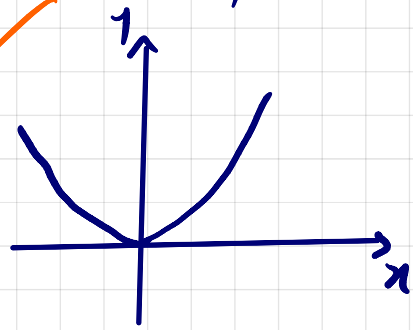
31/08/23

CILINDROS: No espaço ($\mathbb{R}^3$) chama-se CILINDRO toda superfície obtida quando, dada uma curva traçada no $\mathbb{R}^3$ e uma reta fixa, traçarmos uma coleção de retas paralelas à reta dada, todas elas passando pela curva.

EXEMPLOS:

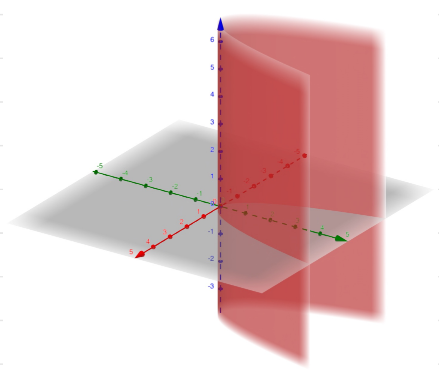
01) em $\mathbb{R}^3$: $y = x^2$, uma parábola no plano xy.

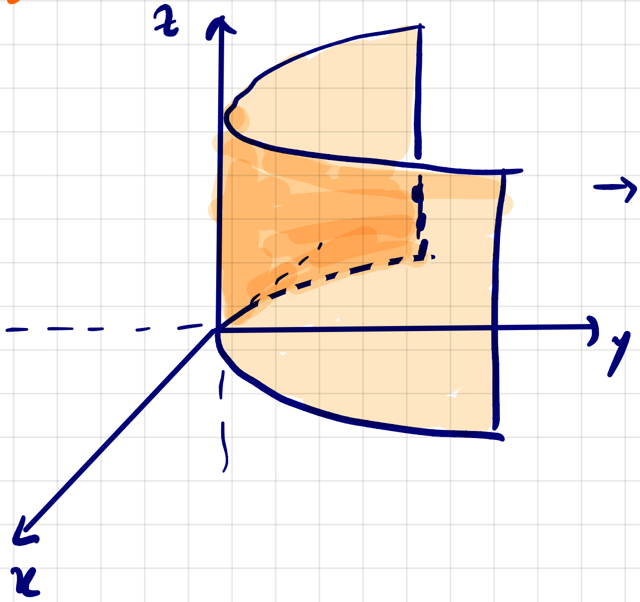


traçado de $y = x^2$ no plano xy

nesta eq. temos que a variável z está ausente, ou seja, é uma variável livre.

PELO GEOGEBRA.



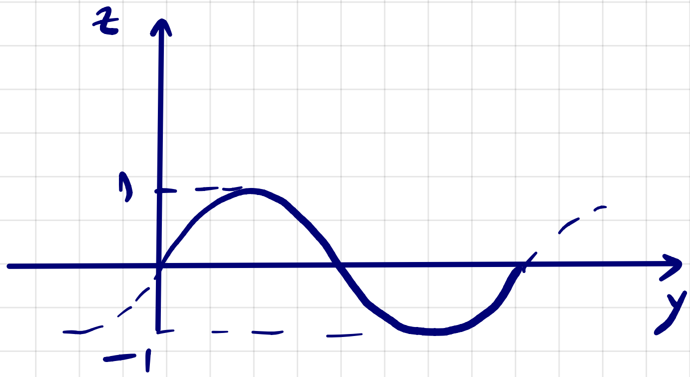


→ CILINDRO PARABÓLICO (POIS O TRAÇO É UMA PARÁBOLA)

