

01) $\int \frac{(x^2+x)}{(x-1)(x^2+1)} dx = ?$

$$\frac{x^2+x}{(x-1)(x^2+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1}$$

$$\frac{x^2+x}{(x-1)(x^2+1)} = \frac{A(x^2+1) + (Bx+C)(x-1)}{(x-1)(x^2+1)}$$

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$$x^2+x = Ax^2 + A + Bx^2 - Bx + Cx - C$$

$$\left. \begin{array}{l} A+B=1 \\ -B+C=1 \\ A-C=0 \end{array} \right\} \quad \left. \begin{array}{l} A=C \\ C=1 \end{array} \right\} \quad \left. \begin{array}{l} A+B=1 \\ B=1-A \\ B=1-1 \\ \Rightarrow B=0 \end{array} \right\}$$

$$2A=2 \Rightarrow A=1$$

Assum, temos:

$$\frac{x^2+x}{(x-1)(x^2+1)} = \frac{1}{x-1} + \frac{0 \cdot x + 1}{x^2+1}$$

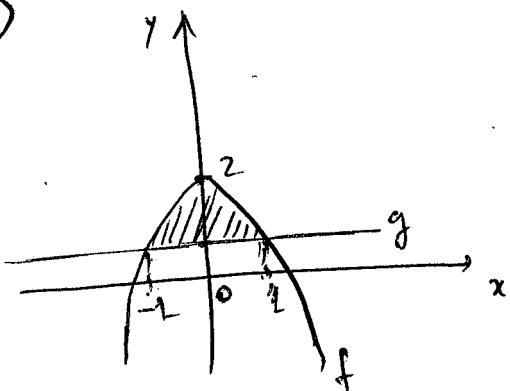
$$\Rightarrow \int \frac{x^2+x}{(x-1)(x^2+1)} dx = \int \left(\frac{dx}{x-1} \right) + \int \frac{1}{x^2+1} dx$$

$$= \ln|x-1| + \frac{1}{1} \arctan\left(\frac{x}{1}\right) + C$$

$$= \ln|x-1| + \arctan(x) + C$$

01

02)



$$f(x) = 2 - x^2$$

$$g(x) = 1.$$

x-intercept(s):

$$f(x) = g(x) \Leftrightarrow 2 - x^2 = 1$$

$$\Leftrightarrow x^2 = 1 \Leftrightarrow x = \pm 1$$

$$(a) A = \int_{-1}^1 (f(x) - g(x)) dx = \int_{-1}^1 (2 - x^2 - 1) dx$$

$$= \int_{-1}^1 (1 - x^2) dx = \left(x - \frac{x^3}{3} \right) \Big|_{-1}^1 =$$

$$= 1 - \frac{1}{3} - \left(-1 - \frac{(-1)^3}{3} \right) = 1 - \frac{1}{3} + 1 - \frac{1}{3}$$

$$= 2 - \frac{2}{3} = \frac{6-2}{3} = \frac{4}{3} \text{ u.a.}$$

$$(b) V = \pi \int_{-1}^1 ([f(x)]^2 - [g(x)]^2) dx$$

$$= \pi \int_{-1}^1 [(2 - x^2)^2 - (1)^2] dx = \pi \int_{-1}^1 (4 - 4x^2 + x^4 - 1) dx$$

$$= \pi \int_{-1}^1 (3 - 4x^2 + x^4) dx = \pi \cdot \left(3x - \frac{4x^3}{3} + \frac{x^5}{5} \right) \Big|_{-1}^1 =$$

$$= \pi \cdot \left[\left(3 - \frac{4}{3} + \frac{1}{5} \right) - \left(-3 + \frac{4}{3} - \frac{1}{5} \right) \right] =$$

02

$$= \pi \left[3 - \frac{4}{3} + \frac{1}{5} + 3 - \frac{4}{3} + \frac{1}{5} \right]$$

$$= \pi \left[6 - \frac{8}{3} + \frac{2}{5} \right] = \pi \cdot \left(\frac{90 - 40 + 6}{15} \right) = \frac{56\pi}{15} \text{ u.r.}$$

o3) $f(x) = \frac{1}{x^2}$ tem uma singularidade em $x=0$.

