RESPOSTAS DA LISTA 09
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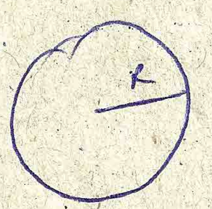
01)



$$\frac{dV}{dt} = +8\ cm^3/min.$$

$$V = \frac{4\pi}{3} R^3 .$$

Achar: $\frac{dR}{dt}$ em $R = 2cm$

$$\frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3} R^2 \cdot \frac{dR}{dt}$$

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt} \Rightarrow \frac{dR}{dt} = \frac{\frac{dV}{dt}}{4\pi R^2} = \frac{8}{4\pi R^2} = \frac{2}{\pi R^2}$$

Assim: $\frac{dV}{dt}(R=2cm) = \frac{\cancel{2}}{\pi \cdot (2)^{\cancel{2}}} = \frac{1}{2\pi}\ cm/min.$

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03) $\frac{dV}{dt} = +10\ m^3/min.$

$$h = 2R \Rightarrow R = \frac{h}{2}$$

$$\frac{dh}{dt}(h = 8m) = ?$$

$$V = \frac{Ab \cdot h}{3} = \frac{\pi R^2 h}{3} = \frac{\pi \cdot \frac{h^2}{4} \cdot h}{3} = \frac{\pi}{12} h^3$$

$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$$

$$\Rightarrow \frac{dV}{dt} = \frac{\pi}{4} \cdot h^2 \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{4 \cdot \frac{dV}{dt}}{\pi h^2} = \frac{4 \cdot 10}{\pi h^2} = \frac{40}{\pi h^2}$$

Assim:

$$\frac{dV}{dt}(h=8m) = \frac{40}{\pi \cdot (8)^2} = \frac{40}{64\pi} = \frac{5}{8\pi}\ m/min.$$

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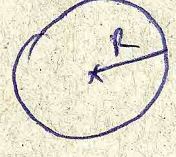
04)



$R = 0{,}5\ cm$

$\frac{dR}{dt} = 0{,}001\ cm/dia.$

Achar $\frac{dV}{dt}$ quando $R = 0{,}5\,cm$.

$$V = \frac{4\pi R^3}{3} \Rightarrow \frac{dV}{dt} = \frac{4\pi}{\cancel{3}} \cdot \cancel{3}R^2 \cdot \frac{dR}{dt}$$

$$\Rightarrow \frac{dV}{dt} = 4\pi R^2 \cdot \frac{dR}{dt}.$$ Logo:

$$\frac{dV}{dt}(0{,}5\,cm) = 4\pi \cdot (0{,}5)^2 \cdot \frac{1}{1000} = \frac{\pi \cdot 0{,}25}{250} = \frac{\pi}{1000}\ cm^3/dia$$

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05) $PV = c \Rightarrow P = c \cdot V^{-1}$. Logo:

$$\frac{dP}{dt} = -c \cdot V^{-2} \cdot \frac{dV}{dt}$$

$$\Rightarrow \frac{dP}{dt} = -\frac{c}{V^2} \cdot \frac{dV}{dt}$$; sabe-se que $\frac{dV}{dt} = +1$.

Quando $P = 150$ e $V = 1{,}5$ vamos obter o valor de c: $c = P \cdot V = 150 \cdot 1{,}5 = 225$

Assim:

$$\frac{dP}{dt} = -\frac{225}{V^2} \cdot \frac{dV}{dt}$$

No instante em que $V = 1{,}5$ temos $\frac{dV}{dt} = +1$,

e daí:

$$\frac{dP}{dt} = -\frac{225}{(1{,}5)^2} \cdot 1 = -100\ Kg/m^2/min.$$

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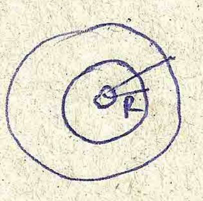
06)



$$\frac{dR}{dt} = +16 \text{ cm/s}$$

$$\frac{dA}{dt}\ (R = 4\text{cm}) = ?$$

$$A = \pi R^2 \Rightarrow \frac{dA}{dt} = 2\pi R \cdot \frac{dR}{dt}$$. Assim:

$$\frac{dA}{dt}\ (4\text{cm}) = 2\pi \cdot 4 \cdot 16 = 128\pi \text{ cm}^2/\text{s}.$$