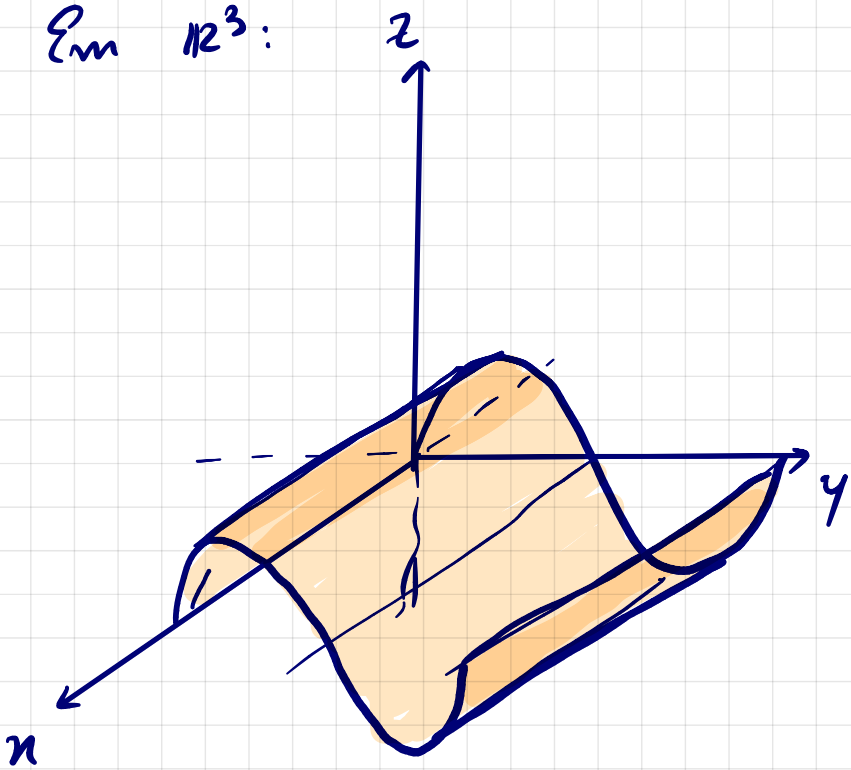
Quando $z = k$, temos uma parábola desenhada sobre o plano $z = k$. Em particular, quando $k = 0$ (plano xy)

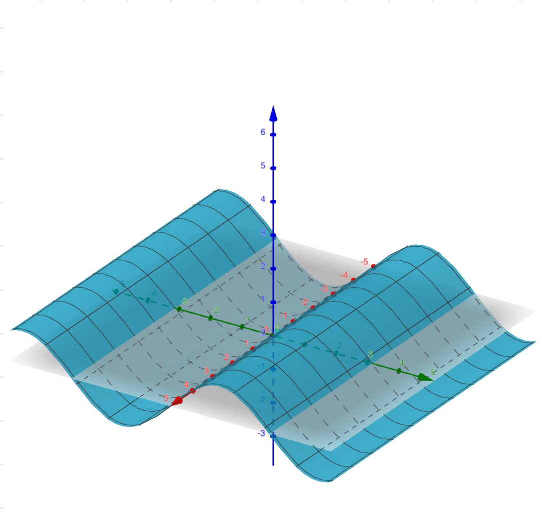
02) $z = \operatorname{sen} y$

. no plano zy teremos o traço:



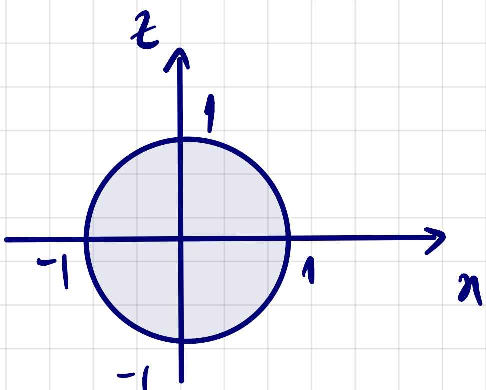
Em $\mathbb{R}^3$:



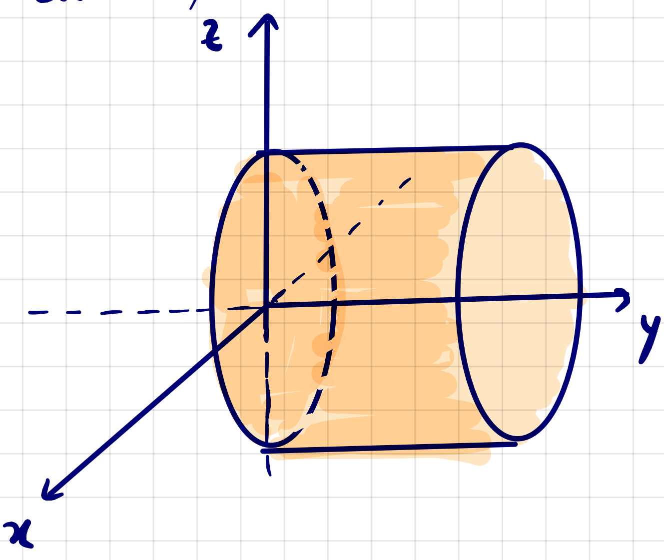


PELO GEOGEBRA.

