Lista 01

7. Se $u = f(x,y)$ é uma função diferenciável de x e y com $x = \rho\cos\theta$ e $y = \rho\sin\theta$, mostre que

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\cos\theta - \frac{\partial u}{\partial \theta}\frac{\sin\theta}{\rho}$$

e

$$\frac{\partial u}{\partial y} = \frac{\partial u}{\partial \rho}\sin\theta + \frac{\partial u}{\partial \theta}\frac{\cos\theta}{\rho}$$

$$x = \rho\cdot\cos\theta \Rightarrow \begin{cases} \dfrac{\partial x}{\partial \rho} = \cos\theta \\ \dfrac{\partial x}{\partial \theta} = -\rho\,\text{sen}\,\theta \end{cases}$$

$$y = \rho\cdot\text{sen}\,\theta \Rightarrow \begin{cases} \dfrac{\partial y}{\partial \rho} = \text{sen}\,\theta \\ \dfrac{\partial y}{\partial \theta} = \rho\cos\theta \end{cases}$$

$$\frac{\partial u}{\partial \rho} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \rho} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \rho}$$

$$\boxed{\frac{\partial u}{\partial \rho} = \frac{\partial u}{\partial x}\cdot\cos\theta + \frac{\partial u}{\partial y}\cdot\text{sen}\,\theta} \quad (*)$$

Além disso:

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \theta}$$

$$\boxed{\frac{\partial u}{\partial \theta} = -\frac{\partial u}{\partial x}\,\rho\,\text{sen}\,\theta + \frac{\partial u}{\partial y}\,\rho\cos\theta} \quad (**)$$

$$\frac{\partial u}{\partial \theta} + \rho\cdot\text{sen}\,\theta\,\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}\cdot\rho\cdot\cos\theta$$

$$\frac{1}{\rho}\cdot\sec\theta\cdot\frac{\partial u}{\partial \theta} + \frac{\cancel{\rho}}{\cancel{\rho}}\cdot\tan\theta\cdot\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y}$$

leva para (*)

$$\frac{\partial u}{\partial \rho} = \frac{\partial u}{\partial x}\cdot\cos\theta + \left(\frac{1}{\rho}\cdot\sec\theta\cdot\frac{\partial u}{\partial \theta} + \tan\theta\cdot\frac{\partial u}{\partial x}\right)\cdot\text{sen}\,\theta$$

$$\frac{\partial u}{\partial \rho} = \frac{\partial u}{\partial x}\cdot\cos\theta + \frac{1}{\rho}\cdot\tan\theta\cdot\frac{\partial u}{\partial \theta} + \frac{\text{sen}^2\theta}{\cos\theta}\cdot\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial \rho} - \frac{1}{\rho}\tan\theta\cdot\frac{\partial u}{\partial \theta} = \left(\cos\theta + \frac{\text{sen}^2\theta}{\cos\theta}\right)\cdot\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial \rho} - \frac{1}{\rho}\cdot\tan\theta\,\frac{\partial u}{\partial \theta} = \frac{1}{\cos\theta}\cdot\frac{\partial u}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\cdot\cos\theta - \frac{1}{\rho}\cdot\frac{\sin\theta}{\cancel{\cos\theta}}\cdot\cancel{\cos\theta}\,\frac{\partial u}{\partial \theta}$$

$$\boxed{\frac{\partial u}{\partial x} = \frac{\partial u}{\partial \rho}\cdot\cos\theta - \frac{\sin\theta}{\rho}\cdot\frac{\partial u}{\partial \theta}}$$

De forma encontra-se a outra relação.

LISTA 05

8. Seja a função vetorial $f : \mathbb{R}^2 \to \mathbb{R}^2$ definida por

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x^2 - y^2 \\ 2xy \end{pmatrix}.$$

Quais são as funções coordenadas de f? Considere o espaço domínio sendo o plano xy e o espaço imagem como sendo o plano uv. Assim:

(a) Determine a imagem do segmento da reta $y = x$ entre $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ e $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(b) Determine a imagem da região definida por $0 < x$, $0 < y$ e $x^2 + y^2 < 1$.

(c) Determine o ângulo entre as imagens das retas $y = 0$ e $y = \frac{1}{\sqrt{3}}x$. (Resp.: $\frac{\pi}{3}$) .