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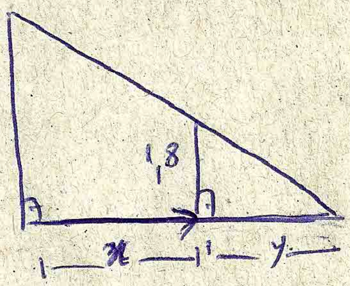
07)



Dado: $\frac{dx}{dt} = 3$ km/h.

sombra: y

achar $\frac{dy}{dt}$ ~~quando~~

$$\frac{y}{1{,}8} = \frac{x+y}{4{,}5}$$

$$1{,}8x + 1{,}8y = 4{,}5y \Rightarrow 1{,}8x = 2{,}7y$$

$$\Rightarrow 1{,}8 \cdot \frac{dx}{dt} = 2{,}7 \cdot \frac{dy}{dt}$$

$$\Rightarrow \frac{dy}{dt} = \frac{1{,}8 \cdot \frac{dx}{dt}}{2{,}7} = \frac{1{,}8 \cdot 3}{2{,}7}$$

$$= 2 \text{ km/h}.$$

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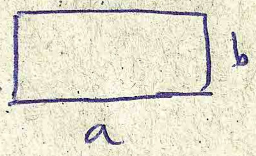
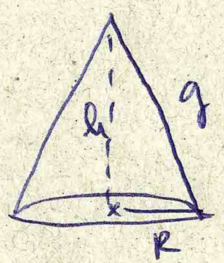
08)



$\frac{da}{dt} = m$ e $\frac{db}{dt} = n$. Como a área é $A = a \cdot b$, temos que:

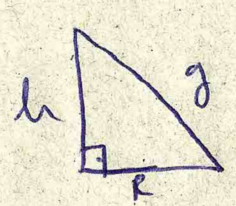
$$\frac{dA}{dt} = \frac{d(a \cdot b)}{dt} = a \cdot \frac{db}{dt} + b \cdot \frac{da}{dt} = an + bm.$$

09) 

$$\frac{dR}{dt} = 3 \text{ cm/h}$$

$$\frac{dh}{dt} = -4 \text{ cm/h}.$$

A área total é dada por: $A = A_l + A_b = \pi R g + \pi R^2$



Como $g^2 = R^2 + h^2$, segue que

$g = \sqrt{R^2+h^2}$ e assim:

$$A = \pi R \cdot \sqrt{R^2+h^2} + \pi R^2.$$

Derivando em função de t, vem:

$$\frac{dA}{dt} = \pi R \cdot \frac{1}{2} \cdot (R^2+h^2)^{-\frac{1}{2}} \cdot \left(2R \cdot \frac{dR}{dt} + 2h \frac{dh}{dt}\right) + $$

$$+ \pi \cdot \frac{dR}{dt} \cdot \sqrt{R^2+h^2} + 2\pi R \cdot \frac{dR}{dt}$$

$$\Rightarrow \frac{dA}{dt} = \frac{\pi R}{\sqrt{R^2+h^2}} \cdot \left(R \cdot \frac{dR}{dt} + h \cdot \frac{dh}{dt}\right) + \pi \cdot \sqrt{R^2+h^2} \cdot \frac{dR}{dt} + 2\pi R \cdot \frac{dR}{dt}$$

Assim, quando $R = 7$ cm e $h = 24$ cm, teremos:

$$\left.\frac{dA}{dt}\right|_{R=7;\, h=24} = \frac{\pi \cdot 7}{\sqrt{49+576}} \left(7 \cdot 3 + 24(-4)\right) + \pi \cdot \sqrt{49+576} \cdot 3 + 2\pi \cdot 7 \cdot 3$$

$$= \frac{7\pi}{25} \cdot (-75) + 75\pi + 42\pi = 96\pi \text{ cm}^2/\text{h}.$$

10) $R = E \cdot I^{-1}$; E – constante.

