LISTA 04

AULA 26; 25/09/23

16) $X = \{ (x,y) \in \mathbb{R}^2 : d_\infty((x,y),(0,1)) \leq 1 \}$

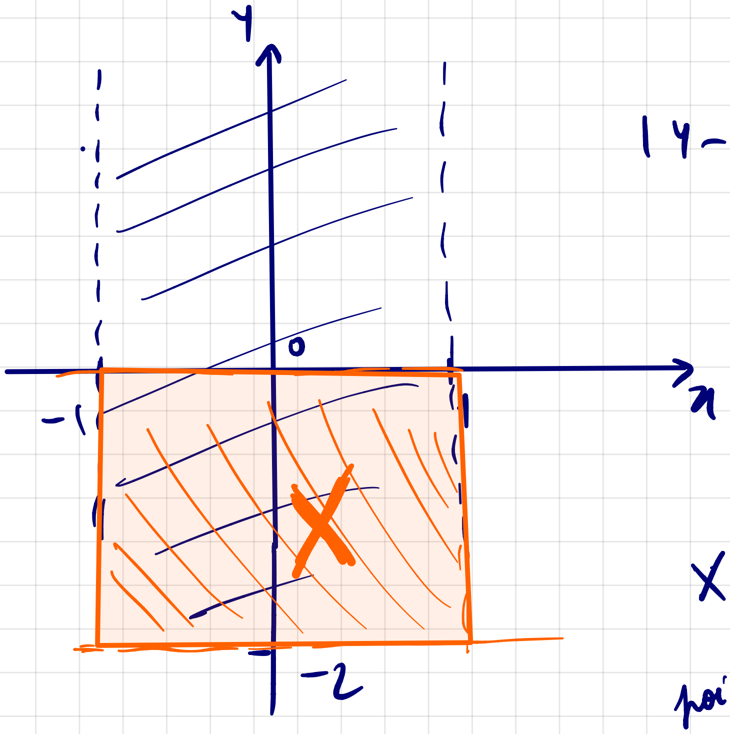
$d_\infty((x,y),(0,1)) = \max\{|x-0|, |y-1|\} \leq 1$

$\max\{|x|, |y-1|\} \leq 1$

$|x| \leq 1 \iff \boxed{-1 \leq x \leq 1}$

$|y-1| \leq 1 \iff -1 \leq y-1 \leq 1$

$\boxed{-2 \leq y \leq 0}$



X é fechado do $\mathbb{R}^2$, pois $\overline{X} = X$; Além disso,

$\partial X = \{(x,0) : x \in [-1,1]\} \cup \{(1,y) : y \in [-2,0]\} \cup$

$\cup \{(x,-2) : x \in [-1,1]\} \cup \{(-1,y) : y \in [-2,0]\}$.

X é compacto pois é limitado e fechado.

LISTA 08 |

02) $f(x,y) = \begin{cases} \dfrac{x^3-y^3}{x^2+y^2}, & \text{se } (x,y) \neq (0,0) \\ 0, & \text{se } (x,y) = (0,0) \end{cases}$

Calcule $f_1(0,0)$ e $f_2(0,0)$

• $f_1(0,0) = \dfrac{\partial f}{\partial x}(0,0) = \lim_{\Delta x \to 0} \dfrac{f(0+\Delta x, 0) - f(0,0)}{\Delta x}$; $\Delta x \neq 0$.

$$= \lim_{\Delta x \to 0} \frac{\dfrac{\Delta x^3 - 0^3}{\Delta x^2 + 0^2} - 0}{\Delta x} = \lim_{\Delta x \to 0} \frac{\dfrac{\Delta x^{\cancel{3}}}{\cancel{\Delta x^2}}}{\Delta x}$$

$$\lim_{\Delta x \to 0} \frac{\cancel{\Delta x}}{\cancel{\Delta x}} = 1.$$

• $f_2(0,0) = \dfrac{\partial f}{\partial y}(0,0) = \lim_{\Delta y \to 0} \dfrac{f(0, 0+\Delta y) - f(0,0)}{\Delta y} =$ $\Delta y \neq 0$

$$= \lim_{\Delta y \to 0} \frac{\dfrac{0-\Delta y^3}{0+\Delta y^2} - 0}{\Delta y} = \lim_{\Delta y \to 0} \frac{-\Delta y^{\cancel{3}}}{\cancel{\Delta y^2}} \times \frac{1}{\Delta y} = -1$$

LISTA 08 |

03) $f(x,y) = \begin{cases} \dfrac{xy \cdot (x^2-y^2)}{x^2+y^2}; & \text{se } (x,y) \neq (0,0) \\ 0, & \text{se } (x,y) = (0,0); \end{cases}$

mostrar que $\dfrac{\partial f}{\partial x}(0,y) = -y$ e $\dfrac{\partial f}{\partial y}(x,0) = x$, $\forall x, \forall y$.

