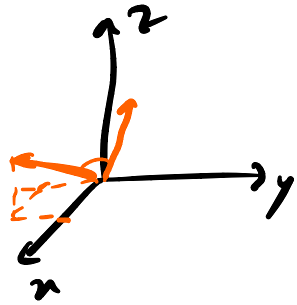
29/08/23

LISTA 05:

24)



$$\vec{u} = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$$

$$\vec{v} = \left(\frac{1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}\right)$$

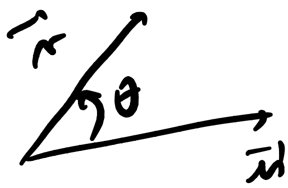
(a)
$$\|\vec{u}\| = \sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + \left(-\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = 1.$$

$$\|\vec{v}\| = \sqrt{\left(\frac{1}{3\sqrt{3}}\right)^2 + \left(\frac{5}{3\sqrt{3}}\right)^2 + \left(\frac{1}{3\sqrt{3}}\right)^2}$$

$$= \sqrt{\frac{1}{27} + \frac{25}{27} + \frac{1}{27}} = 1.$$

(b)



(i) Por um teorema,

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| . \|\vec{v}\| . \operatorname{sen}\theta$$

$$\Rightarrow \operatorname{sen}\theta = \frac{\|\vec{u} \times \vec{v}\|}{\underbrace{\|\vec{u}\|}_{=1} . \underbrace{\|\vec{v}\|}_{=1}} = \|\vec{u} \times \vec{v}\|;$$

onde:

$$\vec{u}\times\vec{v} = \left|\begin{matrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{3\sqrt{3}} & \frac{5}{3\sqrt{3}} & \frac{1}{3\sqrt{3}} \end{matrix}\right| \begin{matrix} \vec{i} & \vec{j} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} \\ \frac{1}{3\sqrt{3}} & \frac{5}{3\sqrt{3}} \end{matrix}$$

$$- \quad - \quad - \quad + \quad + \quad +$$

$$= -\frac{1}{9}\vec{i} + \cancel{\frac{1}{9}\vec{j}} + \frac{5}{9}\vec{k} + \frac{1}{9}\vec{k} - \frac{5}{9}\vec{i} - \cancel{\frac{1}{9}\vec{j}}$$

$$= -\frac{2}{3}\vec{i} + 0\vec{j} + \frac{2}{3}\vec{k} \implies \vec{u}\times\vec{v} = \left(-\frac{2}{3}, 0, \frac{2}{3}\right)$$

$$\implies \|\vec{u}\times\vec{v}\| = \sqrt{\left(-\frac{2}{3}\right)^2 + 0^2 + \left(\frac{2}{3}\right)^2}$$

$$= \sqrt{\frac{4}{9} + \frac{4}{9}} = \sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

$$\implies \operatorname{sen}\theta = \|\vec{u}\times\vec{v}\| = \frac{2\sqrt{2}}{3}$$

(ii) usando o produto escalar:

$$\vec{u}\cdot\vec{v} = \|\vec{u}\|.\|\vec{v}\|.\cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{\underbrace{\|\vec{u}\|}_{=1}.\underbrace{\|\vec{v}\|}_{=1}} = \vec{u}\cdot\vec{v};$$

onde:

$$\vec{u}\cdot\vec{v} = \left(\frac{1}{\sqrt{3}}, -\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)\cdot\left(\frac{1}{3\sqrt{3}}, \frac{5}{3\sqrt{3}}, \frac{1}{3\sqrt{3}}\right)$$

$$= \frac{1}{\sqrt{3}}\cdot\frac{1}{3\sqrt{3}} + \left(-\frac{1}{\sqrt{3}}\cdot\frac{5}{3\sqrt{3}}\right) + \frac{1}{\sqrt{3}}\cdot\frac{1}{3\sqrt{3}}$$

$$= \frac{1}{9} - \frac{5}{9} + \frac{1}{9} = -\frac{3}{9} = -\frac{1}{3}$$

$\cos\theta = -\frac{1}{3}$. Pela relação trigonométrica fundamental. $\operatorname{sen}^2\theta + \cos^2\theta = 1$.

$$\Rightarrow \operatorname{sen}^2\theta = 1 - \cos^2\theta$$

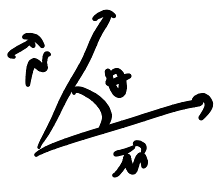
$$\Rightarrow \operatorname{sen}\theta = \pm\sqrt{1 - \left(-\frac{1}{3}\right)^2}$$

$$= \pm\sqrt{1 - \frac{1}{9}} = \pm\frac{2\sqrt{2}}{3}$$

Pelo item (i) sabemos que $\operatorname{sen}\theta = +\frac{2\sqrt{2}}{3}$.

