4} = y$

$y=$ CONSTANTE $\longrightarrow$ no plano $y = k$ temos hipérboles

$k \geq 0$: $\dfrac{x^2}{81} - \dfrac{z^2}{4} = k > 0$

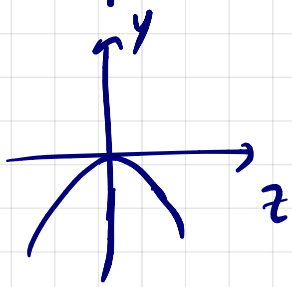


$k < 0$: $-\dfrac{x^2}{81} + \dfrac{z^2}{4} = \underbrace{-k}_{>0}$



traços: • $x=0$ (plano yz)

$y = -\dfrac{z^2}{4}$ (parábola)



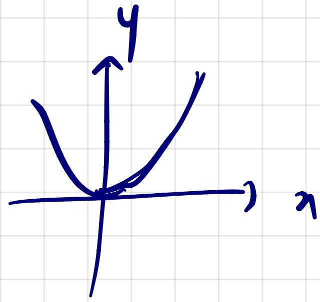
• $y = 0$: (plano xz) (retas)

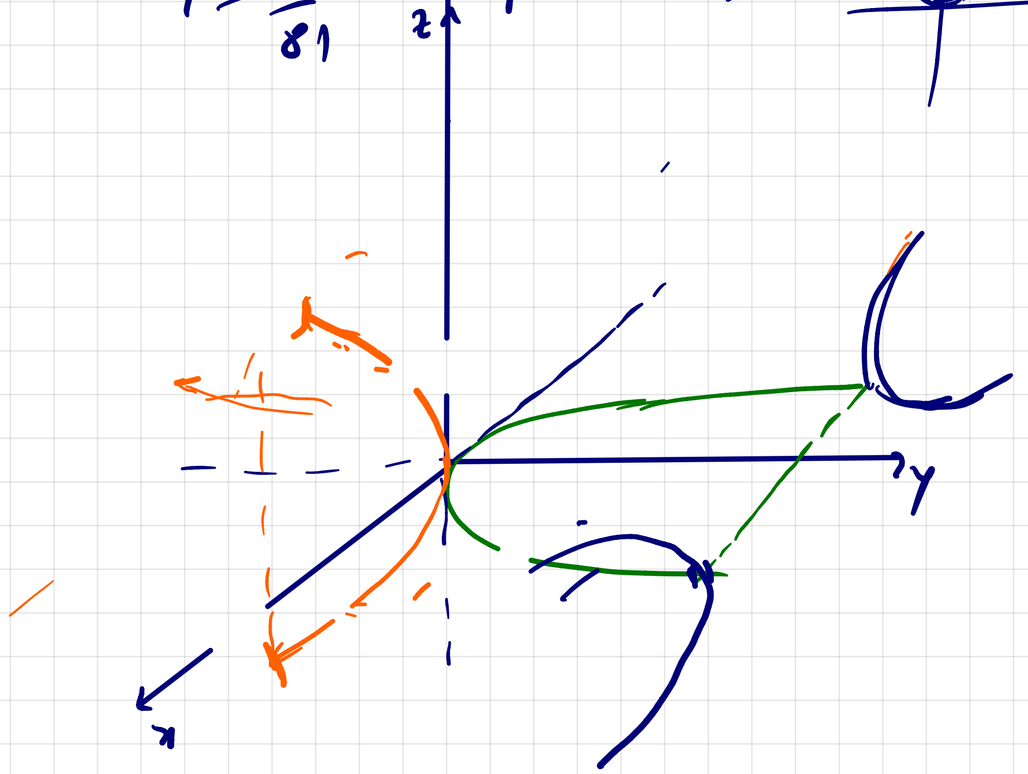
$\dfrac{x^2}{81} - \dfrac{z^2}{4} = 0$

$\left(\dfrac{x}{9} - \dfrac{z}{2}\right) \cdot \left(\dfrac{x}{9} + \dfrac{z}{2}\right) = 0$

• $z=0$ (plano xy)

$$y = \frac{x^2}{81}$$ (parábola)