Derivando em t, obtemos:

$$\frac{dR}{dt} = E \cdot \left(-1 \cdot I^{-2} \cdot \frac{dI}{dt}\right)$$

$$\Rightarrow \frac{dR}{dt} = -\frac{E}{I^2} \cdot \frac{dI}{dt}$$. Logo, o sinal negativo diz que R decresce, e é proporcional ao inverso do quadrado de I.

11) $\frac{dR}{dt} = 0{,}01 \text{ cm/s}$

$A = \pi R^2$

$$\Rightarrow \frac{dA}{dt} = 2\pi R \cdot \frac{dR}{dt}$$

$$\frac{dA}{dt}(2\text{cm}) = 2\pi \cdot 2 \cdot \frac{1}{100} = \frac{\pi}{25} \text{ cm}^2/\text{s}.$$

12) $P \cdot V^{1,4} = c$; c – constante. $P = P(t)$; $V = V(t)$.

Derivando em relação a t; vem:

$$P \cdot 1{,}4 \cdot V^{0,4} \cdot \frac{dV}{dt} + \frac{dP}{dt} \cdot V^{1,4} = 0$$

$$\Rightarrow \frac{dP}{dt} = -\frac{1{,}4 P \cdot V^{0,4}}{V^{1,4}} \cdot \frac{dV}{dt} = -\frac{1{,}4 \cdot P}{V} \cdot \frac{dV}{dt}$$

Assim; sendo $P = 50$ e $V = 10$, obtemos (e $\frac{dV}{dt} = -1$)

$$\frac{dP}{dt} = -\frac{1{,}4 \cdot 50}{10} \cdot (-1) = +7 \text{ lb/pol}^2/\text{s}.$$

15)

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \frac{dR_1}{dt} = 0,3\ \Omega/s\ ;\ \frac{dR_2}{dt} = 0,2\ \Omega/s$$

Determinar $\frac{dR}{dt}$ quando $R_1 = 80\Omega$ e $R_2 = 100\Omega$.

$R^{-1} = R_1^{-1} + R_2^{-1}$. Derivando em t: $\left[\begin{array}{c} R = R(t)\ ;\ R_1 = R_1(t) \\ R_2 = R_2(t) \end{array}\right]$ vem:

$$-R^{-2}\cdot\frac{dR}{dt} = -R_1^{-2}\cdot\frac{dR_1}{dt} - R_2^{-2}\cdot\frac{dR_2}{dt} \qquad (\times -1)$$

$$\frac{1}{R^2}\cdot\frac{dR}{dt} = \frac{1}{R_1^2}\cdot\frac{dR_1}{dt} + \frac{1}{R_2^2}\cdot\frac{dR_2}{dt}$$

$$\Rightarrow \frac{dR}{dt} = \left(\frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}\right)\cdot R^2 \qquad (*)$$

Como $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 \cdot R_2} \Rightarrow \boxed{R = \frac{R_1 \cdot R_2}{R_1 + R_2}}$;

e daí (*) fica:

$$\frac{dR}{dt} = \left(\frac{1}{R_1^2}\frac{dR_1}{dt} + \frac{1}{R_2^2}\frac{dR_2}{dt}\right)\cdot\left(\frac{R_1 \cdot R_2}{R_1 + R_2}\right)^2$$

Queremos $\frac{dR}{dt}$ quando $\frac{dR_1}{dt} = 0,3\ \Omega/s$; $\frac{dR_2}{dt} = 0,2\ \Omega/s$ e $R_1 = 80\Omega$ $R_2 = 100\Omega$:

Logo:

$$\frac{dR}{dt} = \left(\frac{1}{(80)^2}\cdot 0,3 + \frac{1}{(100)^2}\cdot 0,2\right)\cdot\left(\frac{80.100}{80+100}\right)^2$$