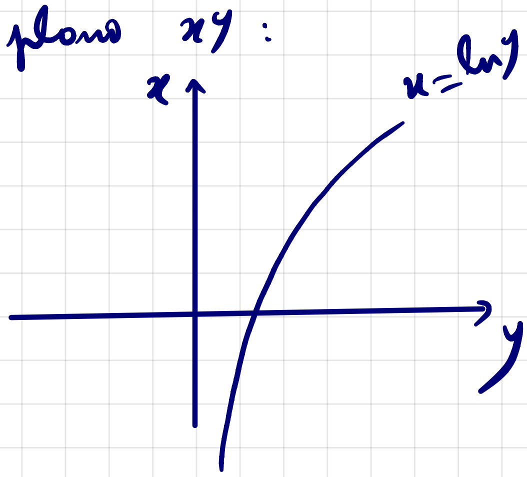
03) $x^2+z^2=1$. $\rightarrow$ no plano xz é uma circunferência de raio unitário centrada na origem. Em $\mathbb{R}^3$, teremos:



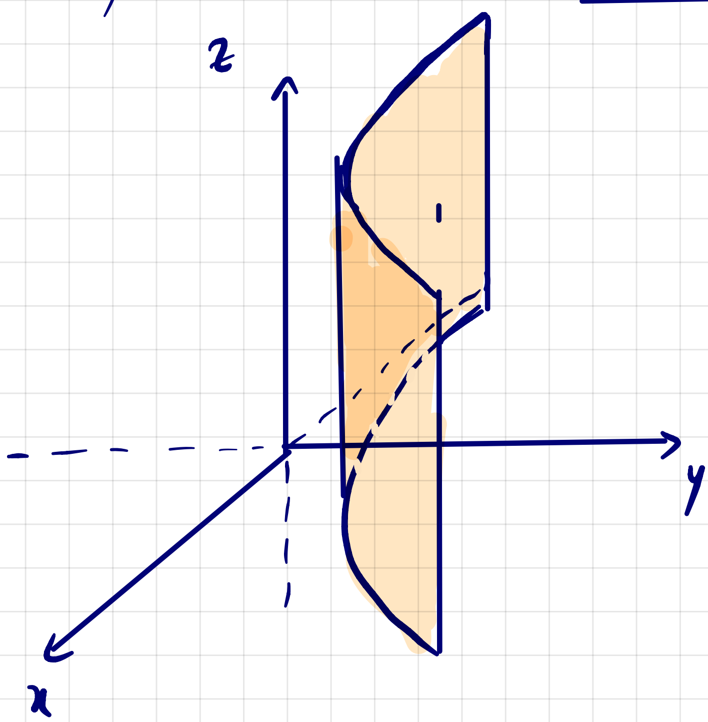
(traço no plano xz)



04) $x = \ln y$. No plano xy:



Em $\mathbb{R}^3$, teremos:



SUPERFÍCIES QUÁDRICAS:

São os gráficos de equações de 2º grau a 3 variáveis.

01) ELIPSÓIDE: são equações do tipo:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \; ; \quad a, b, c \in \mathbb{R} \setminus \{0\}.$$

traços: • $x = 0$: (plano yz):

$$\frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \quad \text{(elipse)}$$

• $y=0$ (plano xz)

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1 \quad \text{(elipse)}$$

• $z=0$ (plano xy):

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \quad \text{(elipse)}$$

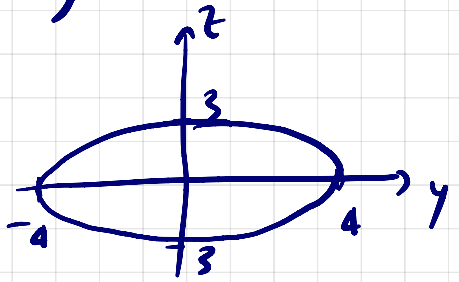
Combinando os 3 traços no $\mathbb{R}^3$ podemos montar um esboço do elipsóide.

