

Universidade Federal de Pelotas
Disciplina de Cálculo II - Turmas T3, T6 e T7
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Lista 07 de Exercícios - Decomposição em frações parciais

1. Calcule cada integral indefinida a seguir, comprovando a resposta indicada ao lado.

- (a) $\int \frac{(4x-2)dx}{x^3-x^2-2x} = \ln \frac{|x^2-2x|}{(x+1)^2} + c$
- (b) $\int \frac{(x+1)dx}{x(x-2)(x+3)} = -\frac{2}{15} \ln|x+3| - \frac{1}{6} \ln|x| + \frac{3}{10} \ln|x-2| + c$
- (c) $\int \frac{4dx}{x^3-x^2-2x} = \frac{4}{3} \ln|x+1| - \ln|x^2| + \frac{2}{3} \ln|x-2| + c$
- (d) $\int \frac{(x^4+2x+1)dx}{x^3-x^2-2x} = \frac{x^2}{2} + x - \frac{1}{2} \ln|x| + \frac{21}{6} \ln|x-2| + c$
- (e) $\int \frac{(x^2-3x+4)dx}{(x-1)(x+2)(x+3)} = \frac{11}{2} \ln|x+3| - \frac{14}{3} \ln|x+2| + \frac{1}{6} \ln|x+1| + c$
- (f) $\int \frac{dx}{x^3-x} = \frac{1}{2} \ln|x-1| + \frac{1}{2} \ln|x+1| - \ln|x| + c$
- (g) $\int \frac{(2x-1)dx}{x^3-2x^2-5x+6} = -\frac{1}{6} \ln|x-1| + \frac{1}{2} \ln|x-3| - \frac{1}{3} \ln|x+2| + c$
- (h) $\int \frac{5x^2-3}{x^3-x} dx = 2 \ln|2x+3| + \ln|x| - \frac{1}{2} \ln|2x+1| + c$
- (i) $\int \frac{(4x+3)dx}{4x^3+8x^2+3x} = -\frac{1}{2} \ln|2x+3| + \ln|x| - \frac{1}{2} \ln|2x+1| + c$
- (j) $\int \frac{(4x^2+6x)dx}{(x-1)^2(x+1)} = -\frac{5}{x-1} + \frac{9}{2} \ln|x-1| - \frac{1}{2} \ln|x+1| + c$
- (k) $\int \frac{(x^2+x)dx}{(x-1)(x^2+1)} = \ln|x-1| + \arctan(x) + c$
- (l) $\int \frac{x^2 dx}{(x^2-1)^3} = \frac{1}{16} \left[\frac{1}{x-1} - \ln|x-1| + \ln|x+1| + \frac{1}{(x+1)^2} - \frac{1}{(x+1)} - \frac{1}{(x-1)^2} \right] + c$
- (m) $\int \frac{(4x^2+2x+8)dx}{x(x^2+2)^2} = -\ln(x^2+2) + \frac{x}{2(x^2+2)} + \frac{\sqrt{2}}{4} \arctan\left(\frac{x\sqrt{2}}{4}\right) + 2 \ln|x| + c$
- (n) $\int \frac{dx}{x^3+x^2+x} = -\frac{1}{2} \ln|x^2+x+1| - \frac{\sqrt{3}}{3} \arctan\left(\frac{\sqrt{3}}{3}(2x+1)\right) + \ln|x| + c$
- (o) $\int \frac{(1-x^3)dx}{(x^2+1)(x-1)^2} = -\frac{3}{2} \ln|x-1| + \frac{1}{4} \ln(x^2+1) - \frac{1}{2} \arctan(x) + c$
- (p) $\int \frac{(1-x^3)dx}{(x^2+1)^2(x-1)^2} = -\frac{3}{4} \ln|x-1| - \frac{x+1}{4x^2+4} + \frac{1}{2} \arctan(x) + \frac{3}{8} \ln(x^2+1) + c$
- (q) $\int \frac{(x+2)dx}{x(x^4-1)} = \frac{1}{16} \left[\frac{1}{x+1} - \frac{5}{x-1} - 11 \ln|x-1| + \frac{2x+4}{x^2+1} + 6 \arctan(x) - 8 \ln(x^2+1) + \right.$
 $\left. + 32 \ln|x| - 5 \ln|x+1| \right] + c$
- (r) $\int \frac{dx}{(x^3-1)^2} = \frac{1}{9} \left[-\frac{1}{x-1} - 2 \ln|x-1| + \ln|x-1| + \ln|x^2+x+1| + \right.$
 $\left. + 2\sqrt{3} \arctan\left(\frac{\sqrt{3}}{3}(2x+1)\right) + \frac{x-1}{x^2+x+1} \right] + c$

2. Mostre que:

(a) $\int_2^4 \frac{x^3-2}{x^3-x^2} dx = \frac{5}{2} + \ln \frac{4}{3}$

(b) $\int_0^5 \frac{(x^2-3)dx}{(x+2)(x+1)^2} = \ln \frac{7}{2} - \frac{5}{3}$