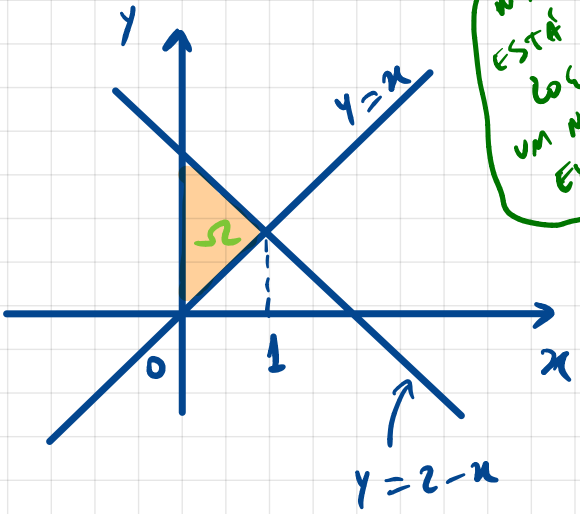
LISTA02 (ALGUMAS RESOLUÇÕES)

02) (b) $$\int_0^1 \int_x^{\overset{2x}{\cancel{2-x}}} (x^2-y)\,dy\,dx = \int_{x=0}^{x=1} \left(x^2y - \frac{y^2}{2}\right)\Bigg|_{y=x}^{\boxed{y=2x}} dx =$$

NA LISTA ESTÁ 2−x, LOGO, ESTE É UM NOVO EXERCÍCIO



$$= \int_0^1 \left[x^2\cdot 2x - \frac{4x^2}{2} - \left(x^2\cdot x - \frac{x^2}{2}\right)\right] dx =$$

$$= \int_0^1 \left(2x^3 - 2x^2 - x^3 + \frac{x^2}{2}\right) dx$$

$$= \int_0^1 \left(x^3 - \frac{3x^2}{2}\right) dx = \left(\frac{x^4}{4} - \frac{\cancel{3}}{2}\,\frac{x^3}{\cancel{3}}\right)\Bigg|_0^1 =$$

$$= \frac{1}{4} - \frac{1}{2} - 0 = -\frac{1}{2} \,//$$

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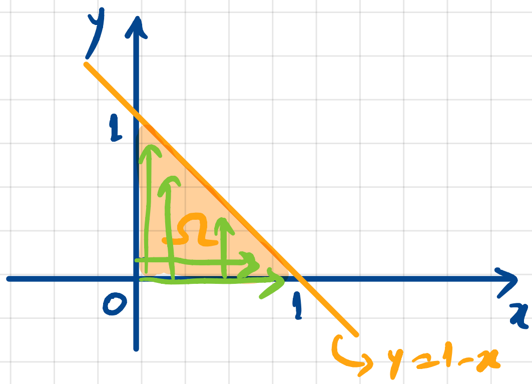
03) (a) $$\iint_{\Omega} x^2 y\, dy\, dx \text{ , onde:}$$

$$\Omega = \{(x,y) \in \mathbb{R}^2 :\ x \geq 0;\ y \geq 0;\ x+y \leq 1\}$$

↪ $y = 1-x$



$$\iint_{\Omega} x^2 y \, dx\, dy = \int_{x=0}^{x=1} \int_{y=0}^{y=1-x} x^2 y \, dy\, dx =$$

$$= \int_{x=0}^{x=1} \left(x^2 \cdot \frac{y^2}{2} \right) \Bigg|_{y=0}^{y=1-x} dx = \int_0^1 \frac{x^2}{2} \left[(1-x)^2 - 0 \right] dx$$

$$= \int_0^1 \frac{x^2}{2} \cdot (1 - 2x + x^2)\, dx = \int_0^1 \left(\frac{x^2}{2} - x^3 + \frac{x^4}{2} \right) dx =$$

