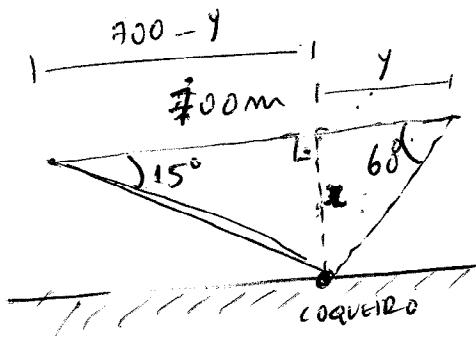


GABARITO PROVA I

29/11/2017.

01)



$$\tan 60^\circ = \frac{x}{y}$$

$$\sqrt{3} = \frac{x}{y} \rightarrow x = y\sqrt{3}$$

$$\tan 15^\circ = \frac{y}{700-y}$$

onde:

$$\tan 15^\circ = \tan(45^\circ - 30^\circ) =$$

$$= \frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \cdot \tan 30^\circ}$$

$$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + \frac{1 \cdot \sqrt{3}}{3}} = \frac{3 - \sqrt{3}}{3 + \sqrt{3}}$$

Então, temos:

$$\tan 15^\circ = \frac{y\sqrt{3}}{700-y}$$

$$\frac{3-\sqrt{3}}{3+\sqrt{3}} = \frac{y\sqrt{3}}{700-y} \Rightarrow (3-\sqrt{3})(700-y) = y\sqrt{3}(3+\sqrt{3})$$

$$2100 - 3y - 700\sqrt{3} + y\sqrt{3} = 3y\sqrt{3} + 3y$$

$$2100 - 700\sqrt{3} = 6y - 2y\sqrt{3}$$

$$\Rightarrow y = \frac{2100 - 700\sqrt{3}}{6 - 2\sqrt{3}}$$

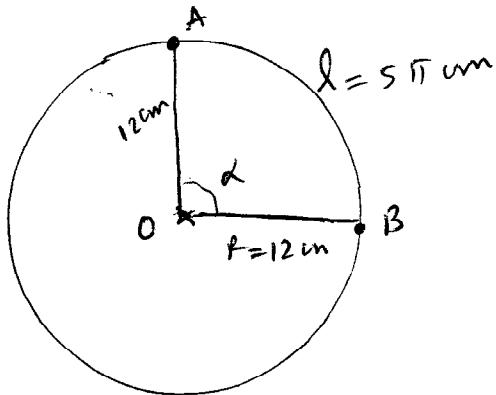
$$\sim \frac{700(3-\sqrt{3})}{2(3-\sqrt{3})} = \underline{\underline{350 \text{ m}}}$$

(N.S.)

$$\Rightarrow x = y\sqrt{3} \Rightarrow x = 350\sqrt{3} \text{ m}$$

01

02)



$$a) \alpha_{(\text{rad})} = \frac{l}{R} = \frac{5\pi}{12} \text{ rad.}$$

$$\frac{2\pi \text{ rad}}{\alpha} \quad \cancel{\frac{\pi R^2}{A \text{ setor}}} \\$$

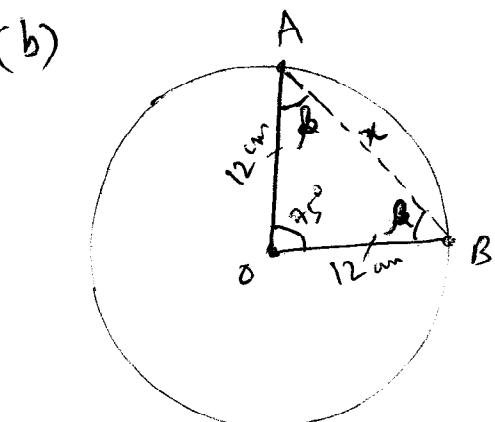
$$A_{\text{setor}} = \frac{\alpha \cdot \pi R^2}{2\pi} = \frac{\alpha R^2}{2}$$

$$A_{\text{setor}} = \frac{5\pi}{12} \cdot \frac{(12)^2}{2} = 30\pi \text{ cm}^2$$

$$A_{\text{setor}} = 30\pi \text{ cm}^2$$

0,3

(b)



$$\frac{5\pi}{12} \text{ rad} = 75^\circ$$

$$A = \frac{1}{2} \cdot 12 \cdot 12 \cdot \sin 75^\circ$$

$$A = 6 \cdot 12 \cdot \sin 75^\circ, \text{ onde:}$$

$$\sin 75^\circ = \sin(30^\circ + 45^\circ) =$$

$$= \sin 30^\circ \cdot \cos 45^\circ + \sin 45^\circ \cdot \cos 30^\circ$$

$$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} = \frac{\sqrt{2} + \sqrt{6}}{4}$$

