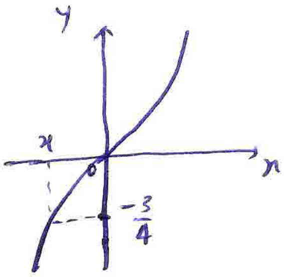
LISTA 02 :

01) a) $\operatorname{senh} x = -\frac{3}{4}$; $x < 0$.



$$\operatorname{csch} x = \frac{1}{\operatorname{senh} x} = -\frac{4}{3}$$

$$\cosh^2 x - \operatorname{senh}^2 x = 1 \Rightarrow \cosh^2 x = 1 + \operatorname{senh}^2 x$$

$$\Rightarrow \cosh x = \pm\sqrt{1 + \left(-\frac{3}{4}\right)^2} = \pm\sqrt{1 + \frac{9}{16}} = \pm\frac{5}{4},$$

e como $x < 0$, segue que $\cosh x = -\frac{5}{4}$.

$$\operatorname{sech} x = \frac{1}{\cosh x} = \frac{1}{-\frac{5}{4}} = -\frac{4}{5}.$$

$$\tanh x = \frac{\operatorname{senh} x}{\cosh x} = \frac{-\frac{3}{4}}{-\frac{5}{4}} = \frac{3}{5}.$$

$$\coth x = \frac{1}{\tanh x} = \frac{5}{3}.$$

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b) $\operatorname{sech} x = \frac{3}{5}$, $x < 0$.

$$\cosh x = \frac{1}{\operatorname{sech} x} = \frac{5}{3}.$$

$$\cosh^2 x - \operatorname{senh}^2 x = 1 \Rightarrow \operatorname{senh}^2 x = \cosh^2 x - 1$$

$$\Rightarrow \operatorname{senh} x = \pm\sqrt{\cosh^2 x - 1} = \pm\sqrt{\left(\frac{5}{3}\right)^2 - 1} = \pm\sqrt{\frac{25}{9} - 1}$$

$\Rightarrow \operatorname{senh} x = \pm\frac{4}{3}$, e como $x < 0$, segue que

$$\operatorname{senh} x = -\frac{4}{3}.$$

$$\operatorname{csch} x = \frac{1}{\operatorname{senh} x} = -\frac{3}{4}.$$

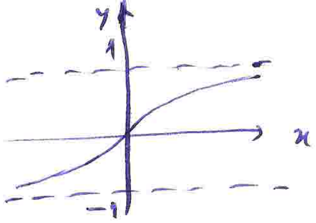
$$\tanh x = \frac{\operatorname{senh} x}{\cosh x} = \frac{-\frac{4}{3}}{\frac{5}{3}} = -\frac{4}{5}.$$

$$\coth x = \frac{1}{\tanh x} = \frac{1}{-\frac{4}{5}} = -\frac{5}{4}$$

c) $\tanh x = -\dfrac{7}{25}$; $x < 0$

$\uparrow$

ARRUMAR NA LISTA!



$$\coth x = \frac{1}{\tanh x} = -\frac{25}{7}.$$

Como

$1 - \tanh^2 x = \operatorname{sech}^2 x$, temos:

$$\operatorname{sech} x = \pm\sqrt{1-\tanh^2 x} = \pm\sqrt{1-\left(-\frac{7}{25}\right)^2} =$$

$$= \pm\sqrt{\frac{625-49}{625}} = \pm\frac{\sqrt{576}}{25} = \pm\frac{\sqrt{2^2\cdot 2^2\cdot 6^2}}{25} = \pm\frac{24}{25};$$

e como $x < 0$, segue que

$$\operatorname{sech} x = -\frac{24}{25}.$$

$$\cosh x = \frac{1}{\operatorname{sech} x} = -\frac{25}{24}.$$

$\Rightarrow \cosh^2 x - \operatorname{senh}^2 x = 1 \Rightarrow \operatorname{senh} x = \pm\sqrt{\cosh^2 x - 1}$

$\Rightarrow \operatorname{senh} x = \pm\sqrt{\dfrac{625-576}{576}} = \pm\dfrac{7}{24}$; e como $x<0$,

segue que $\operatorname{senh} x = -\dfrac{7}{24}$.

$$\operatorname{csch} x = \frac{1}{\operatorname{senh} x} = \frac{1}{-\frac{7}{24}} = \frac{-24}{7}.$$

