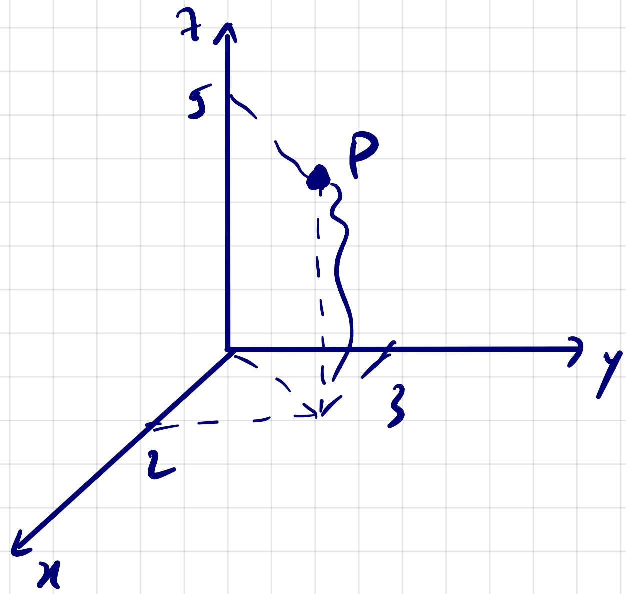
12/09/23

AULA DE EXERCÍCIOS:

LISTA 05. (22) — ANULADA. (erro na digitação do enunciado)

LISTA 06

17) DETERMINE DISTÂNCIA DO PONTO $P(2,3,5)$ A CADA UM DOS EIXOS COORDENADOS.



A distância de um ponto P a uma reta (r):
Seja $A \in (r)$. Então

$$d_{P(r)} = \frac{\|\vec{PA} \times \vec{u}\|}{\|\vec{u}\|};$$

onde $\vec{u}$ é o vetor diretor da reta (r).

1º: (r): eixo OX:

$$(r) \begin{cases} x = 0 + 1t \\ y = 0 \\ z = 0 \end{cases}$$

neste caso o vetor diretor será $\vec{u} = \vec{i} = (1,0,0)$

Seja $A(0,0,0) \in (r)$.

$$\vec{AP} = P - A = (2,3,5) - (0,0,0) = (2,3,5)$$

Assim:

$$\vec{PA} \times \vec{u} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 2 & 3 & 5 & 2 & 3 \\ 1 & 0 & 0 & 1 & 0 \end{array}\right|$$

$$\vec{PA} \times \vec{u} = 0\vec{i} + 5\vec{j} + 0\vec{k} - 3\vec{k} - 0\vec{i} - 0\vec{j}$$

$$= (0, 5, -3)$$

$$\Rightarrow \|\vec{PA} \times \vec{u}\| = \sqrt{(0)^2 + (5)^2 + (-3)^2} = \sqrt{25+9}$$

$$= \sqrt{34}$$

$$\|\vec{u}\| = \sqrt{1^2 + 0^2 + 0^2} = 1.$$

Portanto; $d_{P(r)} = \dfrac{\|\vec{PA} \times \vec{u}\|}{\|\vec{u}\|} = \dfrac{\sqrt{34}}{1} = \sqrt{34}$

Seja (r_1) o eixo z:

$$(r_1): \begin{cases} x = 0 + 0t \\ y = 0 + 0t \\ z = 0 + 1t \end{cases} = \begin{cases} x = 0 \\ y = 0 \\ z = t \end{cases}$$

$\vec{u} = \vec{k} = (0,0,1)$

$A = (0,0,0)$

$$d_{P(r_1)} = \frac{\|\vec{AP} \times \vec{u}\|}{\|\vec{u}\|} \qquad (\ldots)$$

Enfim, a eq. do eixo y: (r_2):

$$(r_2): \begin{cases} x = 0 \\ y = t \\ z = 0 \end{cases} \quad ; \ \vec{u} = \vec{j} = (0,1,0)$$