Assim; $\forall \varepsilon > 0$, tem-se:

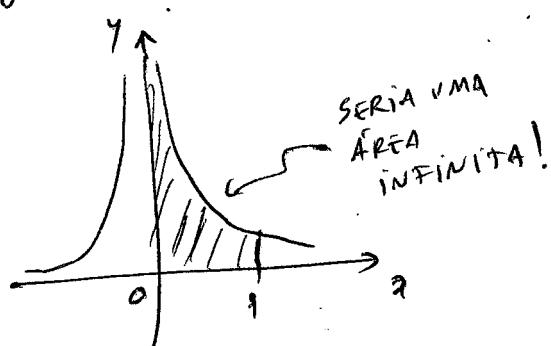
$$\int_0^1 \frac{dx}{x^2} = \lim_{\varepsilon \rightarrow 0^+} \int_{0+\varepsilon}^1 \frac{dx}{x^2} =$$

$$= \lim_{\varepsilon \rightarrow 0^+} \int_{\varepsilon}^1 x^{-2} dx = \lim_{\varepsilon \rightarrow 0^+} \left. \frac{x^{-1}}{-1} \right|_{\varepsilon}^1 = \lim_{\varepsilon \rightarrow 0^+} -\frac{1}{x} \Big|_{\varepsilon}^1$$

$$= \lim_{\varepsilon \rightarrow 0^+} \left(-1 + \frac{1}{\varepsilon} \right) = +\infty.$$

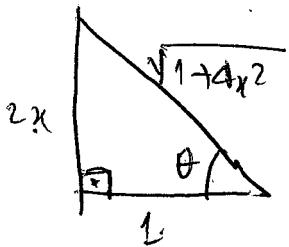
(20)

Logo, a integral é de e divergente.



(3)

$$04) \quad (a) \quad \int f(x) dx = \int \frac{dx}{(1+4x^2)^2} = ?$$



$$\begin{aligned}\tan \theta &= 2x \\ \Rightarrow x &= \frac{1}{2} \tan \theta \\ \Rightarrow dx &= \frac{1}{2} \sec^2 \theta d\theta\end{aligned}$$

$$\cos \theta = \frac{1}{\sqrt{1+4x^2}}$$

$$\Rightarrow \sqrt{1+4x^2} = \sec \theta \Rightarrow 1+4x^2 = \sec^2 \theta$$

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Asumim:

$$\int \frac{dx}{(1+4x^2)^2} = \int \frac{\frac{1}{2} \cdot \sec^2 \theta d\theta}{(\sec^2 \theta)^2} = \frac{1}{2} \int \frac{d\theta}{\sec^2 \theta}$$

$$= \frac{1}{2} \int \cos^2 \theta d\theta = \frac{1}{2} \int \frac{1+\cos 2\theta}{2} d\theta = \frac{1}{4} \int (1+\cos 2\theta) d\theta =$$

$$= \frac{1}{4} \int d\theta + \frac{1}{4} \int \cos 2\theta d\theta = \frac{\theta}{4} + \frac{1}{4} \cdot \frac{1}{2} \int \cos 2\theta \cdot (2d\theta)$$

$$= \frac{\theta}{4} + \frac{1}{8} \sin 2\theta + C \quad \stackrel{\uparrow}{=} \frac{\theta}{4} + \frac{1}{8} \cdot 2 \cdot \sin \theta \cdot \cos \theta + C =$$

$\sin \theta = \frac{2x}{\sqrt{1+4x^2}}$

 $\cos \theta = \frac{1}{\sqrt{1+4x^2}}$

$\tan \theta = 2x$

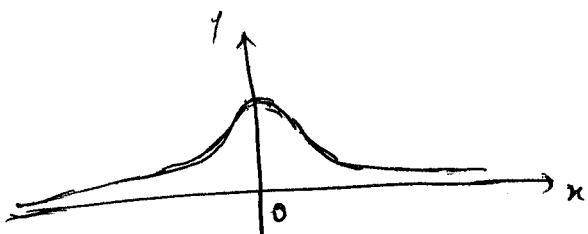
 $\Rightarrow \theta = \arctan(2x)$

04

$$= \frac{1}{4} \cdot \arctan(2x) + \frac{1}{4} \cdot \frac{2x}{\sqrt{1+4x^2}} \cdot \frac{1}{\sqrt{1+4x^2}} + C$$

$$= \frac{1}{4} \arctan(2x) + \frac{x}{2(1+4x^2)} + C$$

(b)



$$A = \int_{-\infty}^{+\infty} f(x) dx = \int_{-\infty}^0 f(x) dx + \int_0^{+\infty} f(x) dx$$

$$= \lim_{A \rightarrow -\infty} \int_A^0 f(x) dx + \lim_{B \rightarrow +\infty} \int_0^B f(x) dx$$