Ex.:

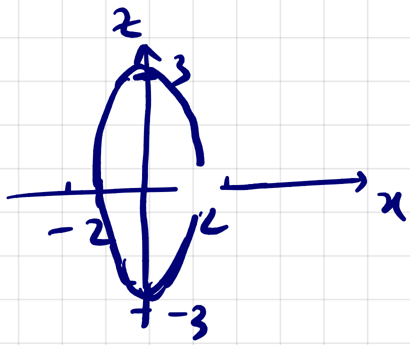
$$\frac{x^2}{4}+\frac{y^2}{16}+\frac{z^2}{9}=1.$$

traços:

• $x=0$ (plano yz): $\frac{y^2}{16}+\frac{z^2}{9}=1$ (elipse)

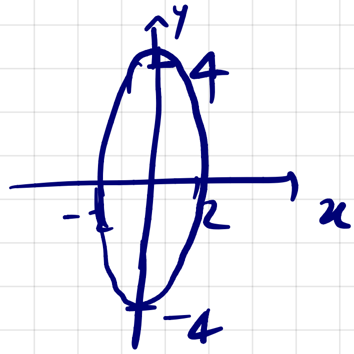


• $y=0$ (plano xz): $\frac{x^2}{4}+\frac{z^2}{9}=1$ (elipse)

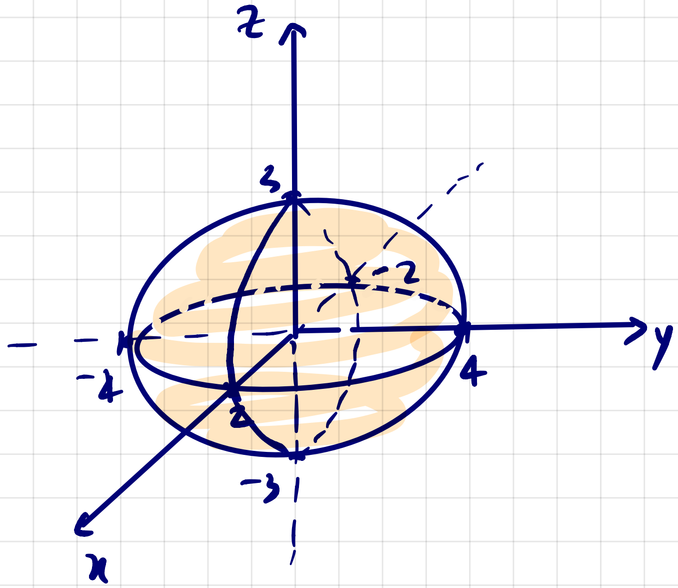


• $z=0$ (plano xy):

$$\frac{x^2}{4}+\frac{y^2}{16}=1 \quad \text{(elipse)}$$



Juntando estes 3 traços no $\mathbb{R}^3$, obtemos:



02) HIPERBOLÓIDE DE UMA FOLHA: Equação do tipo

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1 \quad \text{ou}$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1 \quad \text{ou}$$

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$$

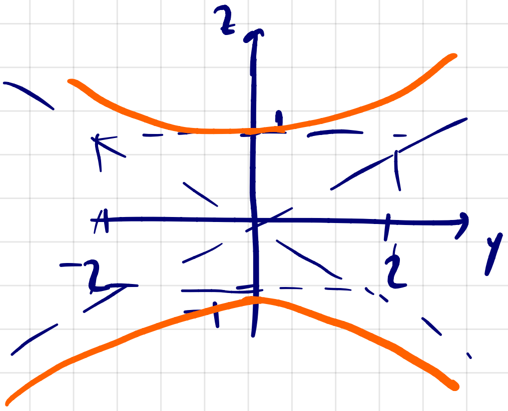
Ex.1 $\quad x^2-4y^2+16z^2=16$

Note que, dividindo por 16, vem:

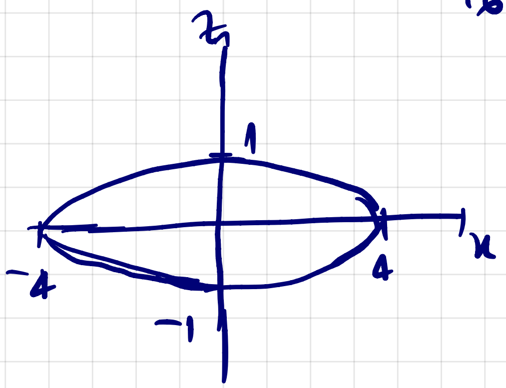
$$\frac{x^2}{16}-\frac{y^2}{4}+\frac{z^2}{1}=1.$$

traços:

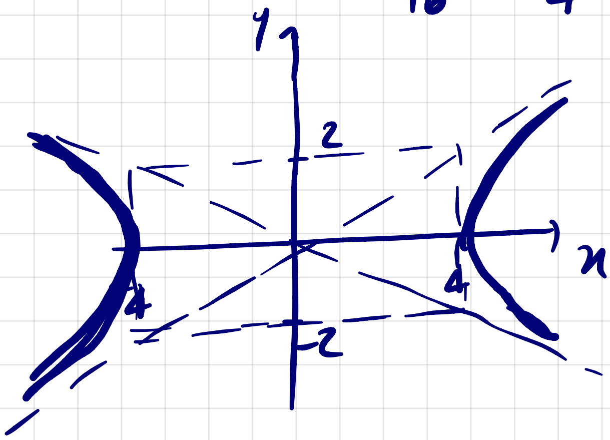
• $x = 0$ (plano yz) $\frac{z^2}{1} - \frac{y^2}{4} = 1$ (hipérbole)



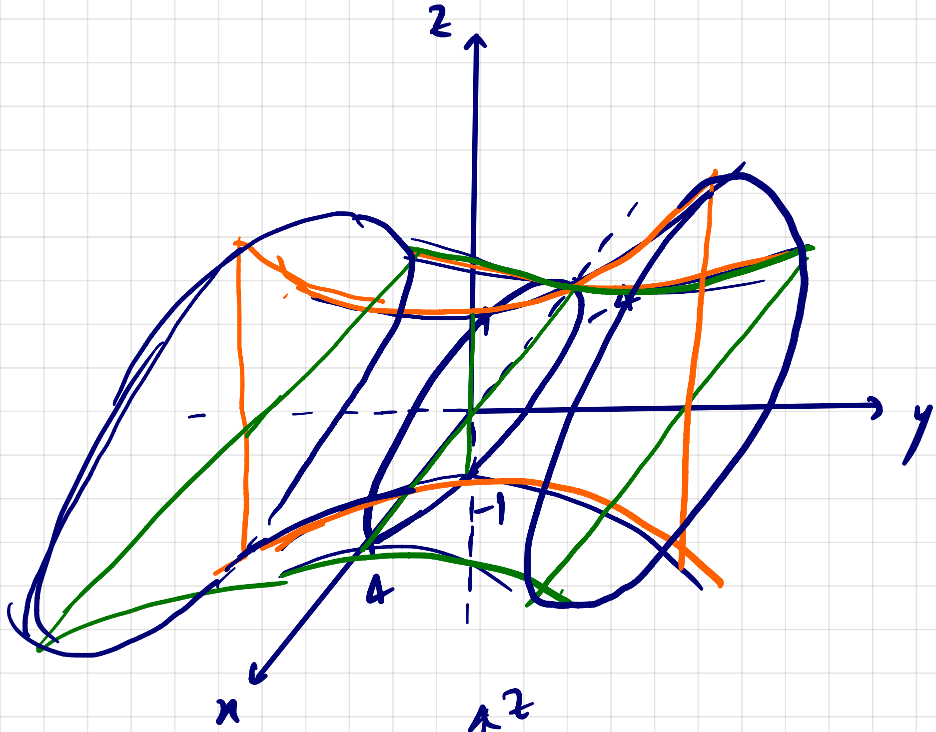
• $y = 0$ (plano xz) $\frac{x^2}{16} + \frac{z^2}{1} = 1$ (elipse)

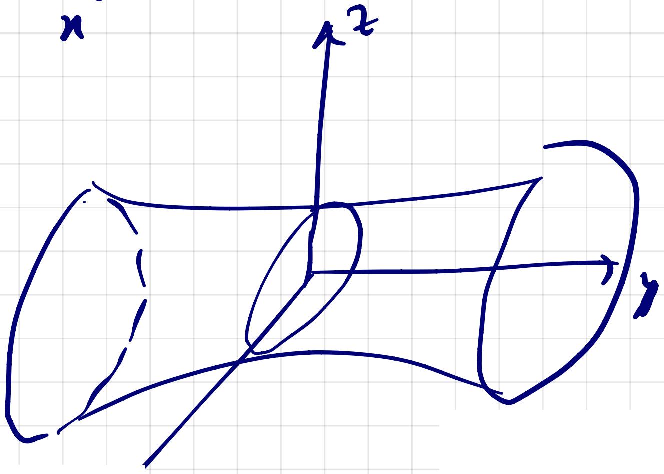


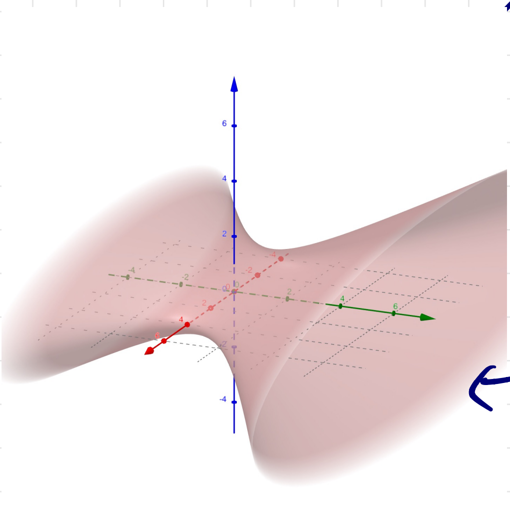
• $z = 0$ (plano xy): $\frac{x^2}{16} - \frac{y^2}{4} = 1$ (hipérbole)