$f : \mathbb{R}^2 \to \mathbb{R}^2$,

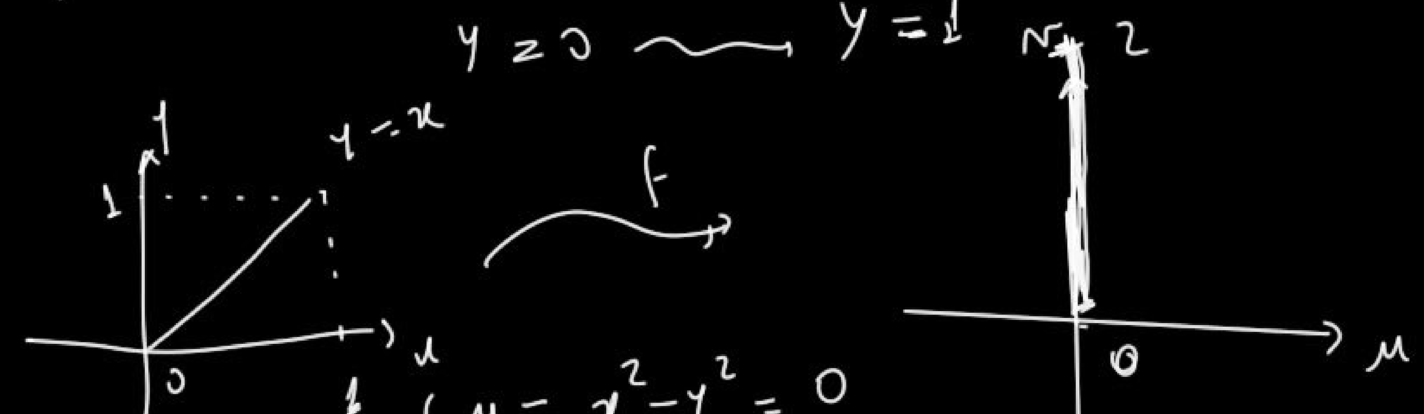
$f(x,y) = (x^2 - y^2, 2xy)$

as funções coordenadas: $f_1(x,y) = x^2 - y^2$

$f_2(x,y) = 2xy$

(a) $y = x$ $\quad x = 0 \rightsquigarrow x = 1$

$y = 0 \rightsquigarrow y = 1$



$$\begin{cases} u = x^2 - y^2 = 0 \\ v = 2xy = 2x^2 \end{cases}$$

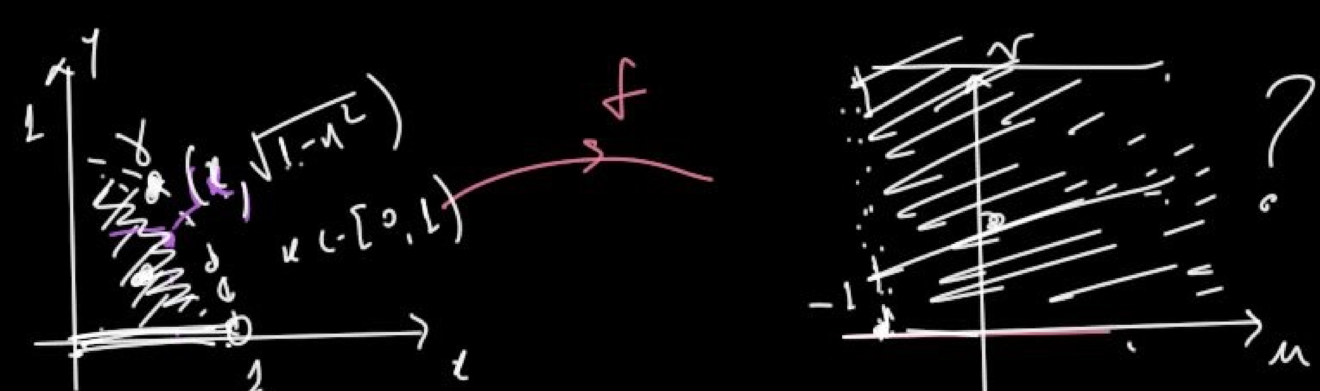
$x = y$

$(u,v) = (0, \underbrace{2x^2}_{\geq 0})$; $x \in [0,1]$

$[0,2]$

Ou seja, a reta $y = x$ no plano xy se transforma mediante f, no intervalo $[0,1]$, no segmento vertical $(0, 2x)$; $x \in [0,1]$, i.e.; no segmento $[0,2]$ vertical.

(b)



$(x,y) \longmapsto (x^2 - y^2, 2xy) \overset{\gamma}{\rightsquigarrow} (x^2 - (1 - x^2), 2x\sqrt{1 - x^2})$

$(-1, \underbrace{2x\sqrt{1 - x^2}}_{\geq 0})$

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