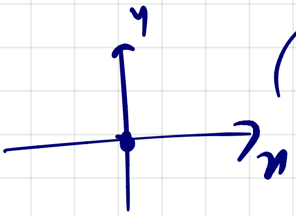


CONE: equação: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 0$

Ex: $x^2 + \dfrac{y^2}{4} - \dfrac{z^2}{9} = 0$

traços: • plano xy: $(z=0)$

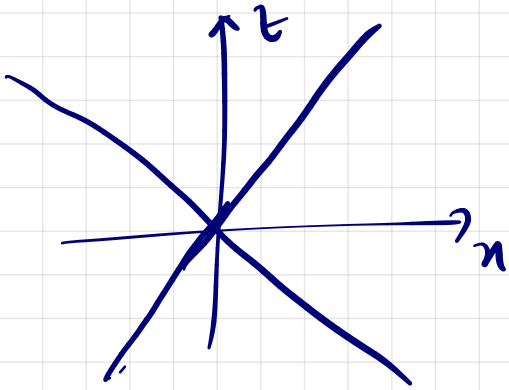
$$x^2 + \frac{y^2}{4} = 0 \iff x = y = 0$$ (origem)



• plano xz $(y=0)$

$$x^2 - \frac{z^2}{9} = 0$$

$$\left(x - \frac{z}{3}\right)\left(x + \frac{z}{3}\right) = 0$$ (retas)



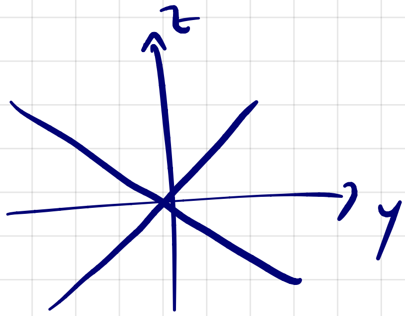
$x = \frac{z}{3}$

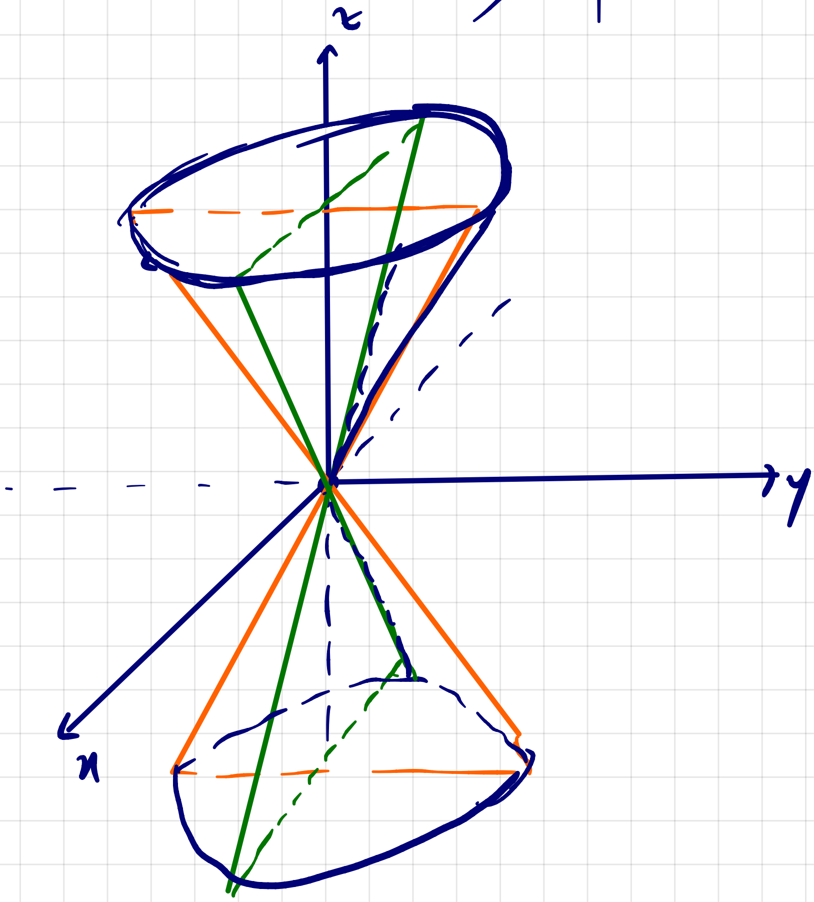
$x = -\frac{z}{3}$

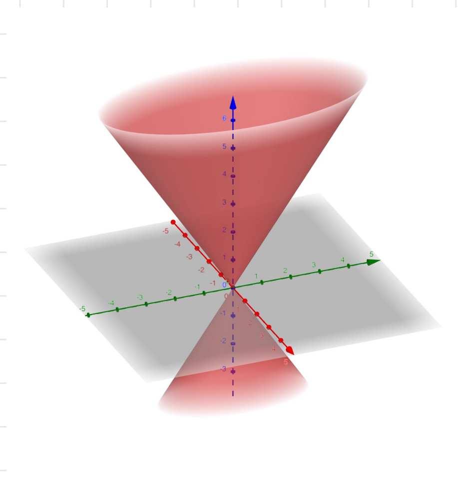
• plano yz $(x=0)$:

$$\frac{y^2}{4} - \frac{z^2}{9} = 0$$

$$\left(\frac{y}{2} - \frac{z}{3}\right)\cdot\left(\frac{y}{2} + \frac{z}{3}\right) = 0 \quad \text{retas)}$$