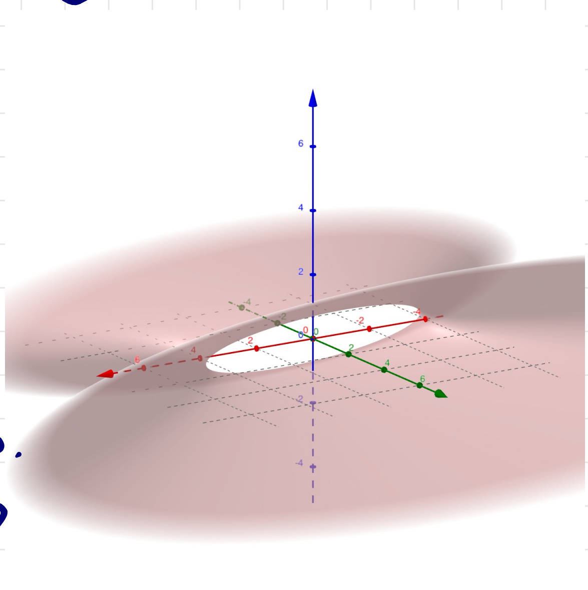
esboço gráfico da superfície:

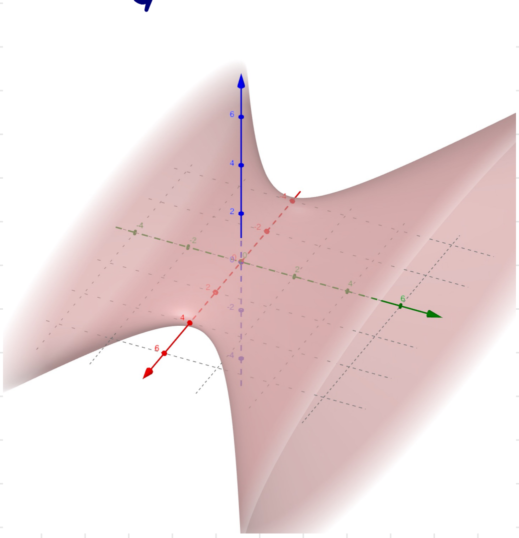






PELO GEOGEBRA, EM 3 POSIÇÕES DE VISUALIZAÇÃO.





03) HIPERBOLÓIDE DE 2 FOLHAS Equações do tipo:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{ou}$$

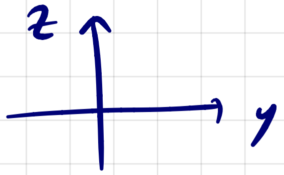
$$-\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1 \quad \text{ou}$$

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

Ex.:
$$\frac{x^2}{4} - \frac{y^2}{9} - \frac{z^2}{16} = 1.$$

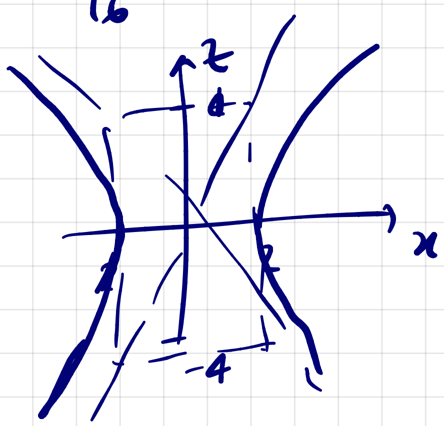
traços:

- $x = 0$: (plano yz) $\quad -\frac{y^2}{9} - \frac{z^2}{16} = 1$ ($\nexists$ traço)

