LISTA 08 – CÁLCULO 1. PROF. MAURÍCIO ZAHN.

01) $f(x) = x^2+1$; $g(x) = e^x$; $h(x) = \operatorname{sen} x$.

(a) $(g\circ f)'(x) = g'(f(x)) . f'(x)$. ; onde

$g'(x) = e^x$ e então $g'(x^2+1) = e^{x^2+1}$,

e $f'(x) = (x^2+1)' = 2x + 0 = 2x$

Logo; $(g\circ f)'(x) = g'(f(x)) . f'(x) = e^{x^2+1} . 2x$

$= 2x . e^{x^2+1}$.

(d) $(h\circ f)'(x) = h'(f(x)) . f'(x)$; onde

$h'(x) = \cos x \rightarrow h'(f(x)) = \cos f(x) = \cos(x^2+1)$,

e $f'(x) = (x^2+1)' = 2x$; logo:

$(h\circ f)'(x) = h'(f(x)) . f'(x) = \cos(x^2+1) . 2x$

$= 2x . \cos(x^2+1)$

---

03) $F = \dfrac{100}{r^2}$ $F'(t) = ?$ $(t = 10s)$

$r = 1 + 0{,}4t^2$

Então $F(t) = \dfrac{100}{[r(t)]^2}$. $F(r(t))$

$= \dfrac{100}{(1+0{,}4t^2)^2} \Rightarrow 100 . (1+0{,}4t^2)^{-2}$

$$\Rightarrow \quad P'(t) = -200 \cdot (1+0{,}4t^2)^{-1} \cdot 0{,}8t$$

$$P'(t) = \frac{-160t}{1+0{,}4t^2}$$

$$\Rightarrow P'(10) = \frac{-1600}{1+40} = -\frac{1600}{41} \text{ N.}$$

---

04)

(b) $$y = \frac{1}{\sqrt{5}} \cdot \ln \frac{2x+3-\sqrt{5}}{2x+3+\sqrt{5}}$$

$$= \frac{1}{\sqrt{5}} \cdot \left(\ln(2x+3-\sqrt{5}) - \ln(2x+3+\sqrt{5}) \right)$$

$$\Rightarrow y' = \frac{1}{\sqrt{5}} \left(\frac{2}{2x+3-\sqrt{5}} - \frac{2}{2x+3+\sqrt{5}} \right)$$

$$y' = \frac{1}{\sqrt{5}} \cdot \left(\frac{\cancel{4x} \cancel{+6} + 2\sqrt{5} - \cancel{4x} \cancel{-6} + 2\sqrt{5}}{[(2x+3)-\sqrt{5}] \cdot [(2x+3)+\sqrt{5}]} \right)$$

$$= \frac{1}{\cancel{\sqrt{5}}} \cdot \frac{4\cancel{\sqrt{5}}}{(2x+3)^2 - (\sqrt{5})^2} = \frac{4}{(2x+3)^2-5}$$

---

05) (a) $y^3 - 3y + 2ax = 0$. Derivando em x, vem:

$$3y^2 \cdot y' - 3y' + 2a = 0$$

$$y' \cdot (3y^2 - 3y) + 2a = 0 \quad \Rightarrow \quad \boxed{y' = \frac{-2a}{3y^2-3y}}$$

(b) $\cos(xy) = x$. Derivando em x:

$$-\operatorname{sen}(xy)\cdot(x\cdot y' + 1\cdot y) = 1.$$

$$-x\cdot\operatorname{sen}(xy)\cdot y' - y\cdot\operatorname{sen}(xy) = 1$$

$$\Rightarrow \boxed{y' = -\frac{1 + y\cdot\operatorname{sen}(xy)}{\operatorname{sen}(xy)}}$$

(c) $x^{\frac{1}{2}} + y^{\frac{1}{2}} = a^{\frac{1}{2}}$. Derivando em x (obs.: a = CONST.):

$$\frac{1}{2}\cdot x^{-\frac{1}{2}}\cdot 1 + \frac{1}{2}\cdot y^{-\frac{1}{2}}\cdot y' = 0$$

$$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0 \quad (\div 2)$$

$$\Rightarrow \frac{y'}{\sqrt{y}} = -\frac{1}{\sqrt{x}} \Rightarrow \boxed{y' = -\frac{\sqrt{y}}{\sqrt{x}}}$$