FUNÇÕES $\mathbb{R} \to \mathbb{R}^3$. (vetoriais)

São funções $\vec{f}: A \subset \mathbb{R} \to \mathbb{R}^3$ dada por

$$\vec{f}(t) = (f_1(t), f_2(t), f_3(t))$$

onde $f_1, f_2, f_3 : A \to \mathbb{R}$ funções de uma variável real, chamadas de FUNÇÕES COORDENADAS.

<u>Ex.:</u> $\vec{f}: \mathbb{R} \to \mathbb{R}^3$

$$\vec{f}(t) = (t, t^2, \cos 2t)$$

neste caso, $\left.\begin{array}{l} x = t \\ y = t^2 \\ z = \cos 2t \end{array}\right\}$ funções coordenadas.

O gráfico de uma $\vec{f}: A \subset \mathbb{R} \to \mathbb{R}^3$ será uma curva no espaço.

$\left\{\begin{array}{l} x = f_1(t) \\ y = f_2(t) \\ z = f_3(t) \end{array}\right.$ é uma parametrização da curva.

<u>Ex.: 01)</u> $\vec{f}(t) = (t, \ln t, 2)$

$D(\vec{f}) = ?$ $\quad x = t.$ $\quad D_1 = \mathbb{R}$

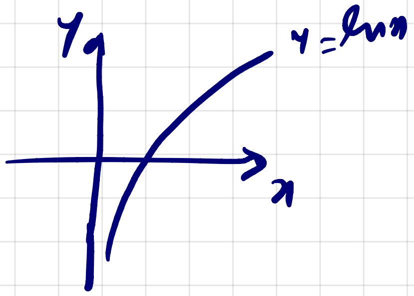
$y = \ln t$; D_2 : $t > 0$

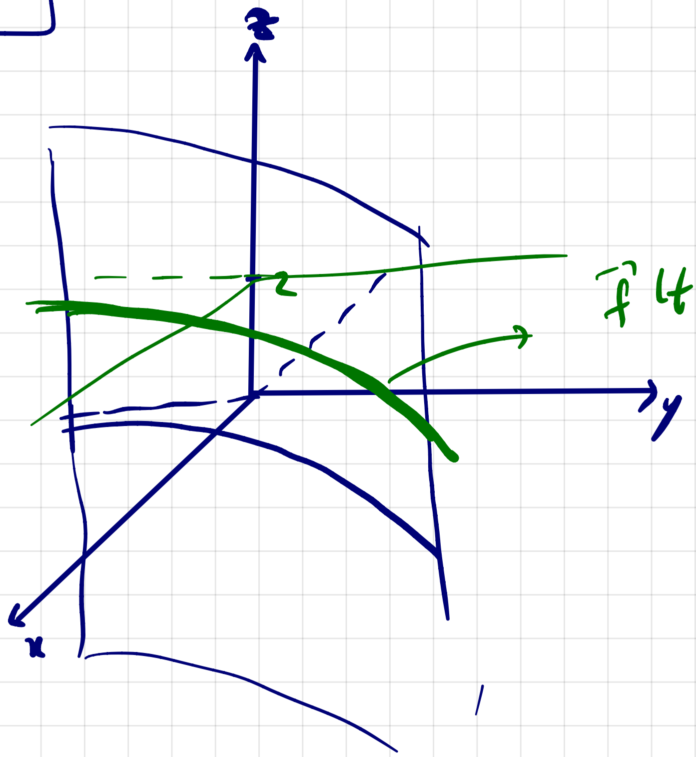
$D_2 : (0, +\infty)$

$z = 2$; $D_3 = \mathbb{R}$.