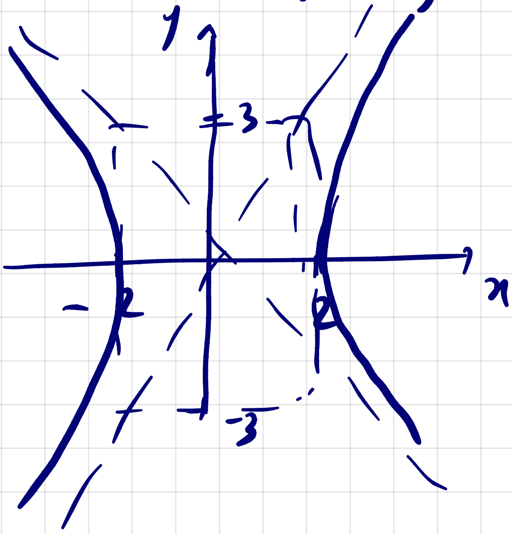


- $y = 0$ (plano xz):

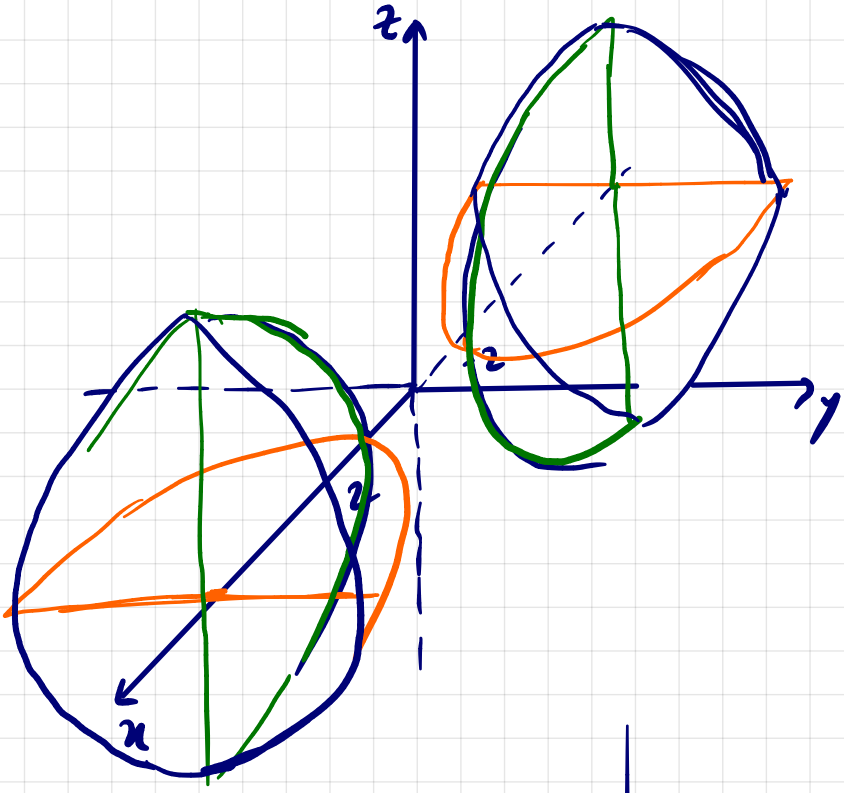
$$\frac{x^2}{4} - \frac{z^2}{16} = 1 \quad \text{(hipérbole)}$$