$$\frac{\partial f}{\partial x}(0,y) = \lim_{\Delta x \to 0} \frac{f(0+\Delta x, y) - f(0,y)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{f(\Delta x, y) - f(0,y)}{\Delta x}, \quad \forall y \in \mathbb{R}.$$

Como f muda de sentença se $y \neq 0$ e $y = 0$, então temos 2 casos a considerar:

CASO I: $y \neq 0$: Então:

$$\frac{\partial f}{\partial x}(0,y) = \lim_{\Delta x \to 0} \frac{f(\Delta x, y) - f(0,y)}{\Delta x} =$$

$$= \lim_{\Delta x \to 0} \frac{\dfrac{\Delta x \cdot y \cdot (\Delta x^2 - y^2)}{\Delta x^2 + y^2} - \underbrace{\dfrac{0 \cdot y \cdot (0^2 - y^2)}{0^2 + y^2}}_{=0}}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{\cancel{\Delta x} \cdot y \cdot (\Delta x^2 - y^2)}{(\Delta x^2 + y^2)\cancel{\Delta x}} = \frac{y \cdot (0^2 - y^2)}{0^2 + y^2}$$

$$= \frac{y \cdot (-\cancel{y^2})}{\cancel{y^2}} = -y$$

CASO II: $y = 0$:

$$\frac{\partial f}{\partial x}(0,y) = \frac{\partial f}{\partial x}(0,0) = \lim_{\Delta x \to 0} \frac{f(0+\Delta x, 0) - f(0,0)}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(\Delta x, 0) - 0}{\Delta x} = \lim_{\Delta x \to 0} \frac{\cancel{\Delta x} \cdot 0(\Delta x^2 - 0^2)}{\Delta x^2 + 0^2} \cdot \frac{1}{\cancel{\Delta x}}$$

$$= 0 = y$$

pois $y = 0$

Em qualquer caso, tem-se que $\frac{\partial f}{\partial x}(0,y) = -y$.

Do mesmo modo se faz a outra parte, i.e., mostrar que $\frac{\partial f}{\partial y}(x,0) = x$.

LISTA 09

01) mostrar que f é diferenciável. Obter $d_a f$.

(c) $f(x,y) = \frac{x^2}{y}$.

LEMBRE DE UM RESULTADO DA AULA: se $\frac{\partial f}{\partial x}$ e $\frac{\partial f}{\partial y}$ forem cont., então f será diferenciável.

$D(f) = \{(x,y) \in \mathbb{R}^2 : y \neq 0\}$

- $\frac{\partial f}{\partial x} = \frac{2x}{y}$ que é cont. no seu domínio.
- $\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}(\underbrace{x^2}_{\text{const.}} \cdot y^{-1}) = x^2 \cdot (-1) y^{-2} = \frac{-x^2}{y^2}$, que é cont. em seu domínio.

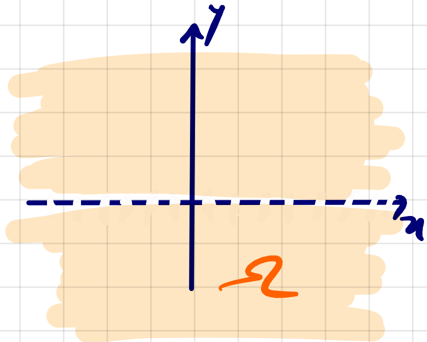
Portanto, f é diferenciável.

$d_a f$: ?

$\boxed{a = (x_0, y_0)}$

$f: \Omega \subset \mathbb{R}^2 \to \mathbb{R}^1$

$\Omega = D(f) = \{(x,y) \in \mathbb{R}^2 : y \neq 0\}$



$d_a f = [L]_{1\times 2} = \left[f_{11} \quad f_{12} \right](a)$,

$= \left[\frac{\partial f}{\partial x}(a) \quad \frac{\partial f}{\partial y}(a) \right] = \left[\frac{2x}{y}(a) \quad \frac{-x^2}{y^2}(a) \right]$

↑

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LISTA 09

$$(\ln m)' = \frac{m'}{m}$$

05)(b) $z = \ln(xy + y^2) = f(x,y)$

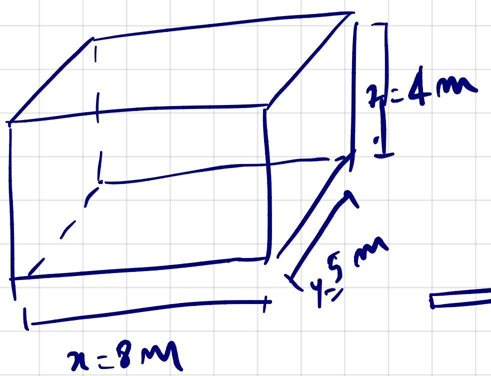
DIFERENCIAL TOTAL DE f.