Tome $A(0,0,0) \in (r_2)$

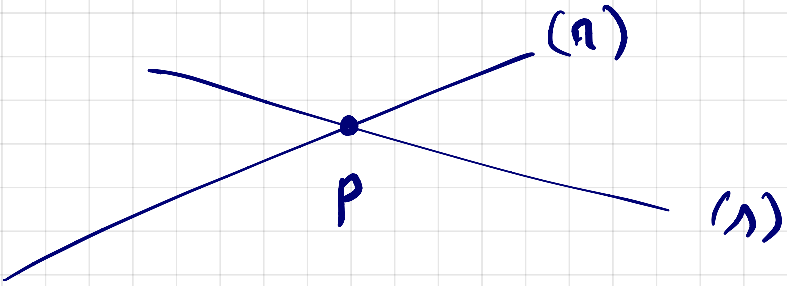
$$d_{P(r_2)} = \frac{\|\vec{AP} \times \vec{u}\|}{\|\vec{u}\|} \qquad (\ldots)$$

LISTA 06

21) eq. da superf. esférica centro C(3,-5,4) e passa pelas retas:

CORRIGIR.

$$(r): \begin{cases} x = 2 + t \\ y = -14 + t \\ z = 3 + t \end{cases} \qquad (s): \begin{cases} x = -4 + 3t \\ y = 1 - 4t \\ z = 2t \end{cases}$$



$P = (r) \cap (s)$

INTERSEÇÃO.

$$x = x$$
$$y = y$$
$$z = z$$

$$x = x:\qquad 2 + t = -4 + 3t \qquad -2t = -6 \qquad \boxed{t = 3}$$

$$y = y:\qquad -14 + t = 1 - 4t \qquad -5t = -15 \qquad \boxed{t = 3}$$

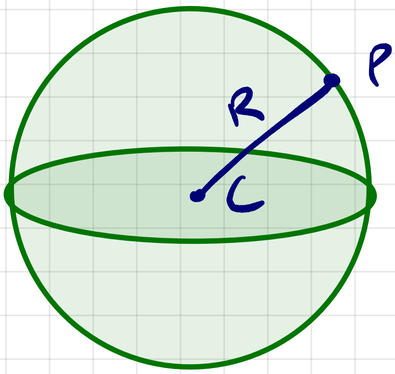
$$z = z:\qquad 2t = 3 + t \qquad \boxed{t = 3}$$

Ou seja, a interseção entre (r) e (s) ocorre quando $t = 3$. Neste caso, o ponto P, interseção entre (r) e (s) será:

$$P(x_p, y_p, z_p);\ \text{onde}$$

$$\begin{cases} x_p = 2 + 3 = 5 \\ y_p = -14 + 3 = -11 \\ z_p = 3 + 3 = 6 \end{cases} \Rightarrow P(5, -11, 6)$$

[USAMOS AQUÍ AS EQUAÇÕES DA RETA (r), MAS PODERIAM SER AS EQUAÇÕES DA RETA (s)]



$$d_{CP} = R$$

$$R = \sqrt{(x_C - x_P)^2 + (y_C - y_P)^2 + (z_C - z_P)^2}$$

$$R = \sqrt{(3-5)^2 + (-5+11)^2 + (4-6)^2}$$

$$R = \sqrt{(-2)^2 + (6)^2 + (-2)^2}$$

$$R = \sqrt{4 + 36 + 4} = \sqrt{44}$$

Por fim, a eq. da superfície esférica será:

$$(x-h)^2+(y-k)^2+(z-l)^2=R^2,$$

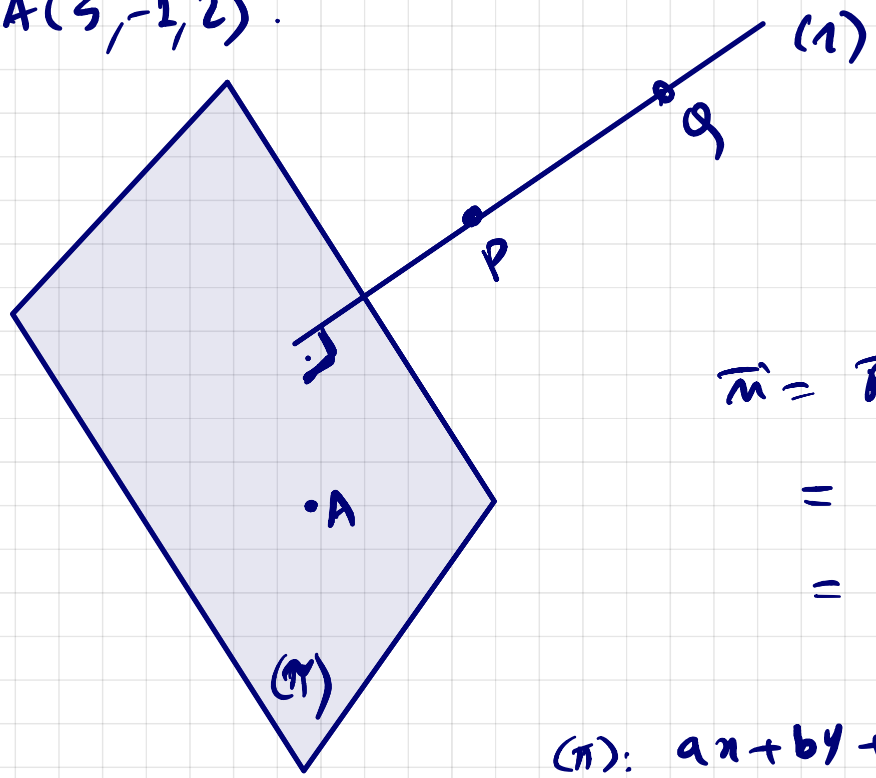
$c(h,k,l)=(3,-5,4)$. Logo, obtemos:

$$(x-3)^2+(y+5)^2+(z-4)^2=(\cancel{\sqrt{44}})^{\cancel{2}}$$

$$\boxed{(x-3)^2+(y+5)^2+(z-4)^2=44}$$

<u>LISTA 06</u>

04) achar a eq. do plano perpendicular à reta que passa por $P(2,2,-4)$ e $Q(7,-1,3)$ e contém o ponto $A(5,-1,2)$.



(π): ?

$$\vec{n}=\vec{PQ}=Q-P$$
$$=(7,-1,3)-(2,2,-4)$$
$$=(5,-3,7)=(a,b,c)$$

(π): $ax+by+cz+d=0$

(π): $5x-3y+7z+d=0$

$A(5,-1,2)$

$A\in(\pi)$:

$$5(5)-3\cdot(-1)+7(2)+d=0$$
$$25+3+14+d=0 \Rightarrow \boxed{d=-42}$$

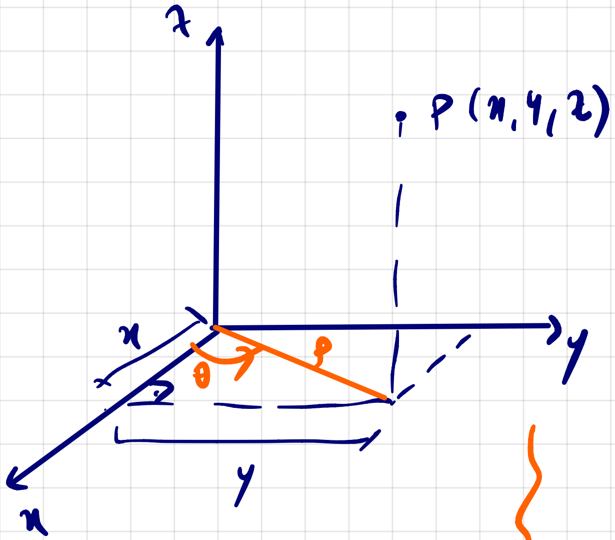
Portanto, obtemos:

$$\boxed{(\pi):\ 5x - 3y + 7z - 42 = 0}$$

LISTA 07

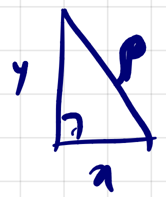
01) represente a eq $x^2+y^2=9$ em coord. cilíndricas. Qual o esboço gráfico?