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02) $\tanh(a+b) = 3$; $\tanh b = 2$. Achar $\cosh a$.

$$\tanh(a+b) = \frac{\tanh a + \tanh b}{1 + \tanh a\cdot\tanh b}.$$

$$3 = \frac{\tanh a + 2}{1 + 2\cdot\tanh a}.$$ Escreva $\tanh a = x$. Assim:

$$\frac{3}{1} = \times \frac{x+2}{1+2x}$$

$$3 + 6x = x + 2$$

$$5x = -1$$

$$x = -\frac{1}{5}$$

$$\Rightarrow \tanh a = -\frac{1}{5}$$

Logo, ~~cosh~~

como $1 - \tanh^2 a = \operatorname{sech}^2 a$, temos que

$$\operatorname{sech} a = \pm\sqrt{1 - \tanh^2 a} = \pm\sqrt{1 - \left(-\frac{1}{5}\right)^2}$$

$$\Rightarrow \operatorname{sech} a = \pm\sqrt{\frac{25-1}{25}} = \pm\frac{2\sqrt{6}}{5};$$

e então

$$\cosh a = \frac{1}{\operatorname{sech} a} = \pm\frac{5}{2\sqrt{6}}.$$

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03) (a) $\tanh x = \frac{1}{2} \Leftrightarrow \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{1}{2} \Leftrightarrow$

$$\Leftrightarrow \frac{e^x - \frac{1}{e^x}}{e^x + \frac{1}{e^x}} = \frac{1}{2} \Leftrightarrow \frac{e^{2x}-1}{e^{2x}+1} = \frac{1}{2}.$$

Seja $e^x = w$, então

$$\frac{w^2-1}{w^2+1} = \frac{1}{2}$$

$$2w^2 - 2 = w^2 + 1$$

$$w^2 = 3 \Rightarrow w = \pm\sqrt{3}, \text{ e daí}$$

$$e^x = w = +\sqrt{3} \Leftrightarrow x = \ln\sqrt{3} = \ln 3^{\frac{1}{2}} = \frac{1}{2}\ln 3.$$

(b) $\cosh x = 2 \Leftrightarrow \frac{e^x + e^{-x}}{2} = 2 \Leftrightarrow e^x + \frac{1}{e^x} = 4 \Leftrightarrow$

$e^{2x} + 1 = 4e^x$. Seja $w = e^x$; então:

$$w^2 + 1 = 4w \Leftrightarrow w^2 - 4w + 1 = 0 \quad ; \quad w > 0.$$

$$\Leftrightarrow w = \frac{4 \pm \sqrt{16-4}}{2} = \frac{4 \pm 2\sqrt{3}}{2} = 2 \pm \sqrt{3}$$

$$\Rightarrow e^x = 2 \pm \sqrt{3} \Rightarrow x = \ln(2 \pm \sqrt{3})$$

04)

(a) $\operatorname{senh} r \cdot \cosh w - \operatorname{senh} w \cdot \cosh r = \dfrac{e^{r}-e^{-r}}{2}\cdot\dfrac{e^{w}+e^{-w}}{2} - \dfrac{e^{w}-e^{-w}}{2}\cdot\dfrac{e^{r}+e^{-r}}{2}$

$$= \frac{\cancel{e^{r+w}} + e^{r-w} - \cancel{e^{-r+w}} - \cancel{e^{-r-w}} - \cancel{e^{r+w}} - e^{-r+w} + e^{r-w} + \cancel{e^{-r-w}}}{4}$$

$$= \frac{2\cdot e^{r-w} - 2e^{-r+w}}{4} = \frac{e^{r-w} - e^{-(r-w)}}{2} = \operatorname{senh}(r-w)$$

(b) $\operatorname{csch} 2x = \frac{1}{2}\cdot \operatorname{sech} x \cdot \operatorname{csch} x$: de fato, basta notar que:

$$\frac{1}{2}\cdot \operatorname{sech} x \cdot \operatorname{csch} x = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{e^{x}+e^{-x}}\cdot\frac{2}{e^{x}-e^{-x}} = \frac{2}{e^{2x}-e^{-2x}} = \operatorname{csch} 2x.$$

