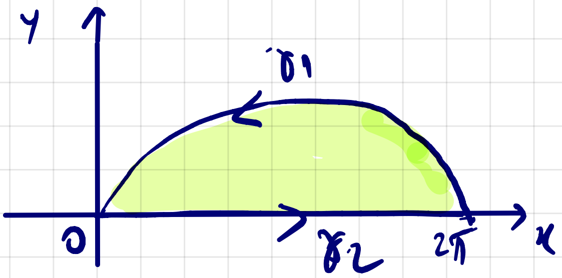
22/09/23 – AULA 26

AULA DE EXERCÍCIOS:

LISTA 06

05) (c) $\gamma(t) = (t - \text{sen}\,t,\ 1 - \cos t)$; $0 \leq t \leq 2\pi$



[AULA 20]

$$A = \frac{1}{2} \oint_{\gamma} x\,dy - y\,dx =$$

- entre 2π e 0:

$$\gamma_1 : \begin{cases} x = t - \text{sen}\,t & \rightsquigarrow dx = (1 - \cos t)\,dt \\ y = 1 - \cos t & \rightsquigarrow dy = +\text{sen}\,t\,dt \end{cases}$$

- entre 0 e 2π:

$$\gamma_2 : \begin{cases} x = t & \rightsquigarrow dx = dt \\ y = 0 & \rightsquigarrow dy = 0 \end{cases}$$

Assim:

$$A = \frac{1}{2}\left(\int_{\gamma_1} + \int_{\gamma_2}\right) =$$

$$= \underbrace{\frac{1}{2}\int_{2\pi}^{0} (t-\operatorname{sen} t)\cdot \operatorname{sen} t\, dt - (1-\cos t)(1-\cos t)\, dt}_{\gamma_1} +$$

$$+ \underbrace{\frac{1}{2}\int_{0}^{2\pi} t\cdot 0 - 0\, dt}_{\gamma_2} = 0$$

$$= \underbrace{\frac{1}{2}\int_{2\pi}^{0} t\cdot \operatorname{sen} t \cdot dt}_{\text{INTEGRAÇÃO POR PARTES}} - \frac{1}{2}\int_{2\pi}^{0} \operatorname{sen}^2 t\, dt - \frac{1}{2}\int_{2\pi}^{0} (1-\cos t)^2\, dt = \dots$$

$\operatorname{sen}^2 t = \dfrac{1-\cos 2t}{2}$

$(1-\cos t)^2 = 1 - 2\cos t + \cos^2 t$

$\cos^2 t = \dfrac{1+\cos 2t}{2}$

LISTA 06

09) Prove:

$$\iint_{\Omega} f\cdot \Delta g \cdot dA = \oint_{\gamma} f\cdot \nabla g\cdot \vec{n}\, ds - \iint_{\Omega} \nabla f\cdot \nabla g \cdot dA,$$

usando o T. de Green rotacional.

T. da divergência:

$$\oint_{\gamma} \vec{F}\cdot \vec{n}\, ds = \iint_{\Omega} \operatorname{div} \vec{F}\cdot dA \qquad (*)$$

Tome o campo vetorial $\vec{F}$ dado por

$$\boxed{\vec{F} = f \cdot \nabla g}$$, (**) onde $f = f(x,y)$

$g = g(x,y)$

$\nabla g = \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right)$

Ou seja, temos:

$$\vec{F} = f \cdot \nabla g = \left(f \cdot \frac{\partial g}{\partial x}, f \cdot \frac{\partial g}{\partial y}\right)$$

Disso, calculando a divergência de $\vec{F}$, vem:

$$\text{div}\,\vec{F} = \frac{\partial}{\partial x}\Big(\underbrace{f \cdot \frac{\partial g}{\partial x}}_{\text{M.M}}\Big) + \frac{\partial}{\partial y}\Big(\underbrace{f \cdot \frac{\partial g}{\partial y}}_{\text{M.M}}\Big)$$

$$= f \cdot \frac{\partial^2 g}{\partial x^2} + \frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial x} + f \cdot \frac{\partial^2 g}{\partial y^2} + \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial y}$$

$$= f \cdot \underbrace{\left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}\right)}_{\Delta g} + \underbrace{\frac{\partial f}{\partial x} \cdot \frac{\partial g}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{\partial g}{\partial y}}_{\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) \cdot \left(\frac{\partial g}{\partial x}, \frac{\partial g}{\partial y}\right) = \nabla f \cdot \nabla g}$$

$$= f \cdot \Delta g + \nabla f \cdot \nabla g$$

Então, (*) fica:

$$\oint_\gamma \underbrace{\vec{F}}_{f \cdot \nabla g} \cdot \vec{n}\, ds = \iint_\Omega \underbrace{\text{div}\,\vec{F}}_{f \cdot \Delta g + \nabla f \cdot \nabla g}\, dA$$, i.e.;

$$\oint_\gamma f \cdot \nabla g \cdot \vec{n}\, ds = \iint_\Omega (f \cdot \Delta g + \nabla f \cdot \nabla g)\, dA$$

