Na aula passada iniciamos o estudo de funções

$$f : \mathbb{R}^m \to \mathbb{R}^n$$

$$x = (x_1, \dots, x_m) \longmapsto f(x) = (f_1(x), \dots, f_n(x))$$

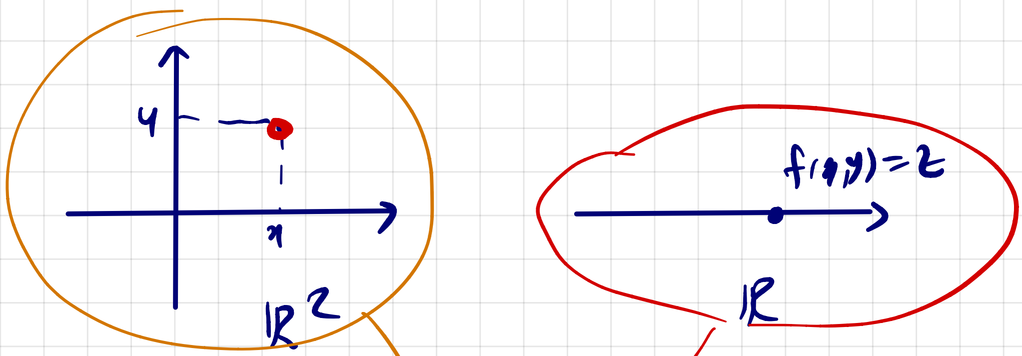
$x = (x_1, \dots, x_m)$

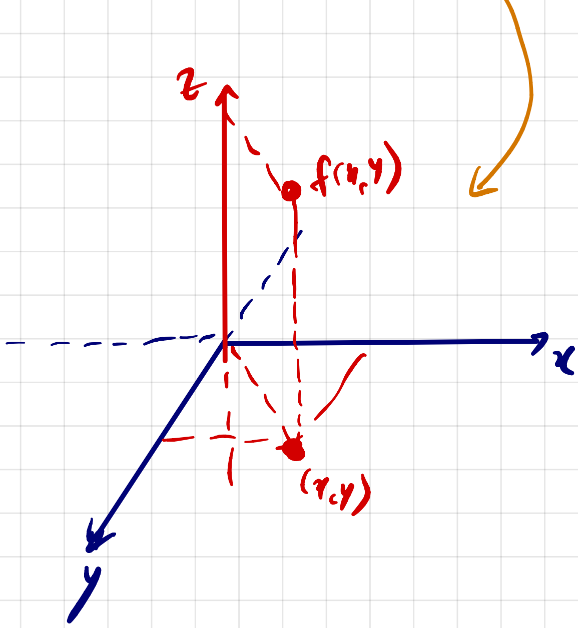
Vimos vários exemplos. Vejamos mais alguns.

05) $f : \mathbb{R}^2 \to \mathbb{R}$ $\qquad f(x,y) = \operatorname{sen} x$

Neste caso, temos $\quad z = f(x,y) = \operatorname{sen} x$

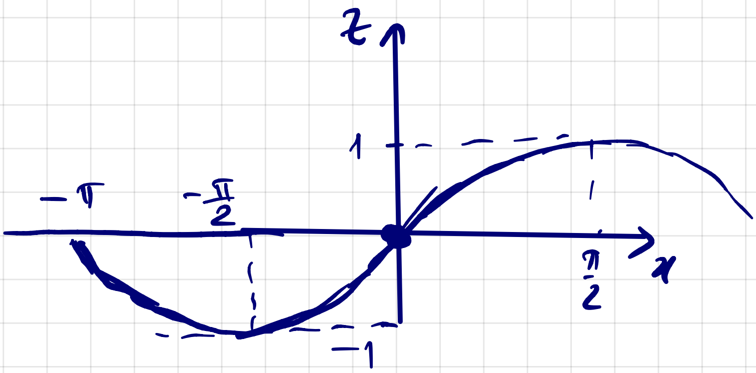
Em $\mathbb{R}^3$ marcamos os pontos $(x, y, f(x,y) = z)$