(c) $\tanh(\ln x) = \dfrac{x^2-1}{x^2+1}$: de fato:

$$\tanh(\ln x) = \frac{e^{\ln x}-e^{-\ln x}}{e^{\ln x}+e^{-\ln x}} = \frac{e^{\ln x}-e^{\ln\frac{1}{x}}}{e^{\ln x}+e^{\ln\frac{1}{x}}} = \frac{x-\frac{1}{x}}{x+\frac{1}{x}} =$$

$$= \frac{\frac{x^2-1}{\cancel{x}}}{\frac{x^2+1}{\cancel{x}}} = \frac{x^2-1}{x^2+1}.$$

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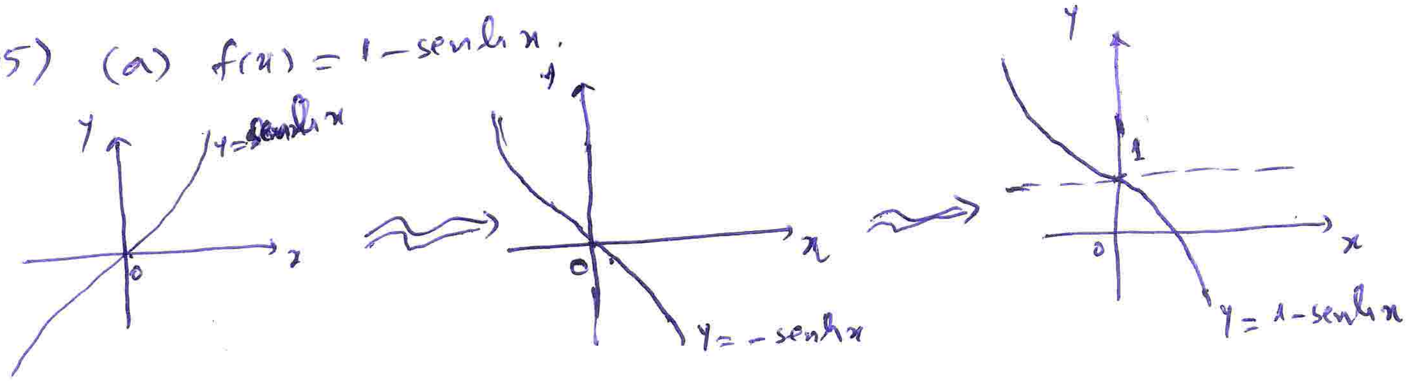
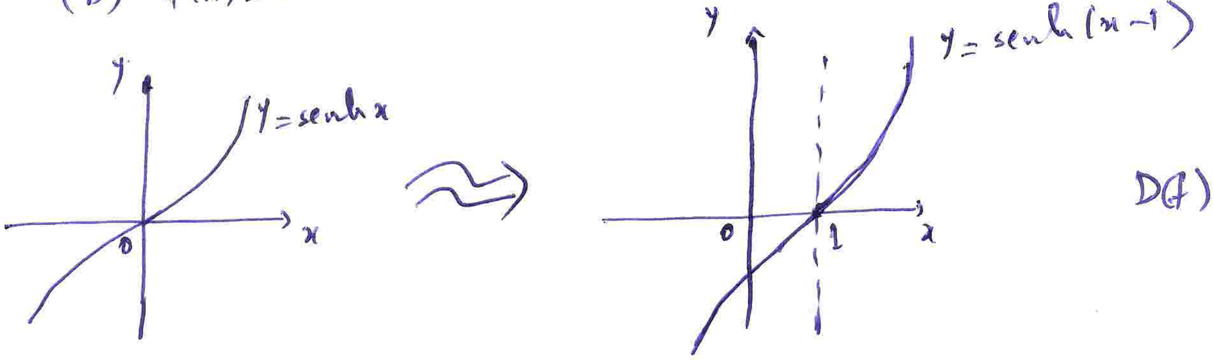
05) (a) $f(x) = 1 - \operatorname{senh} x$.