$x = \rho\cos\theta$
$y = \rho\,\text{sen}\,\theta$
$z = z$



$P(x,y,z) \leadsto P(x(\rho,\theta),\ y(\rho,\theta),\ z)$

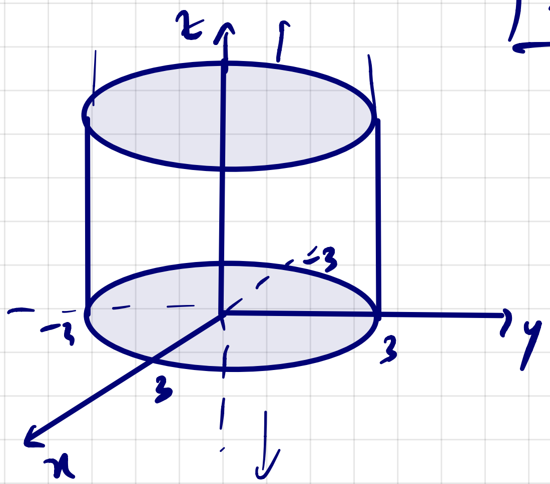
$\rho = \sqrt{x^2+y^2}$

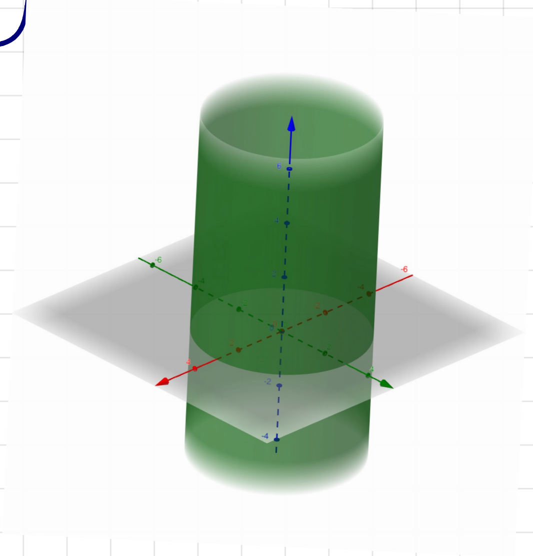


$P(\rho.\cos\theta,\ \rho.\text{sen}\,\theta,\ z)$

$$\overbrace{x^2+y^2}^{=\rho^2}=9 \Rightarrow \rho^2 = 9$$

$$\boxed{\rho = 3}$$

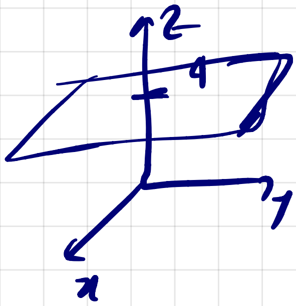




LISTA 07

04) ESFÉRICAS ⟶ CARTESIANAS:

(a) $\underbrace{\rho \cos\varphi}_{z} = 4.$ $\rightsquigarrow$ $\boxed{z = 4}$



$$\left.\begin{aligned} x &= \rho \cos\theta \cdot \operatorname{sen}\varphi \\ y &= \rho \operatorname{sen}\theta \cdot \operatorname{sen}\varphi \\ z &= \rho \cos\varphi \end{aligned}\right\}$$ SIST. DE COORD. ESFÉRICAS