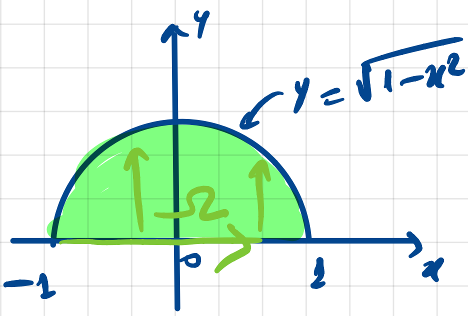
$$= \left(\frac{x^3}{6} - \frac{x^4}{4} + \frac{x^5}{10} \right) \Bigg|_0^1 = \frac{1}{6} - \frac{1}{4} + \frac{1}{10} - 0$$

$$= \frac{10 - 15 + 6}{60} = \frac{1}{60}$$

(e) $\iint_{\Omega} y \, dA$; onde

NA LISTA ESTÁ $x \geq 0$, LOGO ESTE É UM NOVO EXERCÍCIO.

$$\Omega = \{ (x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 , \cancel{y} \geq 0 \}$$



$$\iint_{\Omega} y \cdot dA = \int_{x=-1}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} y \, dy\, dx =$$

$$= \int_{x=-1}^{x=1} \frac{y^2}{2} \Bigg|_{y=0}^{y=\sqrt{1-x^2}} \cdot dx = \int_{-1}^{1} \left(\frac{(\sqrt{1-x^2})^2}{2} - 0 \right) dx =$$

$$= \int_{-1}^{1} \frac{1-x^2}{2}\,dx = \left(\frac{1}{2}x - \frac{1}{2}\frac{x^3}{3}\right)\Big|_{-1}^{1} =$$

$$= \frac{1}{2} - \frac{1}{6} - \left(-\frac{1}{2} + \frac{1}{6}\right) = \frac{1}{2} - \frac{1}{6} + \frac{1}{2} - \frac{1}{6} = 1 - \frac{1}{3} = \frac{2}{3}$$

Obs: Como já estudamos coordenadas polares, sendo Ω região envolvendo arco de círculo, poderíamos fazer:

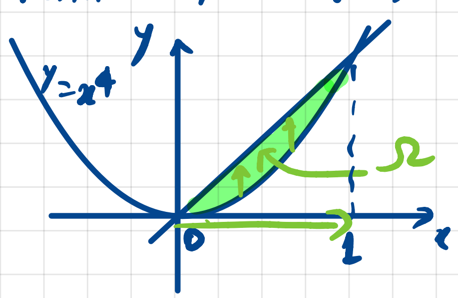
$$\iint_\Omega y\,dy\,dx = \int_{\theta=0}^{\theta=\pi}\int_{\rho=0}^{\rho=1} \underbrace{\rho\,\text{sen}\theta}_{y}\,.\,\underbrace{\rho\,d\rho\,d\theta}_{\substack{dx\,dy\\ \text{ou}\\ dy\,dx}} =$$

$$= \int_{\theta=0}^{\theta=\pi} \text{sen}\theta\,.\,d\theta\,.\int_{\rho=0}^{\rho=1} \rho^2\,.\,d\rho =$$

$$(-\cos\theta)\Big|_0^{\pi}\cdot\left(\frac{\rho^3}{3}\right)\Big|_0^{1} = (-\cos\pi + \cos 0)\,.\left(\frac{1}{3} - 0\right)$$

$$= (1+1)\,.\,\frac{1}{3} = \frac{2}{3}$$

05) PLANO xy (REGIÃO)



INTERCEPTOS:

$$x^4 = x \iff \begin{cases} x = 0 \\ x = 1 \end{cases}$$

Sendo $z = x + 2y$, então:

$$V = \iint_{\Omega} (x+2y)\,dy\,dx = \int_{x=0}^{x=1} \int_{y=x^4}^{y=x} (x+2y)\,dy\,dx =$$

$$= \int_{x=0}^{x=1} (xy+y^2)\Big|_{y=x^4}^{y=x} dx = \int_{x=0}^{x=1} (x^2+x^2-x^5-x^8)\,dx$$

$$= \int_0^1 (2x^2-x^5-x^8)\,dx = \left(\frac{2x^3}{3} - \frac{x^6}{6} - \frac{x^9}{9}\right)\Big|_0^1 =$$

$$\frac{2}{3} - \frac{1}{6} - \frac{1}{9} - 0 = \frac{12-3-2}{18} = \frac{7}{18}$$

