01) $f(x) = \dfrac{x}{x+1}$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\dfrac{x+h}{x+h+1} - \dfrac{x}{x+1}}{h}$$

$$= \lim_{h\to 0} \frac{\dfrac{(x+h)(x+1) - x(x+h+1)}{(x+h+1)(x+1)}}{h} = \lim_{h\to 0} \frac{\dfrac{\cancel{x^2} + \cancel{x} + \cancel{hx} + h - \cancel{x^2} - \cancel{hx} - \cancel{x}}{(x+h+1)(x+1)}}{h}$$

1.0

$$= \lim_{h\to 0} \frac{\cancel{h}}{(x+h+1)(x+1)} \cdot \frac{1}{\cancel{h}} = \frac{1}{(x+1)^2}$$

1,5

PELAS REGRAS:

0,5

$$f'(x) = \frac{(x+1)\cdot 1 - x(1)}{(x+1)^2} = \frac{\cancel{x} + 1 - \cancel{x}}{(x+1)^2} = \frac{1}{(x+1)^2}$$

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02) $f(x) = \ln(1-x) + \sqrt{1+x^2} + 1$; $P(0,2)$

$$f'(x) = \frac{-1}{1-x} + \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \cdot 2x$$

1.0

$$f'(x) = -\frac{1}{1-x} + \frac{x}{\sqrt{1+x^2}}$$

$$m = f'(0) = -\frac{1}{1-0} + \frac{0}{\sqrt{1+0}} = -1$$

Assim, a eq. da reta tangente ao gráfico de f em P será:

$$y - 2 = (-1)\cdot(x - 0) \Rightarrow \boxed{y = -x + 2}.$$

---

03) $f(x) = x\cdot|x| = \begin{cases} x\cdot x, & \text{se } x \geq 0 \\ x\cdot(-x), & \text{se } x < 0 \end{cases} = \begin{cases} x^2, & \text{se } x \geq 0 \\ -x^2, & \text{se } x < 0 \end{cases}$

$$f'_-(0) = \lim_{h\to 0^-} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^-} \frac{f(h)-0}{h}$$

0,5

$$= \lim_{h\to 0^-} \frac{-h^2}{h} = \lim_{h\to 0^-} -h = 0$$

$$f'_+(0) = \lim_{h\to 0^+} \frac{f(0+h)-f(0)}{h} = \lim_{h\to 0^+} \frac{f(h)-0}{h}$$

0,5

$$= \lim_{h\to 0^+} \frac{h^2}{h} = \lim_{h\to 0^+} h = 0$$

1,0

Logo, $\exists\, f'(0) = 0$.

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04) (a) $y = \sec(x - \sqrt{x})$

$$\Longrightarrow y' = \sec(x-\sqrt{x})\cdot\tan(x-\sqrt{x})\cdot\left(1 - \frac{1}{2}x^{-\frac{1}{2}}\right)$$

0,5

$$y' = \left(1 - \frac{1}{2\sqrt{x}}\right)\cdot\sec(x-\sqrt{x})\cdot\tan(x-\sqrt{x})$$

(b) $y = \ln\dfrac{x-1}{\sqrt{x}} = \ln(x-1) - \ln x^{\frac{1}{2}}$

0,5

$$= \ln(x-1) - \frac{1}{2}\cdot\ln x$$

$$\Longrightarrow y' = \frac{1}{x-1} - \frac{1}{2}\cdot\frac{1}{x} = \frac{2x-(x-1)}{2x(x-1)} = \frac{x+1}{2x(x-1)}$$

(c) $y = e^{\tan(x-2x^2)}$ 0,5

$\Rightarrow y' = e^{\tan(x-2x^2)} \cdot \sec^2(x-2x^2) \cdot (1-4x)$

05) $x + \cos(xy) = \arctan(x-y)$. Derivando em x:

$$1 - \operatorname{sen}(xy) \cdot [x \cdot y' + 1 \cdot y] = \frac{1-y'}{1+(x-y)^2}$$

$$1 - x \cdot \operatorname{sen}(xy) \cdot y' - y \operatorname{sen}(xy) = \frac{1}{1+(x-y)^2} - \frac{y'}{1+(x-y)^2}$$

$$\Rightarrow y' = \frac{\frac{1}{1+(x-y)^2} + y \cdot \operatorname{sen}(xy) - 1}{-\frac{1}{1+(x-y)^2} + x \cdot \operatorname{sen}(xy)}$$ 1,0