(b) $\rho \operatorname{sen}\varphi = 4$

$$x^2 + y^2 = \overbrace{\rho^2 \cos^2\theta \operatorname{sen}^2\varphi + \rho^2 \operatorname{sen}^2\theta \cdot \operatorname{sen}^2\varphi}$$

$$x^2 + y^2 = \rho^2 \cdot \operatorname{sen}^2\varphi \cdot \underbrace{(\cos^2\theta + \operatorname{sen}^2\theta)}_{=1}$$

$$x^2 + y^2 = (\rho \underbrace{\operatorname{sen}\varphi)}_{4}{}^2$$

$$\boxed{x^2 + y^2 = 16}$$

(c) $\rho = 9$

$$x^2 + y^2 + z^2 = \overbrace{\rho^2 \cos^2\theta \operatorname{sen}^2\varphi + \rho^2 \operatorname{sen}^2\theta \operatorname{sen}^2\varphi} + \rho^2 \cos^2\varphi$$

$$= \rho^2 \operatorname{sen}^2\varphi \cdot \underbrace{(\cos^2\theta + \operatorname{sen}^2\theta)}_{=1} + \rho^2 \cos^2\varphi$$

$$= \rho^2 \operatorname{sen}^2\varphi + \rho^2 \cos^2\varphi = \rho^2 \underbrace{(\operatorname{sen}^2\varphi + \cos^2\varphi)}_{=1} = \rho^2 = 9^2$$

$$\Rightarrow \boxed{x^2+y^2+z^2=81}$$

(d) $\rho = 6 \cdot \underbrace{\operatorname{sen}\varphi \cdot \operatorname{sen}\theta}_{\frac{y}{\rho}} + 3 \underbrace{\cos\varphi}_{\frac{z}{\rho}}$

pois $y=\rho \operatorname{sen}\varphi \operatorname{sen}\theta$

pois $z=\rho\cos\varphi$

$$\Rightarrow \rho = 6 \cdot \frac{y}{\rho} + 3 \cdot \frac{z}{\rho} \qquad (\times \rho)$$

$$\rho^2 = 6y + 3z$$

$\boxed{x^2+y^2+z^2=\rho^2}$

(no sist. esférico)

por exemplo, isto está deduzido no item anterior)

$$\Rightarrow x^2+y^2+z^2 = 6y+3z$$

$$\boxed{x^2+y^2-6y+z^2-3z=0}$$

LISTA 07 |

08) MOSTRE QUE A INTERSEÇÃO DA SUPERFÍCIE $x^2-4y^2-9z^2=36$ COM O PLANO $x+z=9$ É UMA CIRCUNFERÊNCIA.

ELIPSE

$$\boxed{x=9-z}$$

$$(9-z)^2-4y^2-9z^2=36$$

$$81-18z+z^2-4y^2-9z^2=36$$

$$-8z^2-18z-4y^2=36-81$$

$$8z^2+18z+4y^2=45$$

- $8z^2+18z = 8\cdot\left[(z-A)^2+B\right]$

$$= 8z^2-16Az+8A^2+8B$$

$$-16A=18$$

$$A=\frac{-18}{16}=-\frac{9}{8}$$

$$8A^2+8B=0$$

$$8\cdot\left(-\frac{9}{8}\right)^2+8B=0$$

$$\frac{81}{8}+8B=0$$

$$8B=-\frac{81}{8}$$

$$\boxed{B=-\frac{81}{64}}$$

$$\Longrightarrow\ 8z^2+18z=8\cdot\left[\left(z+\frac{9}{8}\right)^2-\frac{81}{64}\right]$$

Assim:

$$8z^2+18z+4y^2=45$$

$$8\cdot\left[\left(z-\frac{9}{8}\right)^2-\frac{81}{64}\right]+4y^2=45$$

$$\left(z-\frac{9}{8}\right)^2-\frac{81}{8}+4y^2=45$$

$$\left(z-\frac{9}{8}\right)^2+4y^2=45+\frac{81}{8}$$

(elipse)