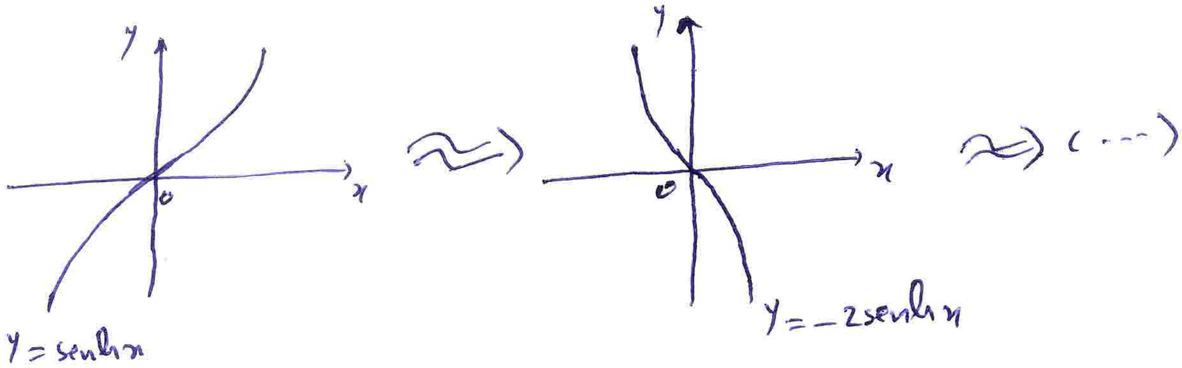
Neste caso, $D(f) = \operatorname{Im}(f) = \mathbb{R}$.

(b) $f(x) = \operatorname{senh}(x-1)$



$D(f) = Im(f) = \mathbb{R}$.

(c) $f(x) = 1 - 2\operatorname{senh}(1-x)$:

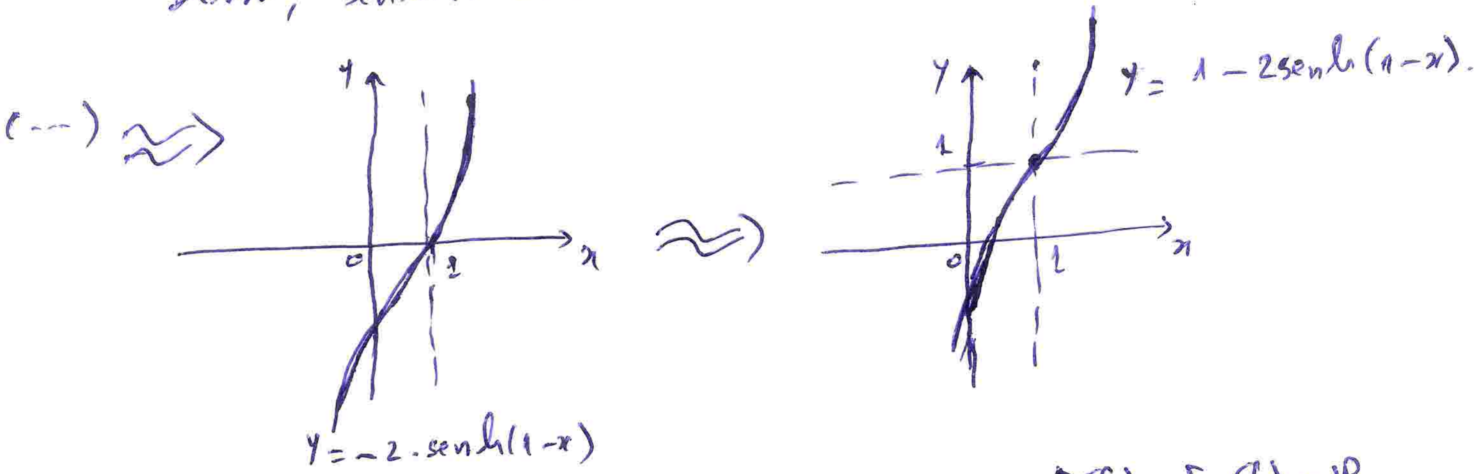


obs.: • o seno hiperbólico é uma função ímpar: de fato, basta notar que

$\operatorname{senh}(-u) = -\operatorname{senh}(u)$; $\forall u \in \mathbb{R}$:

$$\operatorname{senh}(-u) = \frac{e^{-u} - e^{-(-u)}}{2} = -\frac{e^{u} - e^{-u}}{2} = -\operatorname{senh}(u).$$

Disso, $\operatorname{senh}(1-x) = -\operatorname{senh}(x-1)$ e daí :



$D(f) = Im(f) = \mathbb{R}$.

