

RESPOSTAS DA LISTA 09
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01)



$$\frac{dV}{dt} = +8 \text{ cm}^3/\text{min.}$$

$$V = \frac{4\pi}{3} R^3$$

Procurar: $\frac{dR}{dt}$ em $R=2 \text{ cm}$

$$\frac{dV}{dt} = \frac{4\pi}{3} \cdot 3R^2 \cdot \frac{dR}{dt}$$

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{\frac{dV}{dt}}{4\pi R^2} = \frac{8}{4\pi R^2} = \frac{2}{\pi R^2}$$

Assim: $\frac{dV}{dt} (R=2 \text{ cm}) = \frac{2}{\pi \cdot (2)^2} = \frac{1}{2\pi} \text{ cm/min.}$

03)

$$\frac{dV}{dt} = +10 \text{ m}^3/\text{min.}$$



$$h = 2R \Rightarrow R = \frac{h}{2}$$

$$\frac{dh}{dt} (h=8 \text{ m}) = ?$$

$$V = \frac{Ab \cdot h}{3} = \frac{\pi r^2 h}{3} = \frac{\pi \cdot \frac{h^2}{4} \cdot h}{3} = \frac{\pi}{12} h^3$$

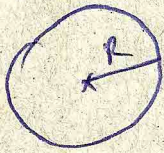
$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{4} \cdot h^2 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4 \cdot \frac{dV}{dt}}{\pi h^2} = \frac{4 \cdot 10}{\pi h^2} = \frac{40}{\pi h^2}$$

Assim: $\frac{dh}{dt} (h=8 \text{ m}) = \frac{40}{\pi \cdot (8)^2} = \frac{40}{64\pi} = \frac{5}{8\pi} \text{ m/min.}$

$$\frac{dV}{dt} (h=8 \text{ m}) = \frac{40}{\pi \cdot (8)^2} = \frac{40}{64\pi} = \frac{5}{8\pi} \text{ m/min.}$$

04.)



$$R = 0,5 \text{ cm}$$

$$\frac{dR}{dt} = 0,001 \text{ cm/dia.}$$

Calcular $\frac{dV}{dt}$ quando $R = 0,5 \text{ cm}$.

$$V = \frac{4\pi R^3}{3} \Rightarrow \frac{dV}{dt} = \frac{4\pi}{3} \cdot 3R^2 \cdot \frac{dR}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi R^2 \cdot \frac{dR}{dt} \quad \text{Logo:}$$

$$\frac{dV}{dt} (0,5 \text{ cm}) = 4\pi \cdot (0,5)^2 \cdot \frac{1}{1000} = \frac{\pi \cdot 0,25}{250} = \frac{\pi}{1000} \text{ cm}^3/\text{dia}$$

05.)

$$PV = c \Rightarrow P = c \cdot V^{-1} \quad \text{Logo:}$$

$$\frac{dP}{dt} = -c \cdot V^{-2} \cdot \frac{dV}{dt}$$

$$\Rightarrow \frac{dP}{dt} = -\frac{c}{V^2} \cdot \frac{dV}{dt}$$

$$; \text{ sabe-se que } \frac{dV}{dt} = +1.$$

Quando $P = 150$ e $V = 1,5$ vamos obter o valor de c :

$$c = P \cdot V = 150 \cdot 1,5 = 225$$

Axim:

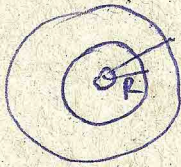
$$\frac{dP}{dt} = -\frac{225}{V^2} \cdot \frac{dV}{dt}$$

No instante em que $V = 1,5$ temos $\frac{dV}{dt} = +1$,

e daí:

$$\frac{dP}{dt} = -\frac{225}{(1,5)^2} \cdot 1 = -100 \text{ Kg/m}^2/\text{min.}$$

06)



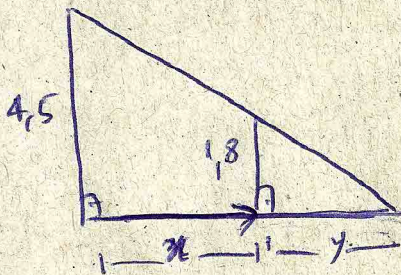
$$\frac{dR}{dt} = +16 \text{ cm/s}$$

$$\frac{dA}{dt} \quad (R=4 \text{ cm}) = ?$$

$$A = \pi R^2 \Rightarrow \frac{dA}{dt} = 2\pi R \cdot \frac{dR}{dt} \quad \text{Annnnn!}$$

$$\frac{dA}{dt} (4 \text{ cm}) = 2\pi \cdot 4 \cdot 16 = 128\pi \text{ cm}^2/\text{s}.$$