$D(f) = D_1 \cap D_2 \cap D_3 = (0, +\infty)$

$$\left\{\begin{array}{l} x = t \\ y = \ln t \\ \boxed{z = 2} \end{array}\right. > y = \ln x \longrightarrow \text{plano } xy:$$





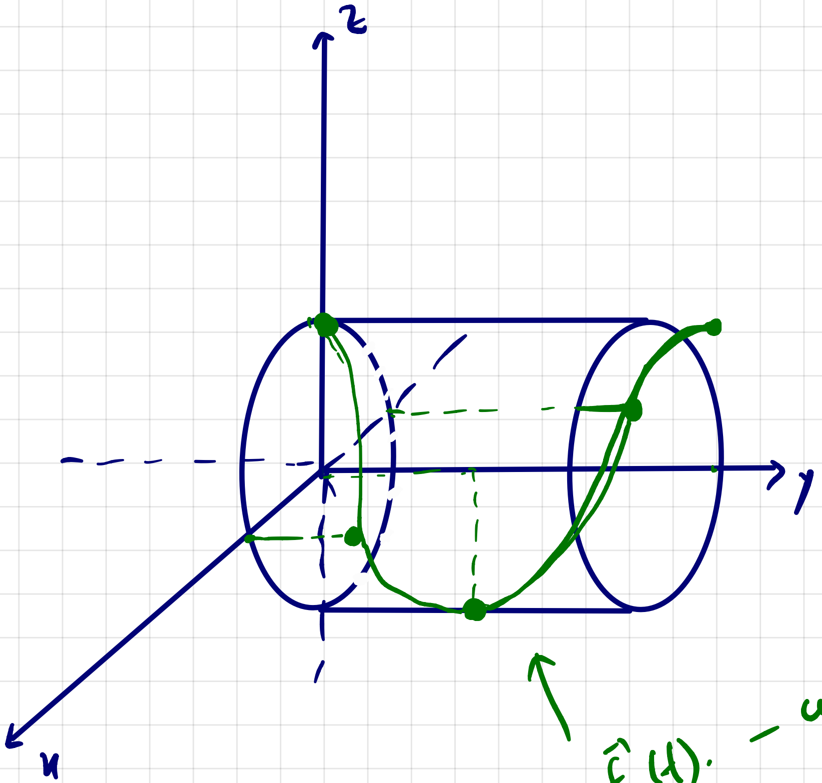
$\vec{f}(t) = (t, \ln t, 2)$

02) $\vec{f}(t) = (\operatorname{sen} t, t, \cos t)$

$$\left\{\begin{array}{l} x = \operatorname{sen} t \\ \boxed{y = t} \\ z = \cos t \end{array}\right. > x^2 + z^2 = \operatorname{sen}^2 t + \cos^2 = 1$$

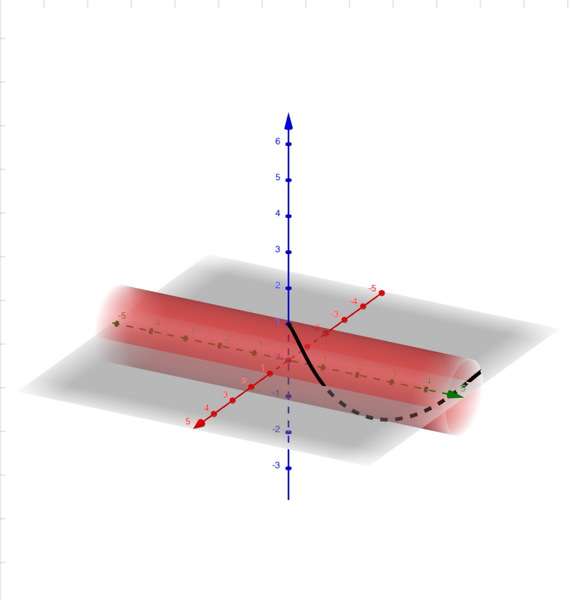
$$\boxed{x^2 + z^2 = 1}$$

$x = \text{sen}\,t$	$y = t$	$z = \cos t$
0	0	1
1	$\frac{\pi}{2}$	0
0	π	-1
-1	$\frac{3\pi}{2}$	0
0	2π	1



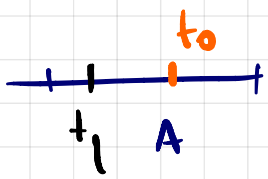
$\vec{f}(t)$. — curva espiral

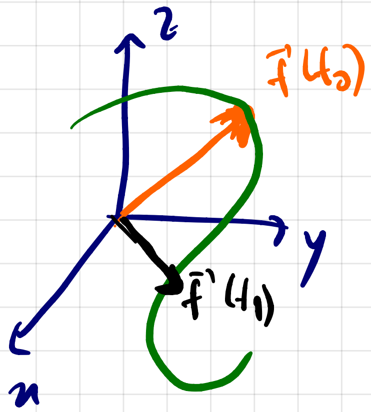
(espiral do caderno lembra a curva)



← pelo geogebra.