N.D.

$$\text{Então: } A = 6 \cdot 12 \cdot \frac{\sqrt{2} + \sqrt{6}}{4} = 18(\sqrt{2} + \sqrt{6}) \text{ cm}^2$$

03) Note que:

- $\tan(\pi - x) = -\tan(\pi - (\pi - x)) = -\tan x$
- $\cot\left(\frac{\pi}{2} + x\right) = -\cot\left(\pi - \left(\frac{\pi}{2} + x\right)\right) = -\cot\left(\frac{\pi}{2} - x\right) = -\tan x$
- $\sin\left(\frac{\pi}{2} + x\right) = +\sin\left(\pi - \left(\frac{\pi}{2} + x\right)\right) = \sin\left(\frac{\pi}{2} - x\right) = \cos x$
- $\csc\left(\frac{3\pi}{2} + x\right) = -\csc\left(2\pi - \left(\frac{3\pi}{2} + x\right)\right) = -\csc\left(\frac{\pi}{2} - x\right) = -\sec x$ .
- $\sec(\pi + x) = -\sec((\pi + x) - \pi) = -\sec x$ .

Afirm; teorema:

$$y = \frac{-\tan x \cdot (-\tan x) - \cos x \cdot (-\sec x)}{-\sec x}$$

$$\sim y = \frac{\tan^2 x + \cos x \cdot \frac{1}{\cos x}}{-\sec x} = \frac{1 + \tan^2 x}{-\sec x} = \frac{\sec^2 x}{-\sec x} = -\sec x.$$

04)  $x+y=300^\circ$ ;  $x \in 1^\circ q$ ;  $y \in 3^\circ q$ ;  $\cot x = \sqrt{2}$

Achar  $\sec 2y$ .

$$\sec 2y = \frac{1}{\cos 2y}; \quad e \quad \cos 2y = \cos^2 y - \sin^2 y.$$

$$y = 300^\circ - x \quad \text{Então:}$$

$$\begin{aligned} \cos y &= \cos(300^\circ - x) = \cos 300^\circ \cdot \cos x - \sin 300^\circ \cdot \sin x \\ &= \cos 60^\circ \cdot \cos x - (-\sin 60^\circ) \cdot \sin x \end{aligned} \Rightarrow$$

02

$$\text{Então: } \cos y = \frac{1}{2} \cdot \cos x + \frac{\sqrt{3}}{2} \cdot \sin x . \quad (*)$$

Como  $\cot x = \sqrt{2}$ , temos:

$$1 + \cot^2 x = \csc^2 x \Rightarrow \csc^2 x = 1 + 2 = 3$$

$$\Rightarrow \csc x = \pm \sqrt{3} \quad \Rightarrow \sin x = \frac{1}{\sqrt{3}} \quad \text{OJ}$$

$x \in 1^{\text{eq}}$

$$\sin^2 x + \cos^2 x = 1 \Rightarrow \cos x = \pm \sqrt{1 - \frac{1}{3}} = \frac{\sqrt{2}}{\sqrt{3}} \Rightarrow \cos x = \frac{\sqrt{2}}{\sqrt{3}} \quad \text{OJ}$$

Assim; voltando em  $(*)$ , temos:

$$\cos y = \frac{1}{2} \cdot \frac{\sqrt{2}}{\sqrt{3}} + \frac{\sqrt{3}}{2} \cdot \frac{1}{\sqrt{3}} = \underbrace{\frac{\sqrt{2} + \sqrt{3}}{2\sqrt{3}}}_{\text{OJ}} ; \text{ e então:}$$

$$\begin{aligned} \sin y &= -\sqrt{1 - \cos^2 y} = -\sqrt{1 - \left(\frac{\sqrt{2} + \sqrt{3}}{2\sqrt{3}}\right)^2} = -\sqrt{1 - \frac{2 + 2\sqrt{6} + 3}{4 \cdot 3}} \\ &\quad \text{OJ} \\ &= -\sqrt{\frac{12 - 5 - 2\sqrt{6}}{4 \cdot 3}} = -\frac{\sqrt{7 - 2\sqrt{6}}}{2\sqrt{3}} \quad \text{OJ} \end{aligned}$$