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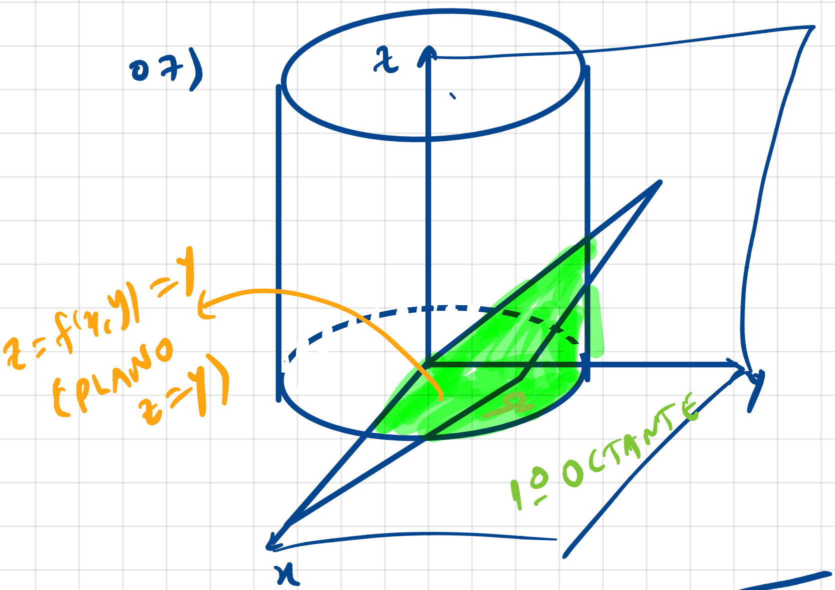
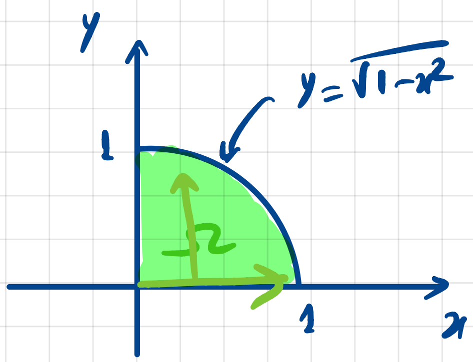
07)





$$V = \iint_{\Omega} f(x,y)\,dy\,dx = \int_{x=0}^{x=1} \int_{y=0}^{y=\sqrt{1-x^2}} y\,.\,dy\,dx = \int_{x=0}^{x=1} \frac{y^2}{2}\Big|_{y=0}^{y=\sqrt{1-x^2}} dx =$$

$$= \int_0^1 \left[\frac{1}{2}(\cancel{\sqrt{1-x^2}})^{\cancel{2}} - 0\right] dx = \frac{1}{2}\left(x - \frac{x^3}{3}\right)\Big|_0^1 =$$

$$= \frac{1}{2}\left(1 - \frac{1}{3} - 0\right) = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} = \frac{1}{3}$$

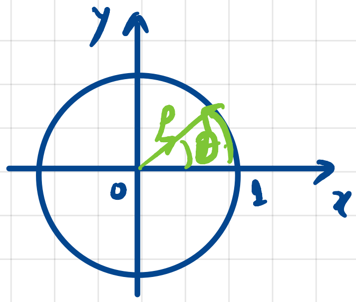
<u>Obs:</u> Também poderíamos ter usado coordenadas polares:

$$V = \iint_{\Omega} \underbrace{y}_{\rho \cdot \operatorname{sen}\theta} \cdot \underbrace{dy\,dx}_{\rho\,d\rho\,d\theta} = \int_{\theta=0}^{\theta=\frac{\pi}{2}} \int_{\rho=0}^{\rho=1} \rho \cdot \operatorname{sen}\theta \cdot \rho\, d\rho \cdot d\theta =$$

$$= \int_{\theta=0}^{\theta=\frac{\pi}{2}} \operatorname{sen}\theta\, d\theta \cdot \int_{\rho=0}^{\rho=1} \rho^2\, d\rho = \left(-\cos\theta\right)\Big|_0^{\frac{\pi}{2}} \cdot \left(\frac{\rho^3}{3}\right)\Big|_0^1 =$$

$$= \left(-\cos\frac{\pi}{2} + \cos 0\right) \cdot \left(\frac{1}{3} - 0\right) = (0+1) \cdot \left(\frac{1}{3}\right) = \frac{1}{3}$$