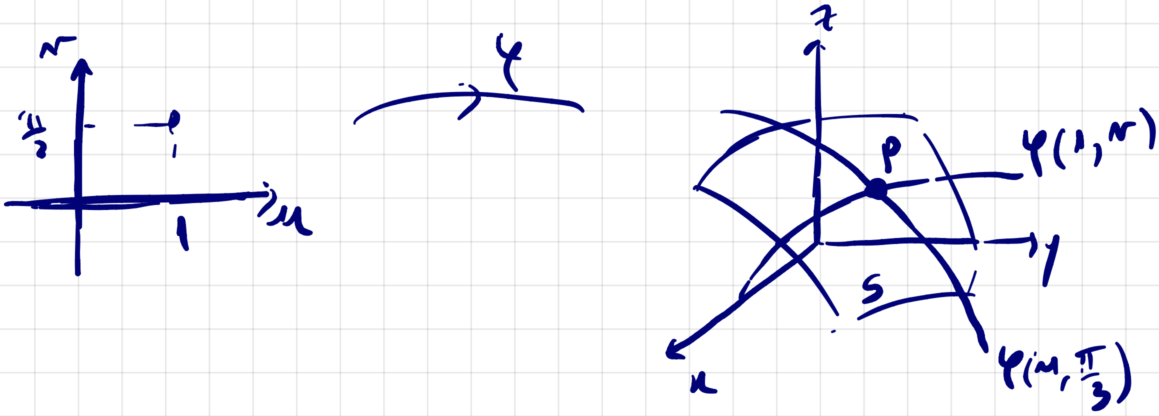
$$\oint_\gamma f\cdot\nabla g\cdot\vec{n}\,ds = \iint_\Omega f\cdot\Delta g\,dA + \iint_\Omega \nabla f\cdot\nabla g\,dA$$

$$\Rightarrow \boxed{\iint_\Omega f\cdot\Delta g\,dA = \oint_\gamma f\cdot\nabla g\cdot\vec{n}\,ds - \iint_\Omega \nabla f\cdot\nabla g\,dA}$$

LISTA 06

15) **b** $\varphi(u,v) = (u\cos v,\ u\,\mathrm{sen}\,v,\ v) = (x(u,v),\ y(u,v),\ z(u,v))$

eq. do plano tang à S dado pela parametrização acima. quando $u=1$ e $v=\frac{\pi}{3}$



$$\begin{cases} x = u\cdot\cos v \\ y = u\cdot \mathrm{sen}\,v \\ z = v \end{cases}$$

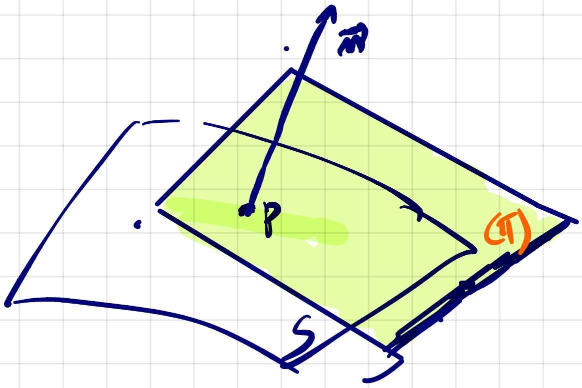
$u=1$; $v=\frac{\pi}{3}$ determina o ponto P.

$x_p = 1\cos\frac{\pi}{3} = \frac{1}{2}$

$y_p = 1\cdot\mathrm{sen}\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

$z_p = \frac{\pi}{3}$

$P\left(\frac{1}{2};\ \frac{\sqrt{3}}{2},\ \frac{\pi}{3}\right)$



$F(x,y,z) = 0$ superficie

$x = u\cos v$

$y = u \cdot \operatorname{sen} v$

$z = v$

$$\frac{y}{x} = \frac{u \operatorname{sen} v}{u \cos v} = \tan v$$

$$\frac{y}{x} = \tan v = \tan z$$

$v = z$

$$\tan z = \frac{y}{x}$$

$$f(x,y) = z = \arctan\left(\frac{y}{x}\right)$$

$$F(x,y,z) = f(x,y) - z$$

$$F(x,y,z) = \arctan\left(\frac{y}{x}\right) - z = \arctan(y \cdot x^{-1}) - z$$

$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\right)$$

$$\nabla F = \left(\frac{-y x^{-2}}{1 + \frac{y^2}{x^2}}, \frac{x^{-1}}{1 + \frac{y^2}{x^2}}, -1\right)$$

$$(\arctan v)' = \frac{v'}{1+v^2}$$

$$\nabla F = \left(\frac{-\frac{y}{\cancel{x^2}}}{\frac{x^2+y^2}{\cancel{x^2}}}, \frac{\frac{1}{\cancel{x}}}{\frac{x^2+y^2}{x^{\cancel{2}}}}, -1\right)$$

$$\nabla F = \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2}, -1\right) ; \quad P\left(\frac{1}{2}, \frac{\sqrt{3}}{2}, \frac{\pi}{3}\right)$$

$$\vec{n} = \nabla F_P = \left(-\frac{\frac{\sqrt{3}}{2}}{\frac{1}{4}+\frac{3}{4}}, \frac{\frac{1}{2}}{\frac{1}{4}+\frac{3}{4}}, -1\right)$$

$$\vec{n} = \nabla F_P = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}, -1\right) = (a, b, c)$$

Portanto, a eq. do plano (π) será:

$$(\pi): ax + by + cz + d = 0$$

$$-\frac{\sqrt{3}}{2}x + \frac{1}{2}y - z + d = 0 \quad . \quad P \in (\pi):$$

$$-\frac{\sqrt{3}}{2} \cdot \cancel{\frac{1}{2}} + \cancel{\frac{1}{2} \cdot \frac{\sqrt{3}}{2}} - \frac{\pi}{3} + d = 0 \Rightarrow \boxed{d = \frac{\pi}{3}}$$

$$\boxed{(\pi): -\frac{\sqrt{3}}{2}x + \frac{1}{2}y - z + \frac{\pi}{3} = 0}$$