$\boxed{\text{(d) e (e) − FAZER}}$

(f) $f(x) = 1 - 2\coth(1-x)$

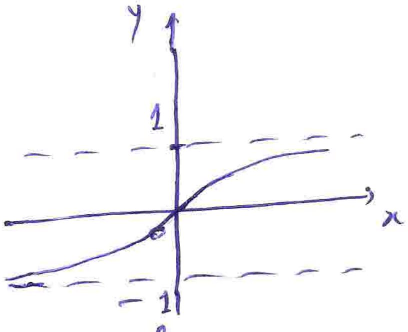
AF! "coth" e "tanh" são funções ímpares:

Verifique que $\coth(-x) = -\coth x$ e
$\tanh(-x) = -\tanh x$

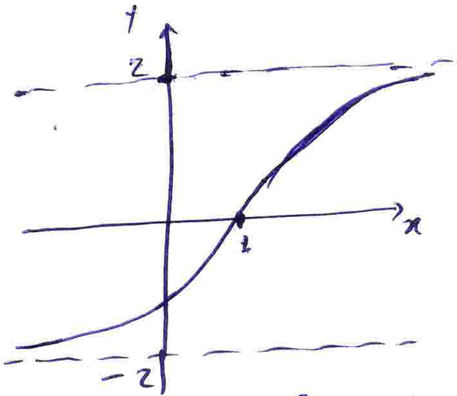
Assim:

$f(x) = 1 + 2\cdot\coth(x-1)$

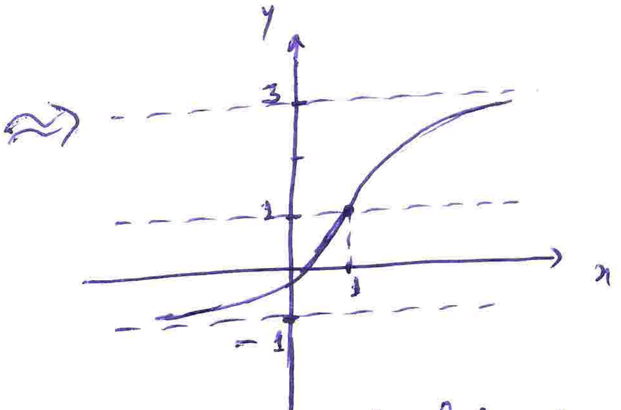
Vamos fazer primeiro o gráfico de $g(x) = 1 + 2\cdot\tanh(x-1)$:



$y = \tanh x$

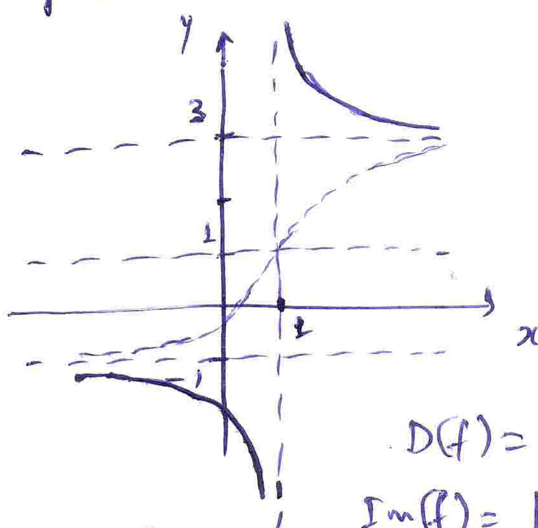


$y = 2\tanh(x-1)$



$y = 1 + 2\cdot\tanh(x-1)$

Agora, tomando o inverso do gráfico ao lado, obtemos o gráfico de $f(x) = 1 + 2\cdot\coth(x-1)$:



$D(f) = \mathbb{R} \setminus \{1\}$

$Im(f) = [-1, 3]^c$.

06) Basta usar que $\text{arcsenh}\, x = \ln\left(x + \sqrt{x^2+1}\right)$ e daí:

$\text{arcsenh}\, 0{,}25 + \text{arcsenh}\, \frac{3}{4} =$

$= \text{arcsenh}\, \frac{1}{4} + \text{arcsenh}\, \frac{3}{4} =$

$= \ln\left(\frac{1}{4} + \sqrt{\left(\frac{1}{4}\right)^2+1}\right) + \ln\left(\frac{3}{4} + \sqrt{\left(\frac{3}{4}\right)^2+1}\right) =$

$= \ln\left(\frac{1}{4} + \sqrt{\frac{17}{16}}\right) + \ln\left(\frac{3}{4} + \sqrt{\frac{25}{16}}\right) =$

