RESOLUÇÃO DE ALGUMAS QUESTÕES DA LISTA 05.

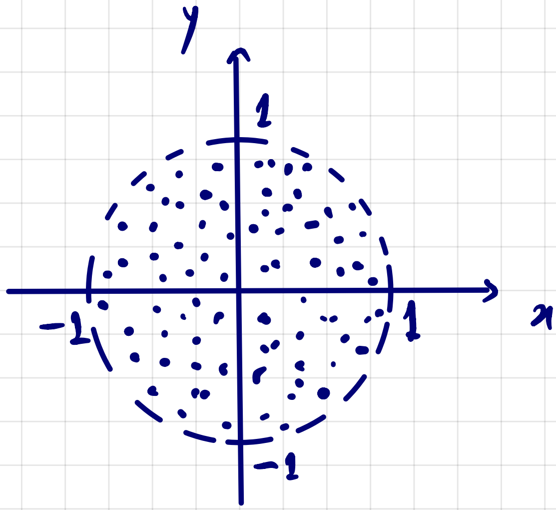
01) (d) $f(x,y) = \dfrac{1}{\sqrt{1-x^2-y^2}}$

condição de existência: $1-x^2-y^2>0$

$\iff x^2+y^2<1.$

$D(f) = \{(x,y) \in \mathbb{R}^2 : x^2+y^2<1\}$

gráfico do domínio:

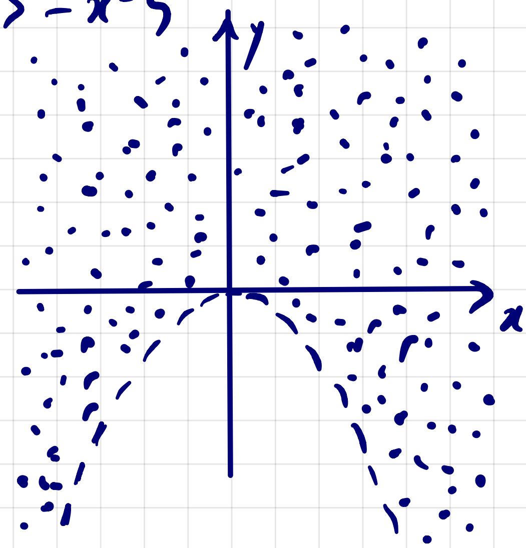


(e) $f(x,y) = \ln(x^2+y)$

condição de existência: $x^2+y>0 \iff y > -x^2$

$D(f) = \{(x,y) \in \mathbb{R}^2 : y > -x^2\}$

gráfico do domínio:



(f) $f(x,y) = \arcsen(x+y)$.

condição de existência: $-\frac{\pi}{2} \leq x+y \leq \frac{\pi}{2}$.

$$\Updownarrow$$

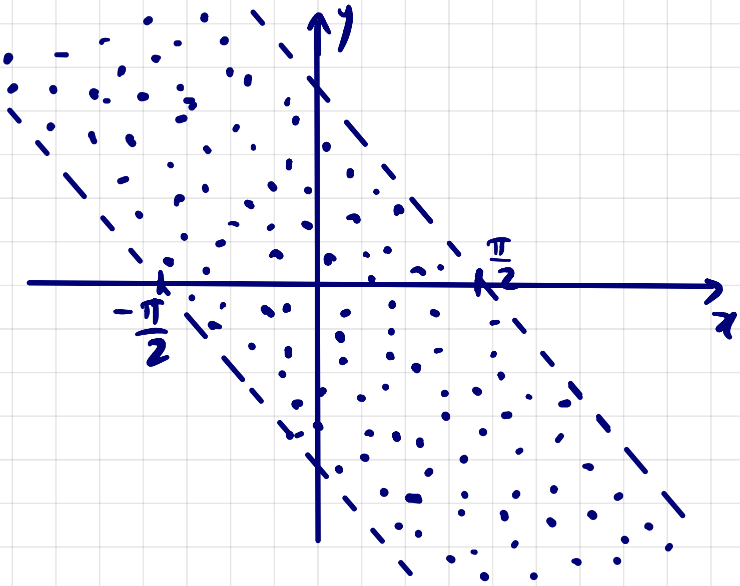
$$-x-\frac{\pi}{2} \leq y \leq -x+\frac{\pi}{2}$$

LEMBRE-SE DA TRIGONOMETRIA:

$x = \arcsen y \iff -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

$D(f) = \{(x,y) \in \mathbb{R}^2 : -\frac{\pi}{2} \leq x+y \leq \frac{\pi}{2}\}$

gráfico do domínio:



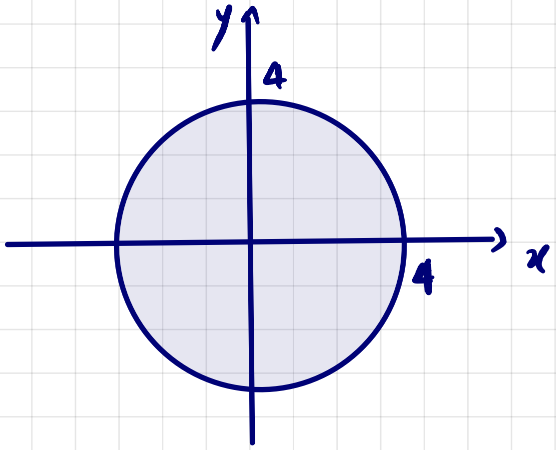
02) (a) $f(x,y) = \sqrt{16-x^2-y^2}$

condição de existência: $16-x^2-y^2 \geq 0$

$$\iff x^2+y^2 \leq 16$$

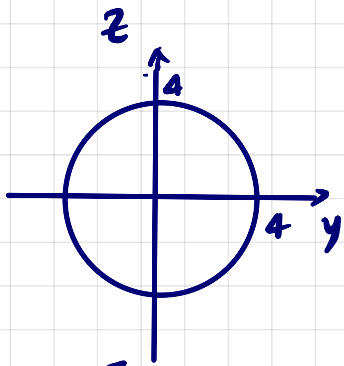
$D(f) = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \leq 16\}$

gráfico do domínio:

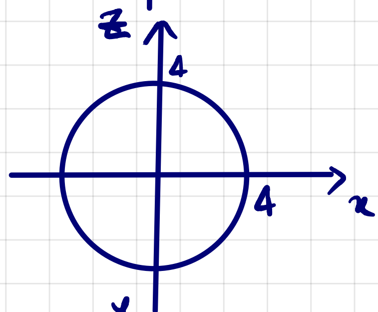


traços: $z=f(x,y)=\sqrt{16-x^2-y^2}$

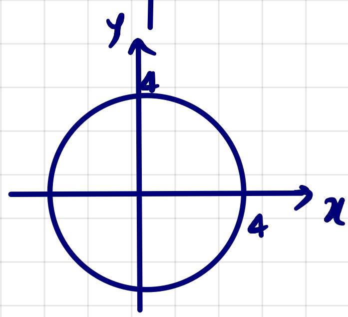
$\Leftarrow \quad x^2+y^2+z^2=16$



- $x=0$: (plano yz): $y^2+z^2=16$



- $y=0$: (plano xz): $x^2+z^2=16$

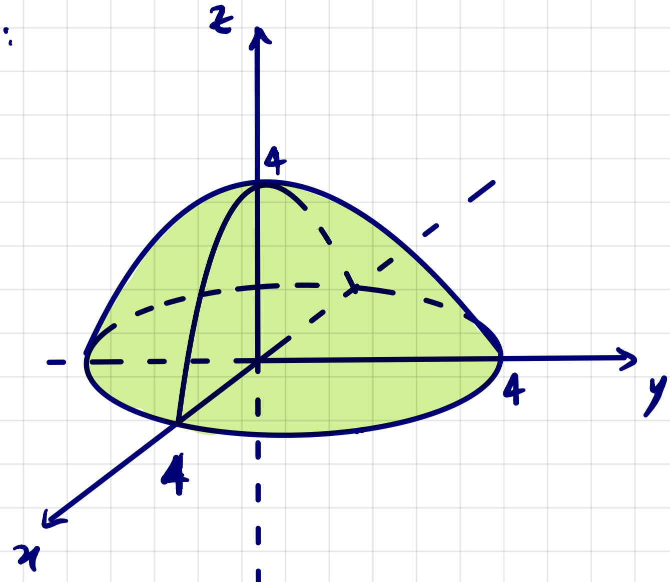


- $z=0$: (plano xy): $x^2+y^2=16$

Esboço gráfico de f:

$f(x,y)=\sqrt{16-x^2-y^2}$

$Im(f)=[0,4]$



07) (b) $f\begin{pmatrix} u \\ v \end{pmatrix} = \begin{pmatrix} \cos u \cdot \operatorname{sen} v \\ \operatorname{sen} u \operatorname{sen} v \\ \cos v \end{pmatrix}$;

$0 \leq u \leq 2\pi$; $0 \leq v \leq \frac{\pi}{2}$.