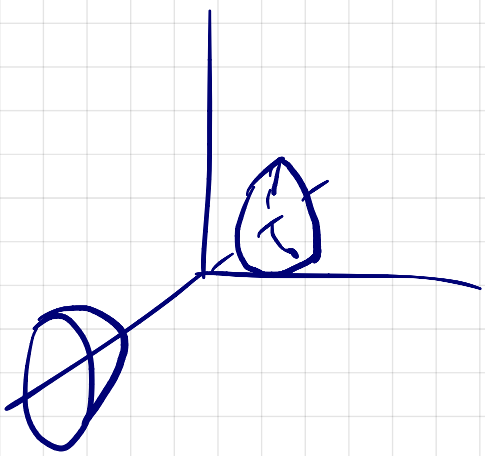


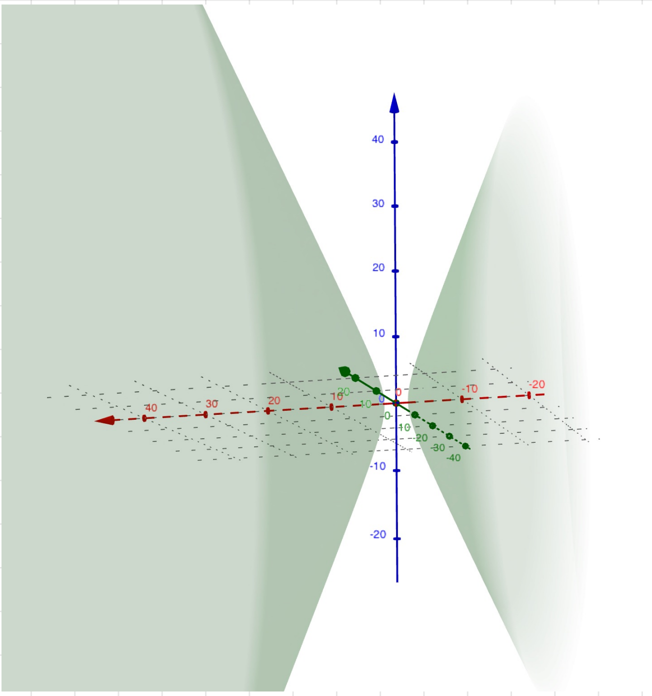
• $z=0$ (plano xy): $\frac{x^2}{4}-\frac{y^2}{9}=1$ (hipérbole)



esboço gráfico da superfície:







→ pelo GEOGEBRA

(Desculpem, não consegui deixar bem centralizado).

04) PARABOLÓIDE ELÍPTICO: são equações do tipo:

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=c\cdot z \quad \text{ou}$$

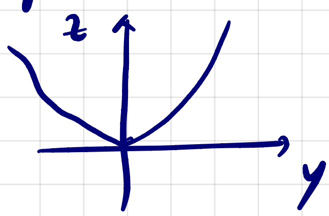
$$\frac{y^2}{b^2}+\frac{z^2}{c^2}=a x \quad \text{ou}$$

$$\frac{x^2}{a^2}+\frac{z^2}{c^2}=b y .$$

Ex.: $x^2+4y^2=z$

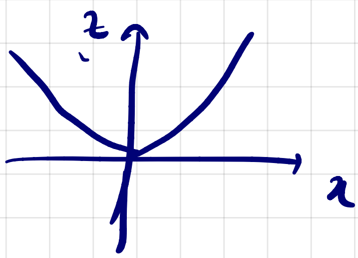
traços: • $x=0$: (plano yz):

$4y^2=z$ (parábola)

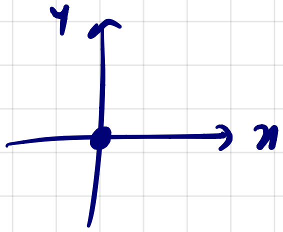


• $y=0$: (plano xz)

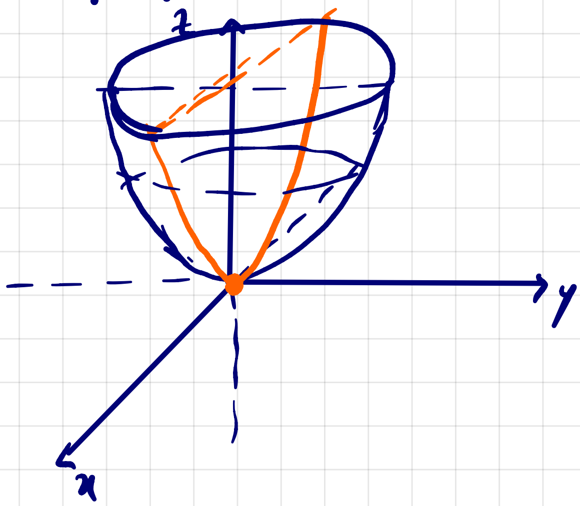
$x^2=z$ (parábola)

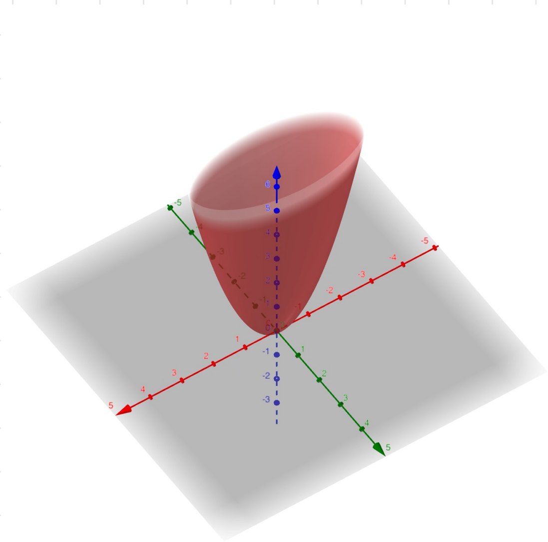


• $z=0$ (plano xy) $x^2+4y^2=0 \Leftrightarrow (x,y)=(0,0)$
origem)



esboço da superfície:





(PELO GEOGEBRA)