07)



$$\text{Dado: } \frac{dx}{dt} = 3 \text{ km/h.}$$

$$\text{compra: } \underline{y}$$

$$\text{achar } \frac{dy}{dt} \quad \text{queremos}$$

$$\frac{y}{1,8} \neq \frac{x+y}{4,5} \quad \text{[scribble]}$$

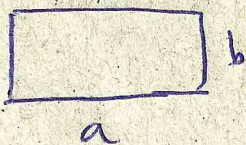
$$1,8x + 1,8y = 4,5y \Rightarrow 1,8x = 2,7y$$

$$\Rightarrow 1,8 \cdot \frac{dx}{dt} = 2,7 \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{1,8 \cdot \frac{dx}{dt}}{2,7} = \frac{1,8 \cdot 3}{2,7}$$

$$= 2 \text{ km/h.}$$

08)

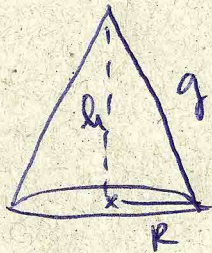


$$\frac{da}{dt} = m \quad \text{e} \quad \frac{db}{dt} = n \quad \text{Como a área}$$

$$\text{e } A = a \cdot b \quad ; \quad \text{temos que:}$$

$$\frac{dA}{dt} = \frac{d(a \cdot b)}{dt} = a \cdot \frac{db}{dt} + b \cdot \frac{da}{dt} = \underline{\underline{am + bm}}$$

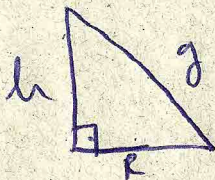
09)



$$\frac{dR}{dt} = 3 \text{ cm/h}$$

$$\frac{dh}{dt} = -4 \text{ cm/h}$$

A área total é dada por: $A = A_l + A_b = \pi R g + \pi R^2$



Como $g^2 = R^2 + h^2$, segue que $g = \sqrt{R^2 + h^2}$ e assim:

$$A = \pi R \cdot \sqrt{R^2 + h^2} + \pi R^2$$

Derivando em função de t , vem:

$$\begin{aligned} \frac{dA}{dt} = & \pi R \cdot \frac{1}{2} \cdot (R^2 + h^2)^{-\frac{1}{2}} \cdot (2R \cdot \frac{dR}{dt} + 2h \cdot \frac{dh}{dt}) + \\ & + \pi \cdot \frac{dR}{dt} \cdot \sqrt{R^2 + h^2} + 2\pi R \cdot \frac{dR}{dt} \end{aligned}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\pi R}{\sqrt{R^2 + h^2}} \cdot \left(R \cdot \frac{dR}{dt} + h \cdot \frac{dh}{dt} \right) + \pi \cdot \sqrt{R^2 + h^2} \cdot \frac{dR}{dt} + 2\pi R \cdot \frac{dR}{dt}$$

Assim, quando $R = 7 \text{ cm}$ e $h = 24 \text{ cm}$, teremos:

$$\left. \frac{dA}{dt} \right|_{R=7; h=24} = \frac{\pi \cdot 7}{\sqrt{49 + 576}} \cdot \left(7 \cdot 3 + 24 \cdot (-4) \right) + \pi \cdot \sqrt{49 + 576} \cdot 3 + 2\pi \cdot 7 \cdot 3$$

$$= \frac{7\pi}{25} \cdot (-75) + 75\pi + 42\pi = 96\pi \text{ cm}^2/\text{h}$$

$$10) \quad R = E \cdot I^{-1} \quad ; \quad E - \text{constante.}$$

Derivando em t , obtemos:

$$\frac{dR}{dt} = E \cdot (-1 \cdot I^{-2} \cdot \frac{dI}{dt})$$

$$\Rightarrow \frac{dR}{dt} = -\frac{E}{I^2} \cdot \frac{dI}{dt} \quad - \text{Logo, o sinal negativo diz que}$$