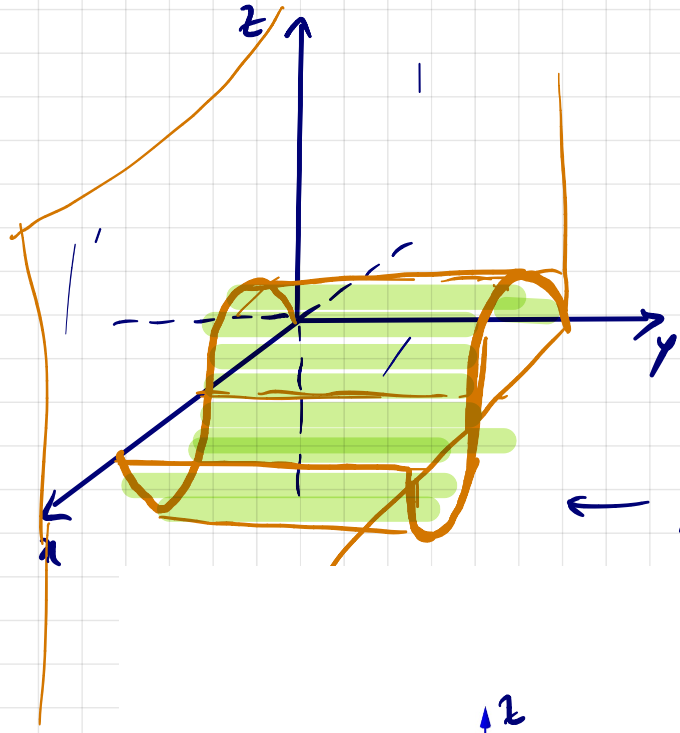




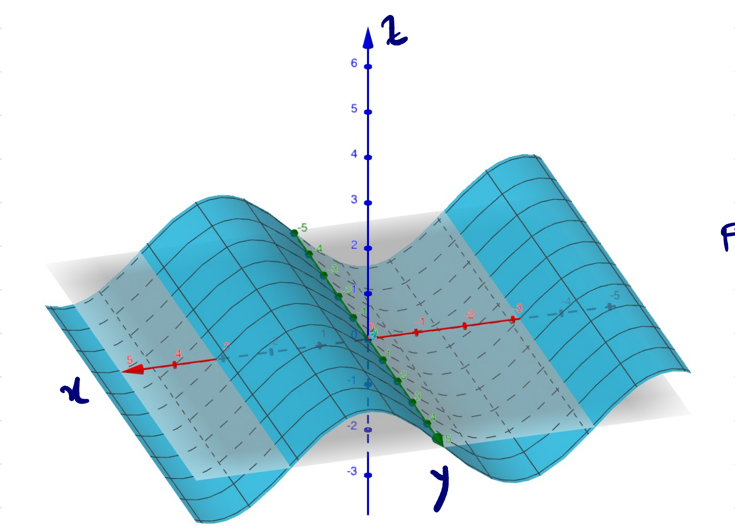
No nosso exemplo; temos

$z = \operatorname{sen} x$





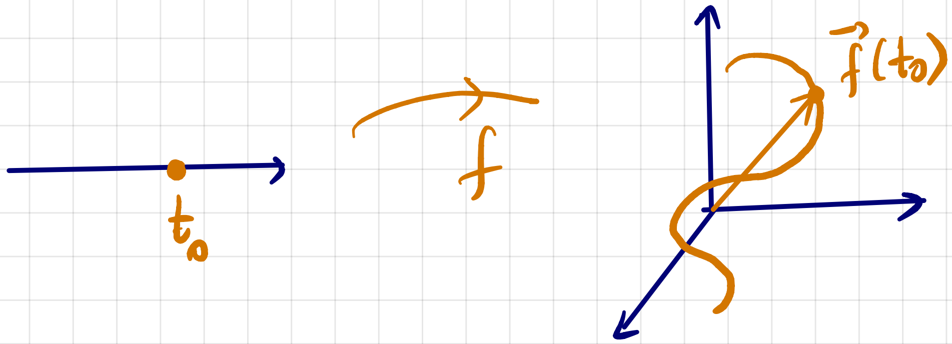
$f(x,y) = \operatorname{sen} x$



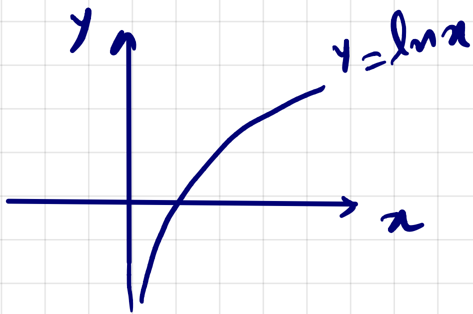
FEITO NO GEOGEBRA.