Note que $f: \mathbb{R}^2 \to \mathbb{R}^3$

$(u, v) \longmapsto (x, y, z)$; onde

$$x = \cos u \operatorname{sen} v$$
$$y = \operatorname{sen} u \cdot \operatorname{sen} v$$
$$z = \cos v$$

Então:

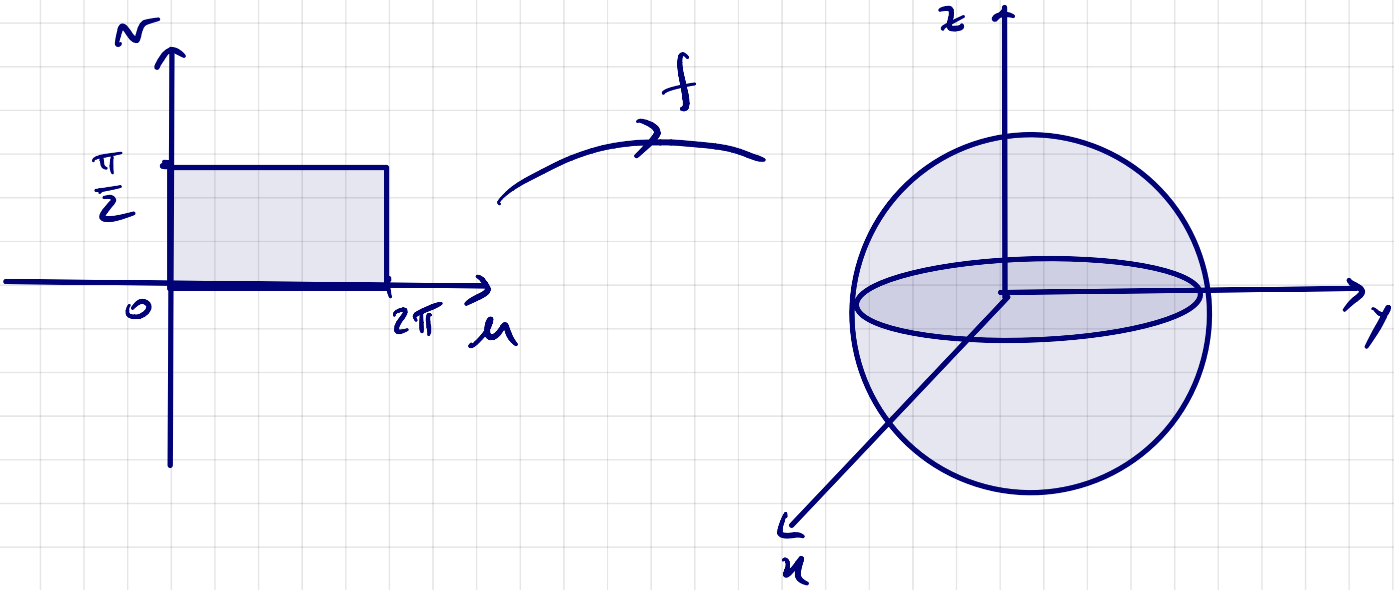
$$x^2 + y^2 + z^2 = \cos^2 u \operatorname{sen}^2 v + \operatorname{sen}^2 u \cdot \operatorname{sen}^2 v + \cos^2 v =$$

$$= \operatorname{sen}^2 v \cdot (\underbrace{\cos^2 u + \operatorname{sen}^2 u}_{=1}) + \cos^2 v$$

$$= \operatorname{sen}^2 v + \cos^2 v = 1.$$

$$\Rightarrow x^2 + y^2 + z^2 = 1.$$

Ou seja, $f: \mathbb{R}^2 \to \mathbb{R}^3$ transforma a região retangular do plano uv na superfície esférica do $\mathbb{R}^3$, c.f. esquema abaixo:



11) $d(x,y) = x^2 - 2y^2 - x + 1$;

$-1 \leq x \leq 1$; $-1 \leq y \leq 1$

$d(x,y) = \frac{7}{4} \iff x^2 - 2y^2 - x + 1 = \frac{7}{4}$

$\iff 4x^2 - 8y^2 - 4x + 4 = 7$

$\iff 4x^2 - 4x \quad - 8y^2 = 3$

$\iff x^2 - x \quad - 2y^2 = \frac{3}{4}$

$\iff x^2 - 2 \cdot \frac{1}{2} x + \frac{1}{4} - \frac{1}{4} \quad - 2y^2 = \frac{3}{4}$

$\iff \left(x - \frac{1}{2}\right)^2 - 2y^2 = \frac{4}{4}$

$\iff \left(x - \frac{1}{2}\right)^2 - 2y^2 = 1.$

A ideia consiste em completar um quadrado perfeito para, com isso, encontrar alguma equação conhecida.

$$\Leftrightarrow \quad \frac{\left(x-\frac{1}{2}\right)^2}{1^2} - \frac{y^2}{\left(\frac{1}{\sqrt{2}}\right)^2} = 1 \qquad \text{(elipse)}$$