R decresce, e é proporcional ao inverso do quadrado de I .

$$11) \quad \frac{dR}{dt} = 0,01 \text{ cm/s}$$

$$A = \pi R^2$$

$$\Rightarrow \frac{dA}{dt} = 2\pi R \cdot \frac{dR}{dt}$$

$$\frac{dA}{dt} (2\text{cm}) = 2\pi \cdot 2 \cdot \frac{1}{100} = \frac{\pi}{25} \text{ cm}^2/\text{s}$$

$$12) \quad P \cdot V^{1,4} = c \quad ; \quad c - \text{constante.} \quad P = P(t); \quad V = V(t).$$

Derivando em relação a t ; vem:

$$\textcircled{\bullet} \quad P \cdot 1,4 \cdot V^{0,4} \cdot \frac{dV}{dt} + \frac{dP}{dt} \cdot V^{1,4} = 0$$

$$\Rightarrow \frac{dP}{dt} = -\frac{1,4 \cdot P \cdot V^{0,4}}{V^{1,4}} \cdot \frac{dV}{dt} = -\frac{1,4 \cdot P}{V} \cdot \frac{dV}{dt}$$

Assim; sendo $P = 50$ e $V = 10$, obtemos (e $\frac{dV}{dt} = -1$)

$$\frac{dP}{dt} = -\frac{1,4 \cdot 50}{10} \cdot (-1) = +7 \text{ lb/pol}^2/\text{s}$$

15)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{dR_1}{dt} = 0,3 \Omega/\text{s} ; \frac{dR_2}{dt} = 0,2 \Omega/\text{s}$$

Determinar $\frac{dR}{dt}$ quando $R_1 = 80 \Omega$ e $R_2 = 100 \Omega$.

$$R^{-1} = R_1^{-1} + R_2^{-1}$$

Derivando em t : $\left[R = R(t) ; R_1 = R_1(t) \right]$ rem: $R_2 = R_2(t)$

$$-R^{-2} \cdot \frac{dR}{dt} = -R_1^{-2} \cdot \frac{dR_1}{dt} - R_2^{-2} \cdot \frac{dR_2}{dt} \quad (*)$$

$$\frac{1}{R^2} \cdot \frac{dR}{dt} = \frac{1}{R_1^2} \cdot \frac{dR_1}{dt} + \frac{1}{R_2^2} \cdot \frac{dR_2}{dt}$$

$$\Rightarrow \frac{dR}{dt} = \left(\frac{1}{R_1^2} \frac{dR_1}{dt} + \frac{1}{R_2^2} \frac{dR_2}{dt} \right) \cdot R^2 \quad (*)$$

Como $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 \cdot R_2} \Rightarrow R = \frac{R_1 \cdot R_2}{R_1 + R_2}$;

e daí (*) fica:

$$\frac{dR}{dt} = \left(\frac{1}{R_1^2} \frac{dR_1}{dt} + \frac{1}{R_2^2} \frac{dR_2}{dt} \right) \cdot \left(\frac{R_1 \cdot R_2}{R_1 + R_2} \right)^2$$

Queremos $\frac{dR}{dt}$ quando $\frac{dR_1}{dt} = 0,3 \Omega/\text{s}$; $\frac{dR_2}{dt} = 0,2 \Omega/\text{s}$ e $R_1 = 80 \Omega$
 $R_2 = 100 \Omega$:

Logo:

$$\frac{dR}{dt} = \left(\frac{1}{(80)^2} \cdot 0,3 + \frac{1}{(100)^2} \cdot 0,2 \right) \cdot \left(\frac{80 \cdot 100}{80 + 100} \right)^2$$

$$= \left(\frac{0,3}{6.400} + \frac{0,2}{10.000} \right) \cdot \frac{64 \cdot 10^6}{324 \cdot 10^2} =$$

$$= \left(\frac{3}{64 \cdot 10^3} + \frac{2}{10^5} \right) \cdot \left(\frac{32}{182} \cdot 10^4 \right) =$$

$$= \frac{1}{2.54} \cdot 10^1 + \frac{32}{810} = \frac{10}{108} + \frac{32}{810} =$$

$$= \frac{5}{54} + \frac{16}{405} = \frac{75 + 32}{810} = \frac{107}{810} \Omega/\text{s}$$

06)