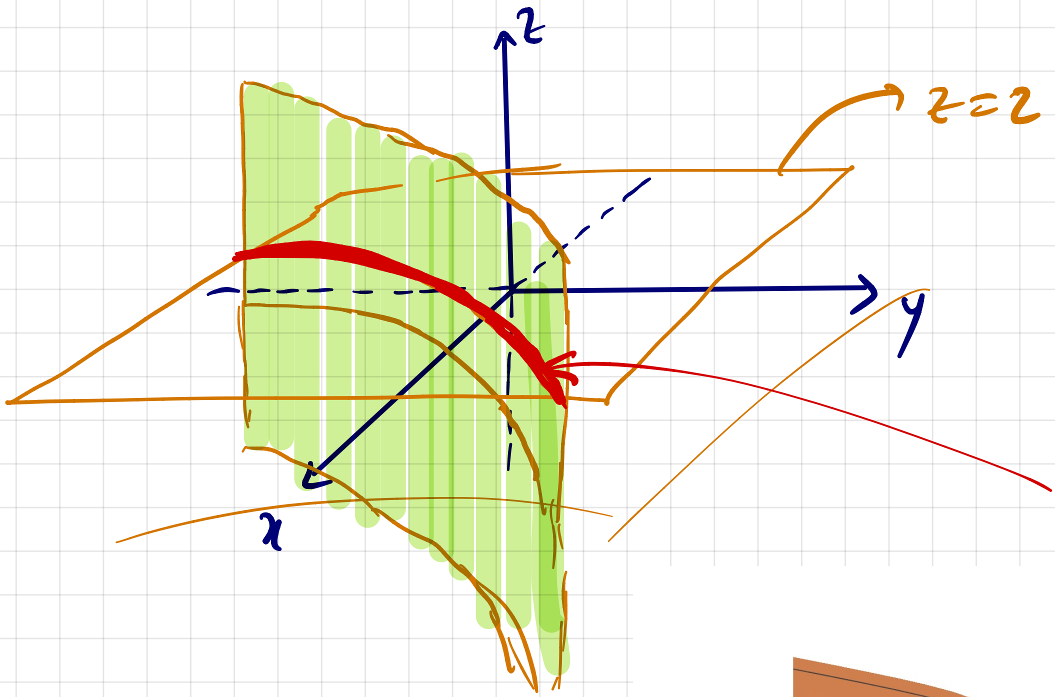
06) $f: \mathbb{R} \to \mathbb{R}^3$

$$\vec{f}(t) = (t, \ln t, 2)$$



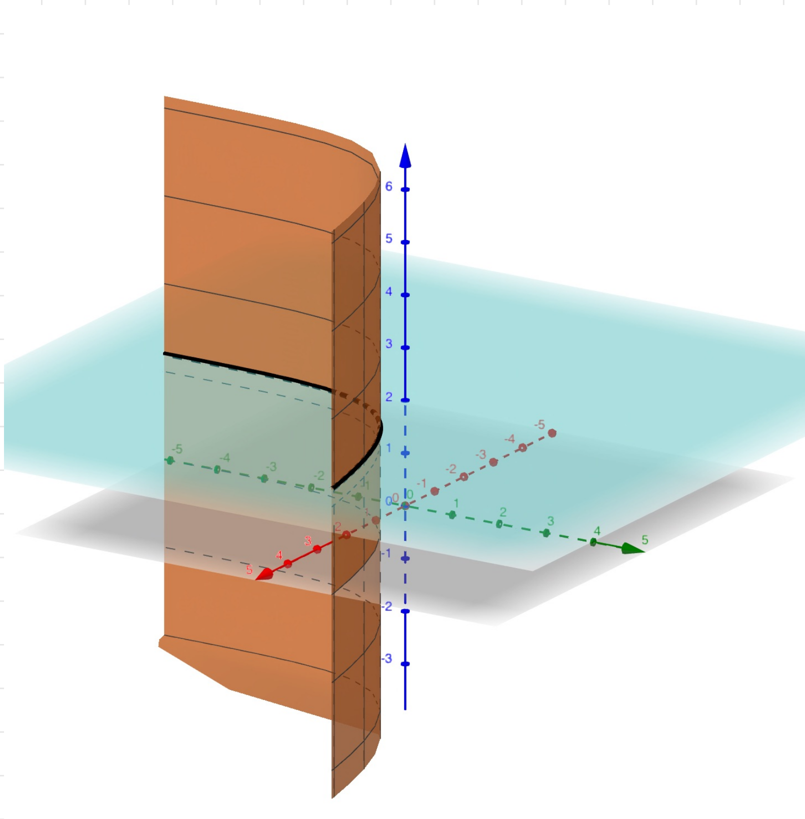
$$\left\{\begin{array}{l} x = t \\ y = \ln t \\ \boxed{z = 2} \end{array}\right. \quad y = \ln x$$