LISTA 05.

17) $\vec{u} = ?$ tal que: $\|\vec{u}\| = 2$



$\theta = 45^\circ$.

$\vec{v} = (1, -1, 0)$

$\vec{u} \perp \vec{w}$; $\vec{w} = (1, 1, 0)$

Seja $\vec{u} = (a, b, c)$.

$$\|\vec{u}\| = \sqrt{a^2+b^2+c^2} = 2$$

$$\Rightarrow \boxed{a^2+b^2+c^2=4} \quad (\bullet)$$

$$\vec{u} \perp \vec{w} \iff \vec{u}\cdot\vec{w} = 0$$

$$(a, b, c)\cdot(1, 1, 0) = 0$$

$$a\cdot 1 + b\cdot 1 + c\cdot 0 = 0$$

$$\boxed{a+b=0} \quad (*)$$

Além disso, como o ângulo θ entre $\vec{u}$ e $\vec{v}$ é $45°$, temos:

$$\underbrace{\cos 45°}_{\frac{\sqrt{2}}{2}} = \frac{\vec{u}\cdot\vec{v}}{\underbrace{\|\vec{u}\|}_{2}\cdot\|\vec{v}\|} = \frac{(a, b, c)\cdot(1, -1, 0)}{2\cdot\sqrt{(1)^2+(-1)^2+0^2}}$$

$$\frac{\sqrt{2}}{\cancel{2}} = \frac{a\cdot 1 + b\cdot(-1) + c\cdot 0}{\cancel{2}\cdot\sqrt{2}}$$

$$\sqrt{2} = \frac{a-b}{\sqrt{2}} \Rightarrow \boxed{a-b=2} \quad (**)$$

De $(*)$ e $(**)$ temos:

$$+\begin{cases} a+\cancel{b}=0 \\ a-\cancel{b}=2 \end{cases}$$
$$2a=2 \Rightarrow a=1$$

$$\begin{cases} a+b=0 \\ b=-a \\ b=-1 \end{cases}$$

De (•) teremos:

$$a^2+b^2+c^2=4$$
$$(1)^2+(-1)^2+c^2=4$$
$$1+1+c^2=4$$

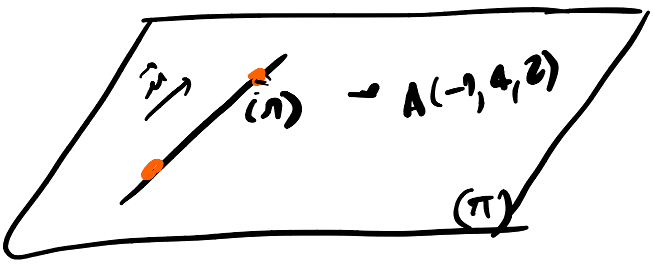
$$\begin{cases} c^2=2 \\ \Rightarrow \boxed{c=\pm\sqrt{2}} \end{cases}$$

Resposta: $\vec{u}=(a,b,c)$ será:

$$\vec{u}=(1,-1,\sqrt{2}) \text{ ou}$$
$$\vec{u}=(1,-1,-\sqrt{2})$$

LISTA 06 :

12) $(\pi)=?$



$(r)=(\alpha)\cap(\beta)$; onde

$(\alpha): 4x-y+z=2$

$(\beta): 2x+y-2z=3$

Assim, para obter a eq. da reta (r) vamos resolver o sist. linear:

$$+\begin{cases} 4x - \cancel{y} + z = 2 \\ 2x + \cancel{y} - 2z = 3 \end{cases}$$

$$6x - z = 5 \rightsquigarrow z = -5 + 6x$$

escreva $x = t$. Assim:

$$z = -5 + 6t$$
$$x = t$$

$$y = 3 - 2x + 2z$$
$$y = 3 - 2.t + 2.(-5 + 6t)$$
$$y = 3 - 2t - 10 + 12t$$
$$y = -7 + 10t.$$