(f) $\ln(x^2+y^2) - 3x^2y^3 = (x+y)^{\frac{1}{2}}$. Derivando em x:

$$\frac{2x + 2y\cdot y'}{x^2+y^2} - 3x^2\cdot 3y^2\cdot y' - 6x\cdot y^3 = \frac{1}{2}(x+y)^{-\frac{1}{2}}\cdot(1+y')$$

$$\Rightarrow \frac{2x}{x^2+y^2} + \frac{2y}{x^2+y^2}\cdot y' - 9x^2y^2\cdot y' - 6xy^3 = \frac{1}{2\sqrt{x+y}} + \frac{1}{2\sqrt{x+y}}\cdot y'$$

$$\Rightarrow \left(\frac{2y}{x^2+y^2} - \frac{1}{2\sqrt{x+y}} - 9x^2y^2\right)\cdot y' = \frac{1}{2\sqrt{x+y}} + 6xy^3 - \frac{2x}{x^2+y^2}$$

$$\Rightarrow \boxed{y' = \frac{\frac{1}{2\sqrt{x+y}} + 6xy^3 - \frac{2x}{x^2+y^2}}{\frac{2y}{x^2+y^2} - \frac{1}{2\sqrt{x+y}} - 9x^2y^2}}$$

06) $x^3 + y^3 = 2xy + 5$ em $P(2,1)$

Derivando em x, obtemos:

$$3x^2 + 3y^2 \cdot y' = 2x \cdot y' + 2 \cdot y + 0$$

$$\Rightarrow 3x^2 + 3y^2 \cdot y' - 2xy' = 2y$$

$$y'(3y^2 - 2x) = 2y - 3x^2$$

$$\Rightarrow y' = \frac{2y - 3x^2}{3y^2 - 2x}$$

A inclinação da reta tangente ao gráfico da curva dada pela função implícita será, no ponto $P(2,1)$:

$$y'\Big|_P = \frac{2.(1) - 3.(2)^2}{3.(1)^2 - 2.(2)} = \frac{2 - 12}{3 - 4} = 10$$

Assim, obtemos:

$$y - y_p = y'\Big|_p \cdot (x - x_p)$$

$$y - 1 = 10.(x - 2)$$

$$\Rightarrow y = 1 + 10x - 20 \Rightarrow \boxed{y = 10x - 19}.$$

---

11) (a) $\begin{cases} x = \operatorname{sen} t \\ y = \cos t \end{cases}$. Então: $x'(t) = \cos t$ e $y'(t) = -\operatorname{sen} t$.

Assim; $f'(x) = \dfrac{y'(t)}{x'(t)} = \dfrac{-\operatorname{sen} t}{\cos t} = -\dfrac{x}{y}$

(c) $\begin{cases} x = (t^2+1)^{\frac{1}{2}} \\ y = \dfrac{t-1}{(t^2+1)^{\frac{1}{2}}} \end{cases}$. Então;

$$x'(t) = \frac{1}{2}\cdot(t^2+1)^{-\frac{1}{2}}\cdot 2t = \frac{t}{\sqrt{t^2+1}} \quad ;$$

e

$$y'(t) = \frac{\sqrt{t^2+1}\cdot 1 - (t-1)\cdot\frac{1}{\cancel{2}}(t^2+1)^{-\frac{1}{2}}\cdot \cancel{2}t}{\left[(t^2+1)^{\frac{1}{2}}\right]^2} =$$

$$= \frac{\sqrt{t^2+1} - \dfrac{t(t-1)}{\sqrt{t^2+1}}}{t^2+1} = \frac{\dfrac{\cancel{t^2}+1-\cancel{t^2}+t}{\sqrt{t^2+1}}}{t^2+1}$$

$$= \frac{t+1}{(t^2+1)\sqrt{t^2+1}}$$

Portanto, teremos:

$$f'(x) = \frac{y'(t)}{x'(t)} = \frac{\dfrac{t+1}{(t^2+1)\cancel{\sqrt{t^2+1}}}}{\dfrac{t}{\cancel{\sqrt{t^2+1}}}} = \frac{t+1}{t(t^2+1)} \,//$$

Se quer continuar, pode deixar em termos de x e de y:

$$x = \sqrt{t^2+1} \Rightarrow x^2 = t^2+1.$$

$$y = \frac{t-1}{\sqrt{t^2+1}} = \frac{t-1}{x} \Rightarrow xy = t-1$$

$$\Rightarrow t = xy+1 \text{, e daí:}$$

$$f'(x) = \frac{t+1}{t(t^2+1)} = \frac{xy+1+1}{(xy+1)\cdot x^2} = \frac{xy+2}{x^2(xy+1)} \,//$$