$$\text{Logo; } \cos 2y = \cos^2 y - \sin^2 y = \left(\frac{\sqrt{2} + \sqrt{3}}{2\sqrt{3}}\right)^2 - \left(-\frac{\sqrt{7 - 2\sqrt{6}}}{2\sqrt{3}}\right)^2$$

$$= \frac{2 + 2\sqrt{6} + 3}{12} - \frac{7 - 2\sqrt{6}}{12} = \frac{5 + 2\sqrt{6} - 7 + 2\sqrt{6}}{12}$$

$$= \frac{2\sqrt{6} - 2}{12} = \frac{\sqrt{6} - 1}{6} \quad \text{OJ}$$

$$\text{Tem; } \sec 2y = \frac{1}{\cos 2y} = \frac{6}{\sqrt{6} - 1} \times \frac{\sqrt{6} + 1}{\sqrt{6} + 1} = \frac{6(1 + \sqrt{6})}{5}$$

20

04

$$05) \cos 2x + \cos x + 1 = 0$$

$$\cos^2 x - 2\cos^2 x + \cos x + 1 = 0$$

$$\cos^2 x - (1 - \cos^2 x) + \cos x + 1 = 0$$

$$2\cos^2 x + \cos x = 0$$

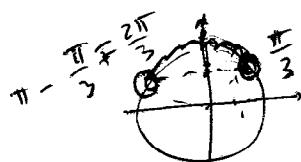
$$\cos x (2\cos x + 1) = 0$$

$$\left. \begin{array}{l} \cos x = 0 \\ \cos x = -\frac{1}{2} \end{array} \right\} \quad \begin{array}{l} \xrightarrow{\text{on circle}} \\ \text{at } x = \frac{\pi}{2}, \frac{4\pi}{3} \end{array} \quad \begin{array}{l} \Leftrightarrow x = k\pi + \frac{\pi}{2}, k \in \mathbb{Z} \\ x = \frac{2\pi}{3} + 2k\pi \end{array}$$

$$S = \left\{ x \in \mathbb{R} : x = \frac{\pi}{2} + k\pi \text{ or } x = \frac{2\pi}{3} + 2k\pi \text{ or } x = \frac{4\pi}{3} + 2k\pi, k \in \mathbb{Z} \right\}$$

$$06) 2 \cdot \sin\left(x - \frac{\pi}{5}\right) - \sqrt{3} \geq 0.$$

$$\sin\left(x - \frac{\pi}{5}\right) \geq \frac{\sqrt{3}}{2}$$



(1.0)

$$\frac{\pi}{3} + 2k\pi < x - \frac{\pi}{5} < \frac{2\pi}{3} + 2k\pi$$

$$\frac{\pi}{3} + \frac{\pi}{5} + 2k\pi < x < \frac{2\pi}{3} + \frac{\pi}{5} + 2k\pi$$

$$\frac{8\pi}{15} + 2k\pi < x < \frac{13\pi}{15} + 2k\pi, k \in \mathbb{Z}$$

$$S = \left\{ x \in \mathbb{R} : \frac{8\pi}{15} + 2k\pi < x < \frac{13\pi}{15} + 2k\pi, k \in \mathbb{Z} \right\}$$

07)

$$(a) \frac{\sin 4\alpha + \sin 2\alpha}{\cos 4\alpha + \cos 2\alpha} = \frac{2 \cdot \sin\left(\frac{4\alpha+2\alpha}{2}\right) \cdot \cos\left(\frac{4\alpha-2\alpha}{2}\right)}{2 \cdot \cos\left(\frac{4\alpha+2\alpha}{2}\right) \cdot \cos\left(\frac{4\alpha-2\alpha}{2}\right)} =$$

(015)

$$= \frac{\sin 3\alpha \cdot \cos \alpha}{\cos 3\alpha \cdot \cos \alpha} = \tan 3\alpha.$$

$$(b) \frac{\sin x}{1 - \cos x} = \frac{\sin x + \sin 0}{\cos 0 - \cos x} = \frac{2 \cdot \sin\left(\frac{x+0}{2}\right) \cdot \cos\left(\frac{x-0}{2}\right)}{-2 \cdot \sin\left(\frac{0+x}{2}\right) \cdot \sin\left(\frac{0-x}{2}\right)}$$

$$= \frac{\sin \frac{x}{2} \cdot \cos \frac{x}{2}}{-\sin \frac{x}{2} \cdot \sin(-\frac{x}{2})} = \frac{\cos \frac{x}{2}}{-(-\sin \frac{x}{2})} = \cot \frac{x}{2}.$$