Logo, uma eq. paramétrica para a reta (1) será:

$$(1): \begin{cases} x = t \\ y = -7 + 10t \\ z = -5 + 6t \end{cases} \qquad \boxed{\vec{u} = (1, 10, 6)}$$

(VETOR DIRETOR DA RETA)

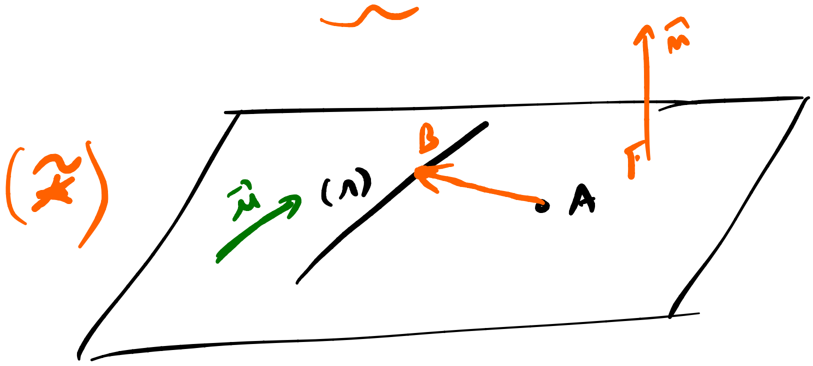
se $t = 0$, temos o ponto $B: \begin{cases} x = 0 \\ y = -7 \\ z = -5 \end{cases}$

$$\Rightarrow B(0, -7, -5)$$

Disso, o vetor $\vec{AB}$ será:

$$\vec{AB} = B - A = (0, -7, -5) - (-1, 4, 2)$$

$$= (1, -11, -7)$$



Então:

$$\vec{n} = \vec{u} \times \vec{AB}$$

$$\Rightarrow \vec{n} = \left|\begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 10 & 6 & 1 & 10 \\ 1 & -11 & -7 & 1 & -11 \end{array}\right|$$

$$= -70\vec{i} + 6\vec{j} - 11\vec{k} - 10\vec{k} + 66\vec{i} + 7\vec{j}$$

$$= -4\vec{i} + 13\vec{j} - 21\vec{k} = \underset{(a,\,b,\,c)}{(-4, 13, -21)}$$

Portanto, a equação do plano (π) será:

$$(\pi): \ ax + by + cz + d = 0$$

$$(\pi): \ -4x + 13y - 21z + d = 0$$

Como $A(-1,4,2) \in (\pi)$; temos:

$$-4\cdot(-1) + 13\cdot(4) - 21\cdot(2) + d = 0$$

$$4 + 52 - 42 + d = 0 \Rightarrow \boxed{d = -14}$$

Portanto, a eq. do plano (π) será:

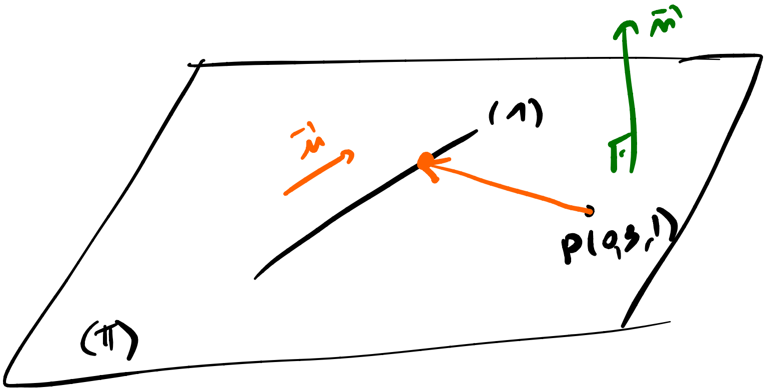
$$\boxed{(\pi): \ -4x + 13y - 21z - 14 = 0}$$

LISTA 06.

18) $(r): (x-2, y+3, z-1) = t\cdot(1,1,2)$. e $P(0,3,1)$

(a) (π) tal que $(r) \in (\pi)$ e $P \in (\pi)$

$$(r): \begin{cases} x = 2 + t \\ y = -3 + t \\ z = 1 + 2t \end{cases} \qquad \vec{u} = (1,1,2)$$



Para obter a eq. do plano (π) basta seguir os mesmos passos da resolução do exercício feito anteriormente, a partir de (*)..