$f(t) = (t, \ln t, 2)$

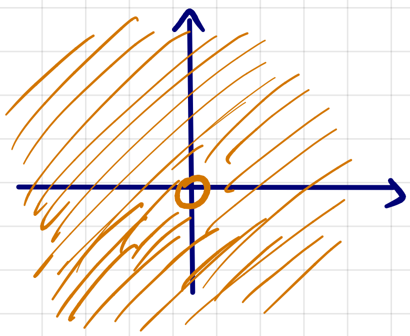
GRÁFICO FEITO NO GEOGEBRA

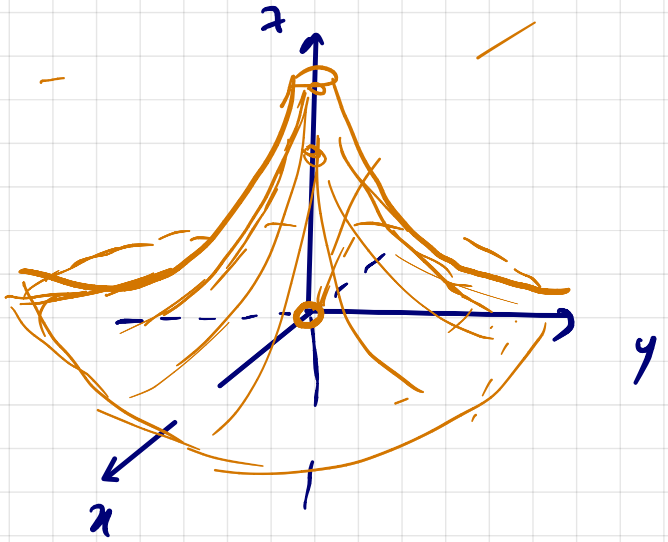


07) $f: \mathbb{R}^2 \to \mathbb{R}$, $\quad f(x,y) = \dfrac{1}{x^2+y^2}$

$D(f) = \{(x,y) \in \mathbb{R}^2 : x^2+y^2 \neq 0\} = \mathbb{R}^2 \setminus \{(0,0)\}$

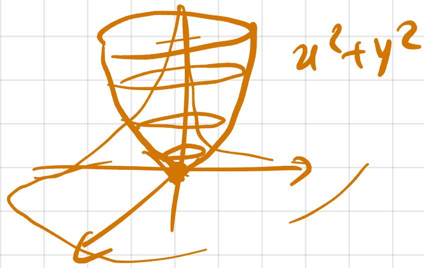
gráfico do domínio:



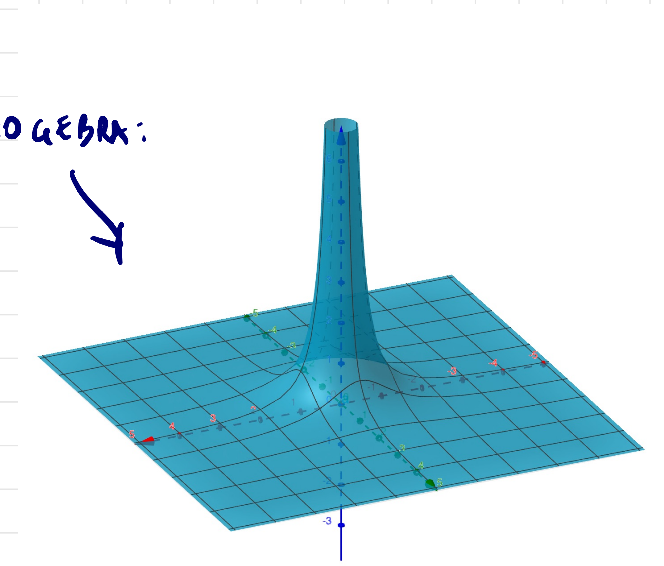


$x^2+y^2 \neq 0$

quanto maior x^2+y^2, menor será $\frac{1}{x^2+y^2}$



PELO GEOGEBRA:



LIMITES DE FUNÇÕES:

Def.: Seja $f: A \to \mathbb{R}^n$, onde $A \subset \mathbb{R}^m$ e $a \in \mathbb{R}^m$ um ponto de acumulação do conj. A [i.e., $\forall x \in A$, $\exists \delta > 0$ tal que $x \in B(a,\delta)\setminus\{a\}$; ou seja, $(B(a,\delta)\setminus\{a\}) \cap A \neq \emptyset$], notando que $\mathbb{R}^m$ está equipado com uma métrica d e $\mathbb{R}^n$ equipado com uma métrica $\tilde{d}$.