08 - b) $\displaystyle\iint_{x^2+y^2 \leq 1} \frac{dx\,dy}{\sqrt{1+x^2+y^2}}$

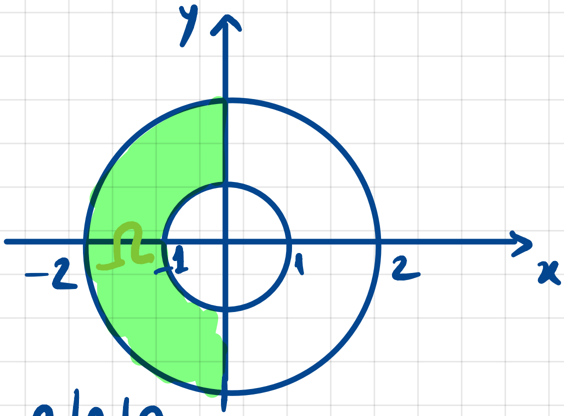


$$\iint_{x^2+y^2\leq 1} \frac{dx\,dy}{\sqrt{1+\underbrace{x^2+y^2}_{=\rho^2}}} = \int_{\theta=0}^{\theta=2\pi} \int_{\rho=0}^{\rho=1} \frac{\rho\, d\rho\, d\theta}{\sqrt{1+\rho^2}} =$$

$$= \int_{\theta=0}^{\theta=2\pi} d\theta \cdot \frac{1}{2} \cdot \int_{\rho=0}^{\rho=1} (1+\rho^2)^{-\frac{1}{2}} \cdot 2\rho \cdot d\rho = \theta\Big|_0^{2\pi} \cdot \frac{1}{\cancel{2}} \cdot \frac{(1+\rho^2)^{\frac{1}{2}}}{\frac{1}{\cancel{2}}}\Bigg|_0^1$$

$$= (2\pi - 0) \cdot \left(\sqrt{1+1^2} - \sqrt{1+0}\right) = 2\pi \cdot (\sqrt{2} - 1)$$

09) $\iint_{\Omega} (x+y)\,dx\,dy$, onde Ω é:



$$\iint_{\Omega} (x+y)\,dx\,dy = \int_{\theta=\frac{\pi}{2}}^{\theta=\frac{3\pi}{2}} \int_{\rho=1}^{\rho=2} (\rho\cos\theta + \rho\,\mathrm{sen}\,\theta)\cdot\rho\,d\rho\,d\theta =$$

$x = \rho\cos\theta$
$y = \rho\,\mathrm{sen}\,\theta$

$$= \int_{\theta=\frac{\pi}{2}}^{\theta=\frac{3\pi}{2}} \int_{\rho=1}^{\rho=2} (\cos\theta + \mathrm{sen}\,\theta)\cdot\rho^2\,d\rho\,d\theta =$$

$$= \int_{\theta=\frac{\pi}{2}}^{\theta=\frac{3\pi}{2}} (\cos\theta + \mathrm{sen}\,\theta)\,d\theta \cdot \int_{\rho=1}^{\rho=2} \rho^2\,d\rho = (\mathrm{sen}\,\theta - \cos\theta)\Big|_{\frac{\pi}{2}}^{\frac{3\pi}{2}} \cdot \frac{\rho^3}{3}\Big|_1^2 =$$

$$= \left[\mathrm{sen}\,\frac{3\pi}{2} - \cos\frac{3\pi}{2} - \left(\mathrm{sen}\,\frac{\pi}{2} - \cos\frac{\pi}{2}\right)\right]\cdot\left(\frac{8}{3} - \frac{1}{3}\right) =$$

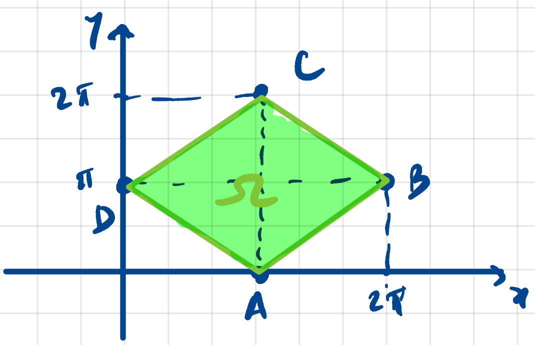
$$= [-1 + 0 - (1 - 0)]\cdot\frac{7}{3} = -\frac{14}{3}$$

→ TEM QUE CORRIGIR NO ENUNCIADO.