De fato:

para $t=0$, vamos obter o ponto

$$B = \begin{cases} x = 2 \\ y = -3 \\ z = 1 \end{cases} \Rightarrow B(2,-3,1) \in (\pi)$$

Assim, $\vec{PB} = B - P$ é paralelo ao plano (π);

onde: $\vec{PB} = B - P = (2,-3,1) - (0,3,1) = (2,-6,0)$

Assim, $\vec{n} = \vec{u} \times \vec{PB}$

$$\vec{n} = \left| \begin{array}{ccc|cc} \vec{i} & \vec{j} & \vec{k} & \vec{i} & \vec{j} \\ 1 & 1 & 2 & 1 & 1 \\ 2 & -6 & 0 & 2 & -6 \end{array} \right|$$

$$\Rightarrow \vec{n} = 0.\vec{i} + 4\vec{j} - 6\vec{k} - 2\vec{k} + 12\vec{i} - 0\vec{j}$$

$$\Rightarrow \vec{n} = 12\vec{i} + 4\vec{j} - 8\vec{k} \Rightarrow \vec{n} = (\underbrace{12}_{a}, \underbrace{4}_{b}, \underbrace{-8}_{c})$$

Então:

(π): $ax + by + cz + d = 0$

(π): $12x + 4y - 8z + d = 0$. $P(0,3,1) \in (\pi)$:

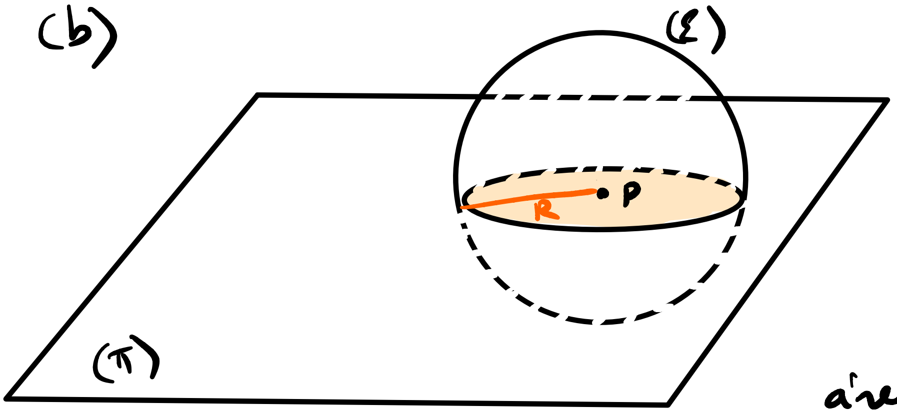
$$12.(0) + 4.(3) - 8.(1) + d = 0 \Rightarrow \boxed{d = -4}$$

Logo:

$$(\pi): 12x + 4y - 8z - 4 = 0 \quad (\div 4)$$

$$\boxed{(\pi): 3x + y - 2z - 1 = 0}$$

(b)



$P(0,3,1)$: centro de (ε)

área $= 16\pi$ u.a.

$\cancel{\pi} \cdot R^2 = 16\cancel{\pi}$

$\boxed{R^2 = 16}$

eq. da (ε):

$$(x-h)^2 + (y-k)^2 + (z-l)^2 = R^2$$

$$(x-0)^2 + (y-3)^2 + (z-1)^2 = 16$$

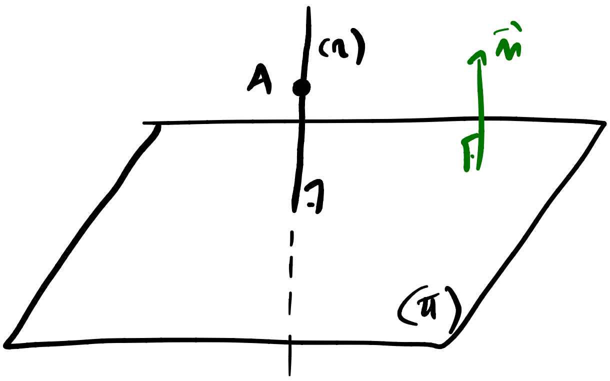
$$\boxed{x^2 + (y-3)^2 + (z-1)^2 = 16}$$

LISTA 06

13) $(r) = ?$

$A(1,3,2) \in (r)$

$(r) \perp (\pi): \; x - 5y + z = 0$



$\vec{m} = (1,-5,1)$ serve como vetor diretor para a reta (r).

A eq. vetorial da (r) será:

$$(r): (x-1, y-3, z-2) = t \cdot (1,-5,1).$$