Dizemos que $\ell \in \mathbb{R}^n$ é o limite de $f(x)$ quando x tende para $a \in \mathbb{R}^m$, e escrevemos

$$\lim_{x \to a} f(x) = \ell$$

$f(x) \in B_\varepsilon(\ell)$

se, e somente se,

$$\forall \varepsilon > 0, \exists \delta > 0 \text{ tal que, } \forall x \in A : \underbrace{0 < d(x,a) < \delta}_{x \in B(a,\delta) \setminus \{a\}} \Rightarrow \tilde{d}(f(x), \ell) < \varepsilon.$$

<u>Vejamos alguns exemplos:</u>

01) $f : A \subset \mathbb{R} \to \mathbb{R}$; neste caso temos que a métrica é a mesma no domínio e no contradomínio, a métrica induzida pelo módulo; i.e.: $d(x,y) = |x-y|$.

Assim, nossa def. de limite fica:

$$\lim_{x \to a} f(x) = \ell \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0 : \forall x : 0 < d(x,a) < \delta \Rightarrow d(f(x), \ell) < \varepsilon$$

$$\Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0 : \forall x : 0 < |x-a| < \delta \Rightarrow |f(x) - \ell| < \varepsilon.$$

02) $f : A \subset (\mathbb{R}^2, d_2) \to (\mathbb{R}^3, d_1)$

$$\lim_{x \to a} f(x) = \ell \Leftrightarrow \forall \varepsilon > 0, \exists \delta > 0 \text{ tal que,}$$

$$\forall x \in \underbrace{B_\delta(a) \setminus \{a\}} \Rightarrow \underbrace{f(x) \in B_\varepsilon(\ell)}.$$

ou seja: $0 < d_2(x,a) < \delta$ $\quad d_1(f(x), \ell) < \varepsilon$

i.e.:

$$\lim_{x\to a} f(x) = l \iff \forall \varepsilon > 0, \exists \delta > 0 \text{ tal que},$$

$$\forall x: 0 < \sqrt{(x_1 - a_1)^2 + (x_2 - a_2)^2} < \delta$$

$$\implies |f_1(x) - l_1| + |f_2(x) - l_2| + |f_3(x) - l_3| < \varepsilon,$$

onde estamos pondo $x = (x_1, x_2)$;

$a = (a_1, a_2)$;

$f(x) = (f_1(x), f_2(x), f_3(x))$;

$l = (l_1, l_2, l_3)$.

03) $f: A \subset \mathbb{R} \to \mathbb{R}^3$, $(\mathbb{R}, |\cdot|)$; $(\mathbb{R}^3, d_2)$

$$f(t) = (f_1(t), f_2(t), f_3(t))$$

$a \in A$ ponto de acumulação de A.

$\lim_{x\to a} f(x) = l \in \mathbb{R}^3$, $l = (l_1, l_2, l_3)$.

$$\Updownarrow$$

$\forall \varepsilon > 0, \exists \delta > 0 : \forall x \in B_\delta(a) \setminus \{a\} \Rightarrow f(x) \in B_\varepsilon(l)$, i.e.,

$$\Updownarrow$$

$\forall \varepsilon > 0, \exists \delta > 0 : \forall x : 0 < |x - a| < \delta \Rightarrow$

$$\Rightarrow \sqrt{(f_1(t) - l_1)^2 + (f_2(t) - l_2)^2 + (f_3(t) - l_3)^2} < \varepsilon .$$

---

04) $f : \mathbb{R}^2 \to \mathbb{R}$; $(\mathbb{R}^2, d_2)$; $(\mathbb{R}, |\cdot|)$

$$\lim_{(x,y) \to (a,b)} f(x,y) = l \in \mathbb{R} \iff$$

$\forall \varepsilon > 0, \exists \delta > 0 : \forall (x,y) \in B_\delta(a,b) \setminus \{(a,b)\} \Rightarrow$

$$\Rightarrow f(x,y) \in B_\varepsilon(l) .$$

$$\Updownarrow$$

$\forall \varepsilon > 0, \exists \delta > 0 : \forall (x,y) : 0 < \sqrt{(x-a)^2 + (y-b)^2} < \delta \Rightarrow |f(x,y) - l| < \varepsilon .$