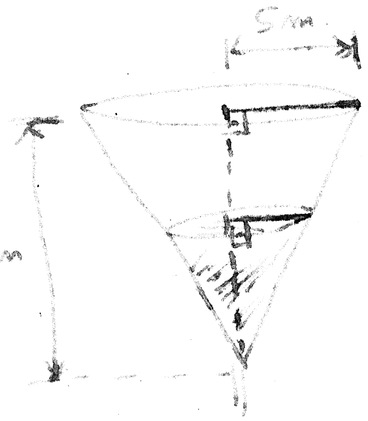
06) $x' = t \cdot e^{1-t} \cdot (-1) + 1 \cdot e^{1-t}$

$= (1-t) \cdot e^{1-t}$ 1,0

$y' = \frac{-1}{1-t}$

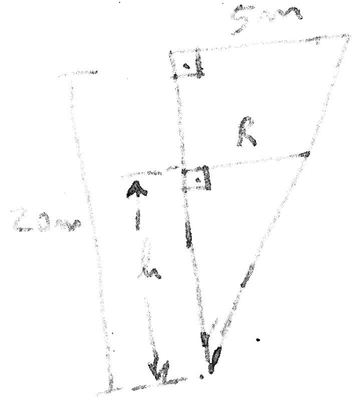
$$\Rightarrow f'(x) = \frac{y'}{x'} = \frac{-1}{1-t} \times \frac{1}{(1-t) \cdot e^{1-t}} = \frac{-1}{(1-t)^2 \cdot e^{1-t}}$$

07)



$\frac{dV}{dt} = -6\,m^3/min.$

$\left.\frac{dh}{dt}\right|_{h=12m} = ?$



$\frac{5}{20} = \frac{R}{h}$

$\frac{1}{4} = \frac{R}{h} \iff \boxed{R = \frac{h}{4}}$

Volume de água: $V = \frac{Ab \cdot h}{3} = \frac{\pi \cdot R^2 \cdot h}{3}$

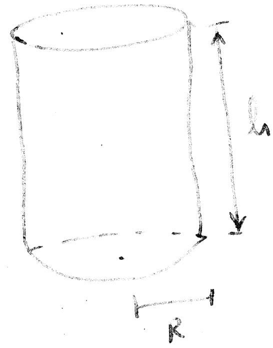
1,5

$V = \frac{\pi \cdot \left(\frac{h}{4}\right)^2 \cdot h}{3} = \frac{\pi \cdot h^3}{48}$

$\Rightarrow \frac{dV}{dt} = \frac{\pi}{48} \cdot 3h^2 \cdot \frac{dh}{dt} \Rightarrow \frac{dh}{dt} = \frac{16 \cdot \frac{dV}{dt}}{\pi h^2}$

$\Rightarrow \left.\frac{dh}{dt}\right|_{h=12m} = \frac{16 \cdot (-6)}{\pi \cdot (12)^2} = \frac{4 \cdot 4 \cdot (-6)}{\pi \cdot 6 \cdot 2 \cdot 6 \cdot 2} = -\frac{2}{3\pi}\ m/min$

08)



$V = 216\pi \, cm^3 = Ab.h$

$216\pi = \pi R^2 h \Rightarrow \boxed{h = \frac{216}{R^2}}$

A deve ser mínima.

$A = A\ell + Ab$

$A = \pi R^2 + 2\pi R . h$

$$\Rightarrow A = \pi R^2 + 2\pi R . \frac{216}{R^2}$$

$$A = \pi R^2 + \frac{432\pi}{R}$$

2,0

$$A'(R) = 2\pi . R - \frac{432\pi}{R^2} = 0 \iff$$

$$2\pi R = \frac{432\pi}{R^2} \iff R^3 = 216$$

$$\iff R = \sqrt[3]{2^3 . 3^3} = 6 \text{ cm}$$

216	2
108	2
54	2
27	3
9	3
3	3
1	

Vejamos se R = 6 cm dá área mínima:

$A' < 0$ ↘ $\quad$ $A' > 0$ ↗

$- \, -$ $\quad$ 6 $\quad$ $+ \, +$

min.

$R = 1 \Rightarrow A'(R) < 0$

$R = 10 \Rightarrow A'(R) > 0$

Portanto, a área será mínima quando R = 6 cm.

Nesse caso, $h = \frac{216}{6^2} = 6$ cm também.

09)

$$y = \frac{8x}{x^2+4}$$

(a) $D(f) = \mathbb{R}$. Logo, não existem assíntotas verticais. Mas $\lim_{x \to \pm\infty} f(x) = 0$. Logo, existe uma assíntota horizontal; $y = 0$.