(015)

$$08) \text{ a) } \sin \frac{\pi}{2m} = \sin\left(\frac{\pi}{2}\right) = \pm \sqrt{\frac{1 - \cos \frac{\pi}{m}}{2}}$$

$$\sqrt{\frac{2-\sqrt{3}}{4}} \Rightarrow \left( \sqrt{\frac{2-\sqrt{3}}{4}} \right)^2 = \left( +\sqrt{\frac{1 - \cos \frac{\pi}{m}}{2}} \right)^2$$

(10)

$$\frac{2-\sqrt{3}}{4} \leftarrow \frac{1 - \cos \frac{\pi}{m}}{2}$$

$$4 - 2\sqrt{3} = 4 - 4 \cos \frac{\pi}{m}$$

$$2 \cos \frac{\pi}{m} = \sqrt{3} \Rightarrow \cos \frac{\pi}{m} = \frac{\sqrt{3}}{2} \Leftrightarrow \begin{cases} \frac{\pi}{m} = \frac{\pi}{6} + k\pi \\ \frac{\pi}{m} = \frac{11\pi}{6} + k\pi \end{cases}$$

$$2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$$

(26)

$$\Leftrightarrow \frac{1}{m} = \frac{1}{6} + 2k, \quad k \in \mathbb{Z} \quad \text{ou}$$

$$\frac{1}{m} = \frac{11}{6} + 2k, \quad k \in \mathbb{Z}$$

Então:

$$m = \frac{1}{\frac{1}{6} + 2k} = \frac{1}{\frac{1+12k}{6}} = \frac{6}{1+12k} \in \mathbb{N} \Leftrightarrow k=0$$

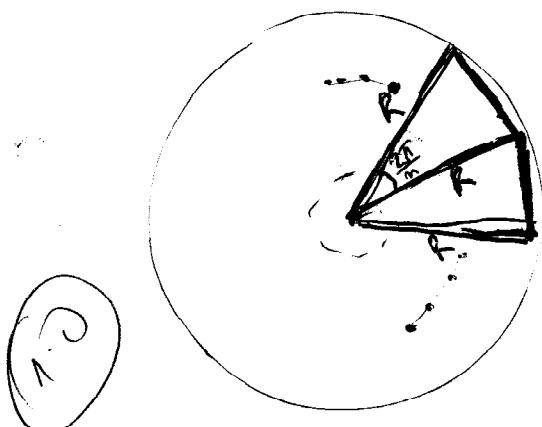
(e então  $m=6$ )

ou

$$m = \frac{1}{\frac{11}{6} + 2k} = \frac{1}{\frac{11+12k}{6}} = \frac{6}{11+12k} \notin \mathbb{N}, \quad \forall k \in \mathbb{Z}.$$

conclusão:  $\boxed{m=6}$ .

(09)



Sendo  $\mathcal{T}$ . de área regular que a área de cada triângulo inscrito seja:

$$A_{\Delta} = \frac{1}{2} \cdot R \cdot R \cdot \sin\left(\frac{2\pi}{n}\right) = \frac{R^2}{2} \cdot \sin\left(\frac{2\pi}{n}\right)$$

Tomando  $A_m = m \cdot A_{\Delta}$ , temos:

$$A_m = \frac{m \cdot R^2}{2} \cdot \sin\left(\frac{2\pi}{n}\right)$$

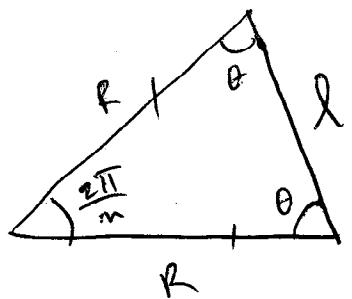
(09)

A área do círculo será:

$$A_0 = \lim_{n \rightarrow \infty} A_m = \lim_{n \rightarrow \infty} \frac{\cancel{m} R^2}{2} \cdot \underbrace{\frac{\sin\left(\frac{2\pi}{n}\right)}{\frac{2\pi}{n}}}_{\xrightarrow{n \rightarrow \infty} 1} \cdot \cancel{\frac{2\pi}{n}} = \overbrace{\pi R^2}^{(09)}$$

(09)