12) – b. $\iint_{\Omega} (x-y)^2 \cdot \mathrm{sen}\,(x+y)\,dx\,dy$, onde Ω

é dado por:

A$(\pi, 0)$; B$(2\pi, \pi)$;
C$(\pi, 2\pi)$; D$(0, \pi)$.



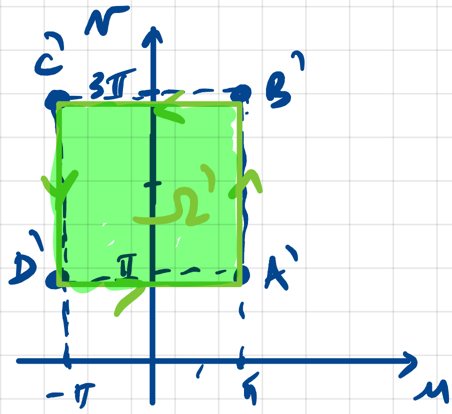
Escreva
$$\begin{cases} u = x - y \\ v = x + y \end{cases}$$
$$+ \quad \overline{u + v = 2x} \Longrightarrow \boxed{x = \frac{1}{2}u + \frac{1}{2}v}$$

Assim:
$$y = v - x = v - \frac{1}{2}u - \frac{1}{2}v =$$
$$\longrightarrow \boxed{y = -\frac{1}{2}u + \frac{1}{2}v}$$

$$\Rightarrow T(u,v) = (x,y) = \left(\frac{1}{2}u + \frac{1}{2}v,\ -\frac{1}{2}u + \frac{1}{2}v\right)$$

$$\det\big(J(T)(u,v)\big) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{1}{4} - \left(-\frac{1}{4}\right) = \frac{1}{2}$$

$$(x,y) \rightsquigarrow (u,v) = (x-y,\ x+y)$$
$$A(\pi, 0) \rightsquigarrow A'(\pi, \pi)$$
$$B(2\pi, \pi) \rightsquigarrow B'(2\pi - \pi,\ 2\pi + \pi) = (\pi, 3\pi)$$
$$C(\pi, 2\pi) \rightsquigarrow C'(\pi - 2\pi,\ \pi + 2\pi) = (-\pi, 3\pi)$$
$$D(0, \pi) \rightsquigarrow D'(0 - \pi,\ 0 + \pi) = (-\pi, \pi)$$



$$\Omega' = T(\Omega)$$

$$\iint_{\Omega} (x-y)^2 \cdot \operatorname{sen}(x+y)\,dx\,dy = \iint_{\Omega'} u^2 \cdot \operatorname{sen} v \cdot \underbrace{\left|\det(J)(T)(u,v)\right|}_{=\frac{1}{2}}\,du\,dv$$

$$= \int_{v=\pi}^{v=3\pi} \int_{u=-\pi}^{u=\pi} u^2 \cdot \operatorname{sen} v \cdot \frac{1}{2} \cdot du\,dv =$$

$$= \frac{1}{2}\int_{v=\pi}^{v=3\pi} \operatorname{sen} v \cdot dv \cdot \int_{u=-\pi}^{u=\pi} u^2\,du = (-\cos v)\Big|_{\pi}^{3\pi} \cdot \left(\frac{u^3}{3}\right)\Big|_{-\pi}^{\pi} =$$

$$= (-\cos 3\pi + \cos\pi)\cdot\left(\frac{\pi^3}{3} - \left(-\frac{\pi^3}{3}\right)\right) =$$

$$\underbrace{(-(-1) + (-1))}_{=0}\left(\frac{2\pi^3}{3}\right) = 0$$

14) $\iint_{\Omega} \cos\left(\frac{y-x}{y+x}\right) dA$; onde Ω é a região trapezoidal dada pelos vértices: $A(1,0)$; $B(2,0)$; $C(0,2)$; $D(0,1)$.