$t_0 \in A \subset \mathbb{R}$. $\vec{f}(t): A \subset \mathbb{R} \to \mathbb{R}^3$

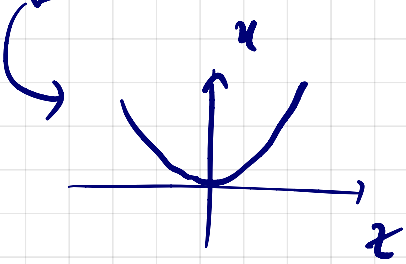




03) $\vec{f}(t) = (t^2, t+1, t)$

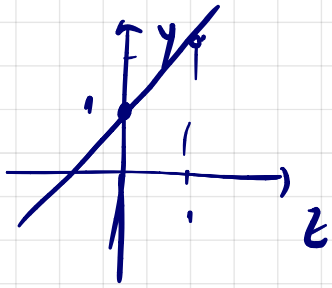
$$\begin{cases} x = t^2 \\ y = t+1 \\ t = z \end{cases} \Rightarrow \boxed{y = z+1}$$

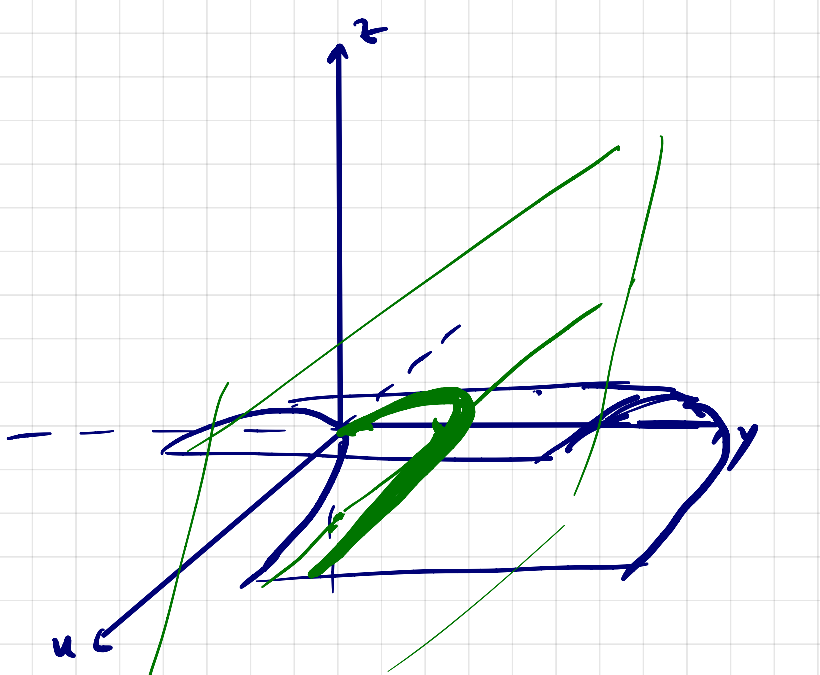
$$\boxed{x = z^2}$$

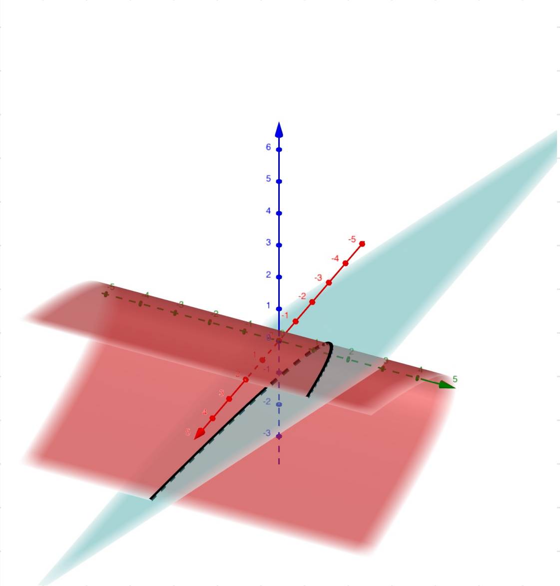


a curva no $\mathbb{R}^3$ dada por $\vec{f}(t)$ será a intersecção do plano $y = z+1$ com o cilindro $x = z^2$

$y = z+1$







← pelo geogebra.