0,5

(b) $$f'(x) = \frac{(x^2+4)\cdot 8 - 8x\cdot(2x)}{(x^2+4)^2} = \frac{8x^2+32-16x^2}{(x^2+4)^2}$$

$$f'(x) = \frac{32-8x^2}{(x^2+4)^2}$$

$f'(x) = 0 \iff -8x^2+32 = 0 \iff x^2 = 4 \iff x = \pm 2$

$f'<0$ $\quad f'>0$

$- - -$ $\quad + + +$ $\quad - - -$

$-2 \qquad 2$

$x = -3$; $f'(x) < 0$
$x = 0$; $f'(x) \geq 0$
$x = 3$; $f'(x) < 0$.

1,5

- Logo, f é cresc. em $(-2, 2)$ e decrescente em $(-\infty, -2) \cup (2, +\infty)$
- $\exists$ mín $(-2, f(-2)) = \left(-2, \frac{-16}{8}\right) = (-2,-2)$
- $\exists$ máx $(2, f(2)) = \left(2, \frac{16}{8}\right) = (2,2)$.

(c) $$f''(x) = \frac{(x^2+4)^2 \cdot (-16x) - (32-8x^2)\cdot 2 \cdot (x^2+4)\cdot 2x}{(x^2+4)^4}$$

$$f''(x) = \frac{-4x\cancel{(x^2+4)}\left[4\cdot(x^2+4) + (32-8x^2)\right]}{(x^2+4)^{\cancel{4}\,3}}$$

$$f''(x) = \frac{-4x\cdot(4x^2+16+32-8x^2)}{(x^2+4)^3}$$

$$f''(x) = \frac{-4x\,(-4x^2+48)}{(x^2+4)^3} = \frac{+16x(x^2-12)}{(x^2+4)^3}$$

$$f''(x) = 0 \iff 16x(x^2-12) = 0$$
$$\iff x = 0 \quad \text{ou} \quad x = \pm\sqrt{12} = \pm 2\sqrt{3}$$

C.P.B		C.P.C		C.P.B		C.P.C
$- -$		$+++$		$- - -$		$++$
	$-2\sqrt{3}$		0		$2\sqrt{3}$	

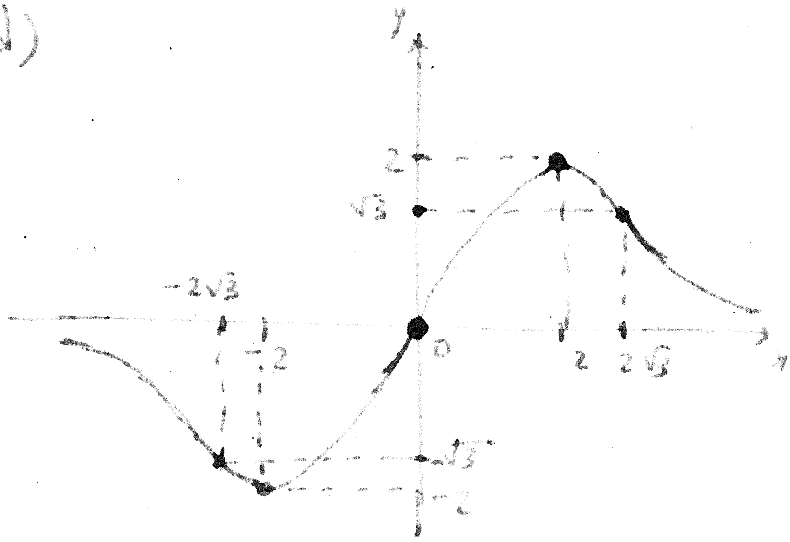
$$\begin{cases} x = -10;\ f''(x) < 0 \\ x = -1;\ f''(x) > 0 \\ x = 1;\ f''(x) < 0 \\ x = 10;\ f''(x) > 0 \end{cases}$$

Assim, f possui:

- C.P.C. em $(-2\sqrt{3}, 0) \cup (2\sqrt{3}, +\infty)$
- C.P.B. em $(-\infty, -2\sqrt{3}) \cup (0, 2\sqrt{3})$
- P.I. $(-2\sqrt{3}, f(-2\sqrt{3})) = (-2\sqrt{3}, -\sqrt{3})$;
- P.I. $(0, f(0)) = (0, 0)$;
- P.I. $(2\sqrt{3}, f(2\sqrt{3})) = (2\sqrt{3}, \sqrt{3})$.

1,5

(d)



0,5