(02)



$$2\theta + \frac{2\pi}{m} = \pi$$

$$\Rightarrow 2\theta = \pi - \frac{2\pi}{m}$$

$$\Rightarrow \boxed{\theta = \frac{\pi}{2} - \frac{\pi}{m}}$$

Ideas del seno, teorema:

$$\frac{l}{\sin\left(\frac{2\pi}{m}\right)} = \frac{R}{\sin\theta}, \text{ así que;}$$

$$\frac{l}{\sin\left(\frac{2\pi}{m}\right)} = \frac{R}{\sin\left(\frac{\pi}{2} - \frac{\pi}{m}\right)} ; \text{ e como } \sin\left(\frac{\pi}{2} - \frac{\pi}{m}\right) = \cos\left(\frac{\pi}{m}\right)$$

segue que:

$$\frac{l}{\sin\left(\frac{2\pi}{m}\right)} = \frac{R}{\cos\frac{\pi}{m}} \Rightarrow \frac{l}{R} = \frac{\sin\left(2\frac{\pi}{m}\right)}{\cos\left(\frac{\pi}{m}\right)} \Rightarrow$$

$$\Rightarrow \underbrace{\frac{l}{R}}_{\sim} = \frac{2 \cdot \sin\left(\frac{\pi}{m}\right) \cdot \cos\left(\frac{\pi}{m}\right)}{\cos\left(\frac{\pi}{m}\right)} = \underbrace{2 \cdot \sin\left(\frac{\pi}{m}\right)}_{\sim}$$

$$\Rightarrow \boxed{\frac{l}{R} = 2 \cdot \sin\left(\frac{\pi}{m}\right)}$$

**Fundação Universidade Federal de Pelotas**  
**Departamento de Matemática e Estatística**  
**Curso de Licenciatura em Matemática**  
**Primeira Prova de Trigonometria**  
**Prof. Dr. Maurício Zahn**

**Nome:**

**Data:** 29/11/2017

**Questão 01.** Um barco navega seguindo uma trajetória retilínea e paralela à costa. Num certo momento, um coqueiro situado na costa da praia é visto do barco sob um ângulo de  $15^\circ$  com a trajetória. Navegando mais 700 m o barco, ainda andando paralelamente à costa, e já afastando-se do coqueiro, passa a vê-lo sob um ângulo de  $60^\circ$ . Qual a distância do barco à costa?

**Questão 02.** Os pontos  $A$  e  $B$  marcados num círculo de raio 12 cm determinam um setor circular de comprimento  $5\pi$  cm.

- (a) Determine a medida do ângulo central  $\alpha$  subtendido, em radianos. Em seguida, obtenha a área do setor circular  $AOB$ , onde  $O$  corresponde ao centro do círculo.
- (b) Calcule a área do triângulo  $AOB$ .

**Questão 03.** Sendo  $x$  um arco do primeiro quadrante simplifique a expressão

$$y = \frac{\tan(\pi - x) \cdot \cot\left(\frac{\pi}{2} + x\right) - \sin\left(\frac{\pi}{2} + x\right) \cdot \csc\left(\frac{3\pi}{2} + x\right)}{\sec(\pi + x)}.$$

**Questão 04.** Sabendo que  $x + y = 300^\circ$ ,  $0^\circ \leq x < 90^\circ$ ,  $180^\circ < y < 270^\circ$  e que  $\cot x = \sqrt{2}$ , determine o valor de  $\sec 2y$ .

**Questão 05.** Resolva, em  $\mathbb{R}$ , a equação trigonométrica  $\cos 2x + \cos x + 1 = 0$ .

**Questão 06.** Determine os valores de  $x \in \mathbb{R}$  para os quais  $2 \sen(x - \frac{\pi}{5}) - \sqrt{3} > 0$ .

**Questão 07.** Prove cada igualdade:

$$(a) \frac{\sen 4\alpha + \sen 2\alpha}{\cos 4\alpha + \cos 2\alpha} = \tan 3\alpha \quad (b) \frac{\sen x}{1 - \cos x} = \cot \frac{x}{2}$$

**Questão 08.** Determine o valor de  $n \in \mathbb{N}$  tal que  $\sen \frac{\pi}{2n} = \sqrt{\frac{2 - \sqrt{3}}{4}}$ .

**Questão 09.** Mostre qua a área  $A_n$  de um polígono regular de  $n$  lados inscrito numa circunferência de raio  $R$  pode ser dada pela fórmula

$$A_n = \frac{nR^2}{2} \cdot \sen\left(\frac{2\pi}{n}\right).$$

Deduza a partir daí a fórmula da área da circunferência de raio  $R$ .

Se  $\ell$  for a medida do lado do polígono regular inscrito na referida circunferência, mostre que

$$\frac{\ell}{R} = 2 \cdot \sen\left(\frac{\pi}{n}\right).$$

Questão	01	02	03	04	05	06	07	08	09
Valor	1,5	1,5	1,0	2,0	1,0	1,0	1,0	1,0	2,0