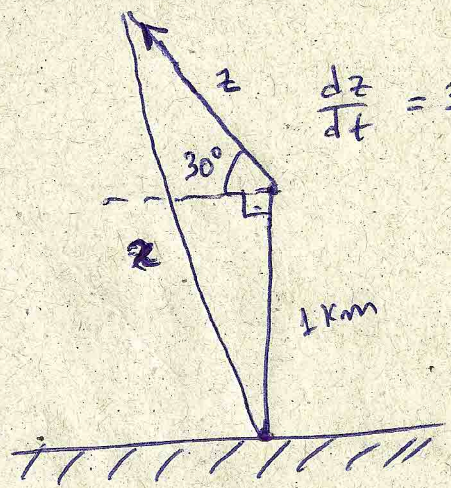
$$= \left(\frac{0,3}{6.400} + \frac{0,2}{10.000}\right)\cdot\frac{64.10^6}{324.10^2} =$$

$$= \left(\frac{3}{64.10^3} + \frac{2}{10^5}\right)\cdot\left(\frac{32}{162}\cdot 10^4\right) =$$

$$= \frac{1}{2.54}\cdot 10^1 + \frac{32}{810} = \frac{10}{108} + \frac{32}{810} =$$

$$= \frac{5}{54} + \frac{16}{405} = \frac{75 + 32}{810} = \frac{107}{810}\ \Omega/s$$

16)



$\frac{dz}{dt} = 300 \text{ Km/h}$

$t = 1 \text{ min} = \frac{1}{60} \text{ h}.$

Achar $\frac{dx}{dt}$ para $t = \frac{1}{60}$ h.

Note que quando $t = \frac{1}{60}$ h o avião atingiu a "altura inclinada" $z = \frac{300 \text{ Km}}{1 \cancel{h}} \times \frac{1}{60} \cancel{h}$

$\boxed{z = 5 \text{ Km}}$

Pela lei dos cossenos, temos que

$x^2 = 1^2 + z^2 - 2 \cdot 1 \cdot z \cdot \cos(120°)$

$x^2 = 1 + z^2 - 2z \cdot (-\cos 60°)$

$x^2 = 1 + z^2 + \cancel{2}z \cdot \frac{1}{\cancel{2}}$

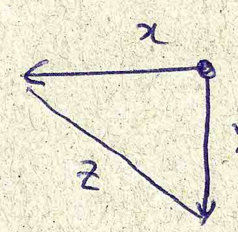
$\boxed{x^2 = z^2 + z + 1}$. Derivando em t $\left(x = x(t);\ z = z(t) \right)$:

$2x \frac{dx}{dt} = 2z \cdot \frac{dz}{dt} + \frac{dz}{dt}$

$\Rightarrow \frac{dx}{dt} = \frac{(2z+1) \cdot \frac{dz}{dt}}{2x} = \frac{(2z+1)\frac{dz}{dt}}{2\sqrt{z^2+z+1}}$

Portanto, $\frac{dx}{dt}(z = 5\text{Km}) = \frac{(2 \cdot 5 + 1) \cdot 300}{2\sqrt{25+5+1}} = \frac{1650}{\sqrt{31}}$ Km/h.

17)



$\frac{dx}{dt} = 72 \text{ km/h}$

$\frac{dy}{dt} = 30 \text{ km/h}.$

Achar $\frac{dz}{dt}$ em $t = 2h$. Quando $t = 2h$, então teremos neste instante:

$$\begin{cases} y = \frac{30 \text{ km}}{\cancel{h}} \cdot 2\cancel{h} = 60 \text{ km} \\ x = \frac{72 \text{ km}}{\cancel{h}} \times 2\cancel{h} = 144 \text{ km} \end{cases}$$

$z^2 = x^2 + y^2$. Derivando em t, onde $x = x(t)$; $y = y(t)$ e $z = z(t)$:

$$2z \cdot \frac{dz}{dt} = 2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt}}{z} = \frac{x \cdot \frac{dx}{dt} + y \cdot \frac{dy}{dt}}{\sqrt{x^2 + y^2}}.$$

Logo:

$$\frac{dz}{dt}(2h) = \frac{144 \cdot 72 + 60 \cdot 30}{\sqrt{(60)^2 + (144)^2}} = 78 \text{ km/h}.$$