Escreva

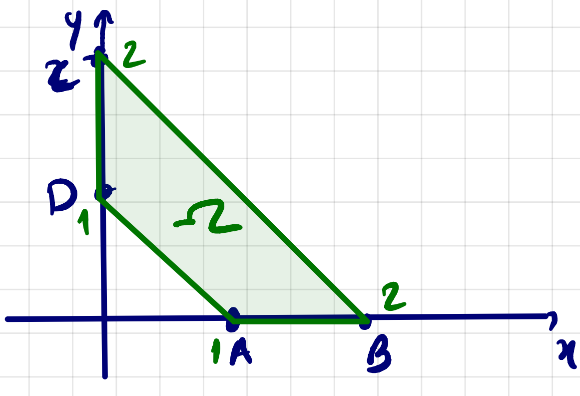
$$\begin{cases} u = y - x \\ v = y + x \end{cases}$$

$$+\quad u + v = 2y \rightarrow \boxed{y = \frac{1}{2}u + \frac{1}{2}v}$$

$$\rightsquigarrow x = u + y = u + \frac{1}{2}u + \frac{1}{2}v$$

$$\Rightarrow \boxed{x = \frac{3}{2}u + \frac{1}{2}v}$$

Logo: $T(u,v) = (x,y) = \left(\frac{3}{2}u + \frac{1}{2}v, \frac{1}{2}u + \frac{1}{2}v\right)$



$$\det J(T)(u,v) = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} = \begin{vmatrix} \frac{3}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{vmatrix} = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

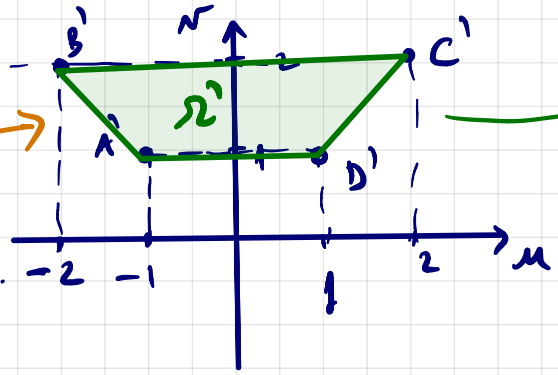
sendo $u = y - x$ e $v = y + x$, então:

$A(\underset{x}{1}, \underset{y}{0}) \rightsquigarrow A'(\underbrace{0-1}_{y-x}, \underbrace{0+1}_{y+x}) \to A'(-1,1)$

$B(2,0) \rightsquigarrow B'(0-2, 0+2) \rightsquigarrow B'(-2,2)$

$C(0,2) \rightsquigarrow C'(2-0, 2+0) \rightsquigarrow C'(2,2)$

$D(0,1) \rightsquigarrow D'(1-0, 1+0) = D'(1,1)$



RETA PASSANDO POR A' E B' É A BISSETRIZ DOS QUADRANTES PARES, OU SEJA, $\boxed{u=-v}$

RETA PASSANDO POR C' E D' É A BISSETRIZ DOS QUADRANTES ÍMPARES, OU SEJA $\boxed{u=v}$.

Assim, pela mudança de variável, teremos:

$$\iint_{\Omega} \cos\left(\frac{y-x}{y+x}\right) dy\,dx = \iint_{\Omega'} \cos\left(\frac{u}{v}\right) \cdot \underbrace{|\det J(T)(u,v)|}_{\left|\frac{1}{2}\right| = \frac{1}{2}} \cdot du\,dv$$

$$= \int_{v=1}^{v=2} \int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \frac{1}{2} \cdot du\,dv = \frac{1}{2} \int_{v=1}^{v=2} v \left(\int_{u=-v}^{u=v} \cos\left(\frac{u}{v}\right) \frac{1}{v}\, du \right) dv$$

$$= \frac{1}{2} \int_{v=1}^{v=2} v \cdot \sin\left(\frac{u}{v}\right)\Big|_{u=-v}^{u=v} \cdot dv = \frac{1}{2} \int_{v=1}^{v=2} v \cdot \left(\sin 1 - \underbrace{\sin(-1)}_{-\sin(1)} \right) dv$$

$$= \frac{1}{\cancel{2}} \int_{r=1}^{r=2} r \cdot (\cancel{2} \cdot \operatorname{sen} 1) \cdot dr = \operatorname{sen} 1 \cdot \left. \frac{r^2}{2} \right|_{r=1}^{r=2}$$

$$= \operatorname{sen} 1 \cdot \left(\frac{4}{2} - \frac{1}{2} \right) = \frac{3}{2} \cdot \operatorname{sen} 1$$