2,0

$$= \lim_{A \rightarrow -\infty} \left(\frac{1}{4} \arctan(2A) + \frac{A}{2(1+A^2)} \right) \Big|_A^0 + \lim_{B \rightarrow +\infty} \left(\frac{1}{4} \arctan(2B) + \frac{B}{2(1+B^2)} \right) \Big|_0^B$$

$$= \lim_{A \rightarrow -\infty} \left(\frac{1}{4} \arctan(0) + 0 - \left[\frac{1}{4} \arctan(2A) + \frac{A}{2(1+4A^2)} \right] \right) +$$

$$+ \lim_{B \rightarrow +\infty} \left(\frac{1}{4} \arctan(2B) + \frac{B}{2(1+4B^2)} - \left[\frac{1}{4} \arctan(0) + 0 \right] \right)$$

$$= -\frac{1}{4} \arctan(-\infty) + 0 + \frac{1}{4} \arctan(+\infty) + 0$$

$$= -\frac{1}{4} \cdot \left(-\frac{\pi}{2} \right) + \frac{1}{4} \cdot \left(\frac{\pi}{2} \right) = \frac{\pi}{8} + \frac{\pi}{8} = \underbrace{\frac{\pi}{4}}_{u-a}$$

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05) (a) $\sum_{m=1}^{\infty} \frac{m^2}{2^m}$. Teorema de Raabe:

$$\lim_{m \rightarrow \infty} \frac{a_{m+1}}{a_m} = \lim_{m \rightarrow \infty} \frac{\frac{(m+1)^2}{2^{m+1}}}{\frac{m^2}{2^m}} = \lim_{m \rightarrow \infty} \frac{(m+1)^2}{2^{m+1}} \cdot \frac{2^m}{m^2}$$

05) $= \lim_{m \rightarrow \infty} \left(\frac{m+1}{m}\right)^2 \cdot \frac{1}{2} = \lim_{m \rightarrow \infty} \underbrace{\left(1 + \frac{1}{m}\right)}_1 \cdot \underbrace{\frac{1}{2}}_{\frac{1}{2}} = \frac{1}{2} < 1.$

Logo, a série converge.

(b) $\sum_{m=1}^{\infty} \frac{3^m - 1}{2^m} = \sum_{m=1}^{\infty} \left(\frac{3^m}{2^m} - \frac{1}{2^m} \right) =$

05) $= \underbrace{\sum_{m=1}^{\infty} \left(\frac{3}{2}\right)^m}_{\substack{\text{série} \\ \text{divergente}}} - \underbrace{\sum_{m=1}^{\infty} \left(\frac{1}{2}\right)^m}_{\substack{\text{série} \\ \text{convergente} \\ (\text{séries geométricas})}} \rightarrow$ a diferença entre séries convergente e divergente resulta em uma série divergente.

(c) $\sum_{m=1}^{\infty} \frac{\ln m}{m}$

05) $\forall m \geq 1$, tem-se: $\ln m < m$, e então

$\frac{1}{m} < \frac{1}{\ln m}$, e como $\sum \frac{1}{m}$ diverge, segue pelo T. de comparação que $\sum \frac{\ln m}{m}$ diverge. 06

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Questão 01. Calcule a integral indefinida

$$\int \frac{(x^2 + x)}{(x - 1)(x^2 + 1)} dx.$$

Questão 02. Seja R a região limitada pelas curvas $f(x) = 2 - x^2$ e $g(x) = 1$.

- (a) Obtenha a área dessa região.
- (b) Obtenha o volume do sólido S obtido ao girar a região R em torno do eixo x .

Questão 03. Calcule $\int_0^1 \frac{dx}{x^2}$, concludendo se esta integral é convergente ou divergente.

Questão 04. Considere $f : \mathbb{R} \rightarrow \mathbb{R}$ a função definida por

$$f(x) = \frac{1}{(1 + 4x^2)^2}$$

- (a) Mostre por integração que $\int f(x)dx = \frac{1}{4} \arctan(2x) + \frac{x}{2(1 + 4x^2)} + c$.
- (b) Mostre que a área compreendida pela curva descrita por f e sua assíntota horizontal $y = 0$ é $\frac{\pi}{4}$ u.a.

Questão 05. Verifique se as séries abaixo são convergentes ou divergentes, justificando.

(a) $\sum_{n=1}^{+\infty} \frac{n^2}{2^n}$

(b) $\sum_{n=1}^{+\infty} \frac{3^n - 1}{2^n}$

(c) $\sum_{n=1}^{+\infty} \frac{\ln n}{n}$