$$df = dz = \frac{\partial f}{\partial x} \cdot dx + \frac{\partial f}{\partial y} dy \quad ; \text{ onde:}$$

$$\frac{\partial f}{\partial x} = \frac{y}{xy + y^2} \quad ; \quad \frac{\partial f}{\partial y} = \frac{x + 2y}{xy + y^2}$$

$$\Rightarrow \boxed{df = \frac{y}{xy + y^2} dx + \frac{x + 2y}{xy + y^2} dy}$$

08)



$$V = x \cdot y \cdot z = V(x,y,z)$$

$$\Delta x = \Delta y = \Delta z = 0{,}04\,m$$

$$dV = \frac{\partial V}{\partial x} \cdot \Delta x + \frac{\partial V}{\partial y} \cdot \Delta y + \frac{\partial V}{\partial z} \Delta z$$

$$\frac{\partial V}{\partial x} = y.z \quad ; \quad \frac{\partial V}{\partial y} = xz \quad ; \quad \frac{\partial V}{\partial z} = xy$$

$$\Rightarrow dV = yz \cdot \Delta x + xz \cdot \Delta y + xy \cdot \Delta z$$

Portanto;

$$dV\Big|_{\substack{x=8\\ y=5\\ z=4,\\ \Delta x=\Delta y=\Delta z=0,04}} = 5\cdot 4\cdot 0{,}04 + 8\cdot 4\cdot 0{,}04 + 8\cdot 5\cdot 0{,}04$$

$$= 20\cdot\frac{4}{100} + 32\cdot\frac{4}{100} + 40\cdot\frac{4}{100}$$

$$= \frac{4}{100}\left(20+32+40\right) = \frac{4\cdot 92}{100}$$

$$= \frac{92}{25} = \underline{3{,}68\ m^3}$$

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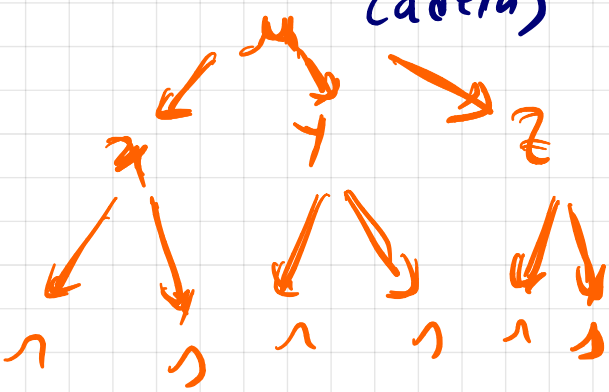
10) (b) $u = x^2yz \qquad x = r\cdot s^{-1}\,;\ y = r\cdot e^{s}\,;\ z = re^{-s}$

$$\frac{\partial u}{\partial r} = ?$$

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial r}\,;$$ onde:

(regra da cadeia)



$\dfrac{\partial u}{\partial x} = 2xyz\,;$

$\dfrac{\partial u}{\partial y} = x^2z\,;\qquad \dfrac{\partial u}{\partial z} = x^2y\,;$

$$\frac{\partial x}{\partial r} = \frac{1}{s} \; ; \quad \frac{\partial y}{\partial r} = e^{s} \; ; \quad \frac{\partial z}{\partial r} = e^{-s} .$$

Portanto,

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial r} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial r} + \frac{\partial u}{\partial z} \cdot \frac{\partial z}{\partial r}$$

$$= 2xyz \cdot \frac{1}{s} + x^2 z \cdot e^{s} + x^2 y \cdot e^{-s}$$

$$= 2 \cdot \frac{r}{s} \cdot r \underbrace{e^{s} \cdot r \cdot e^{-s}}_{1} \cdot \frac{1}{s} + \frac{r^2}{s^2} \cdot r \underbrace{e^{-s} \cdot e^{s}}_{1} + \frac{r^2}{s^2} \cdot r \underbrace{e^{s} \cdot e^{-s}}_{1}$$

$$= 2\frac{r^2}{s^2} + \frac{r^3}{s^2} + \frac{r^3}{s^2} = \frac{2r^2}{s^2} + 2\frac{r^3}{s^2}$$

$$= \frac{2r^2}{s^2} \cdot (1 + r)$$