$= \ln\left(\frac{1+\sqrt{17}}{4}\right) + \ln\left(\frac{8}{4}\right) = \ln\left(\frac{1+\sqrt{17}}{4}\right) + \ln 2 =$

$= \ln\left[\left(\frac{1+\sqrt{17}}{4}\right) \cdot 2\right] = \ln\left(\frac{\sqrt{17}+1}{2}\right)$

$\boxed{\ln(a \cdot b) = \ln a + \ln b}$

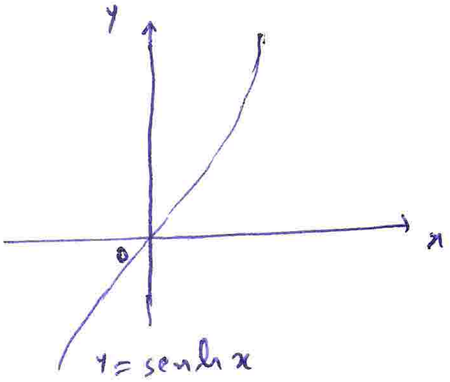
07) (a) $f(x) = 1 - \text{arcsenh}\, x$.

Vamos fazer primeiramente o gráfico de $x = g(y)$. (a função inversa); lembrando que "senh" admite inversa em todo o $\mathbb{R}$.

$$y = 1 - \text{arcsenh}\, x \iff \text{arcsenh}\, x = 1 - y$$
$$\iff x = \text{senh}(1-y) = -\text{senh}(y-1)$$

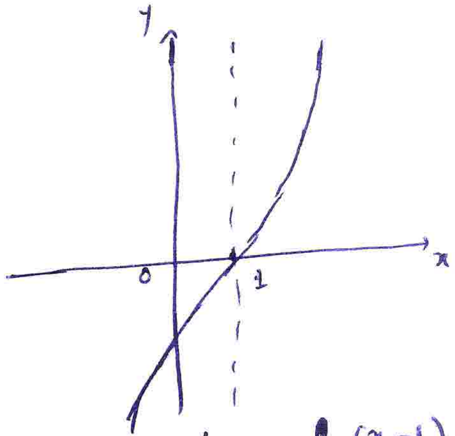
Assim,

$$g(x) = -\text{senh}(x-1)$$



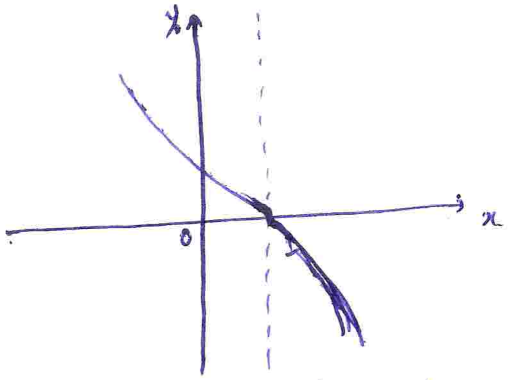
$y = \text{senh}\, x$

$\rightsquigarrow$



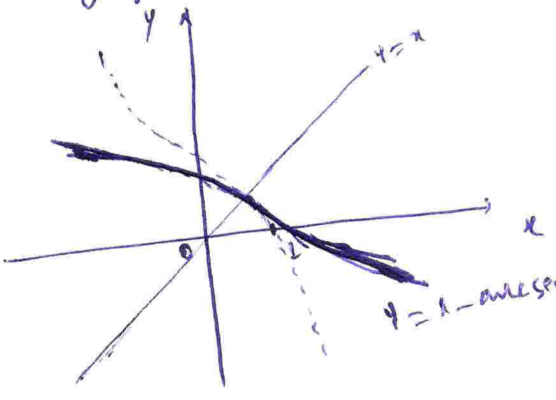
$y = \text{senh}(x-1)$

$\rightsquigarrow$



$y = -\text{senh}(x-1)$

Agora, pela técnica de "espelhamento" sobre a reta bissetriz dos quadrantes ímpares $y = x$, determinemos o gráfico da sua inversa, que será o gráfico de $y = 1 - \text{arcsenh}\, x$,



$y = 1 - \text{arcsenh}\, x$.

(b) $f(x) = 1 + \operatorname{arccosh}(1-x)$

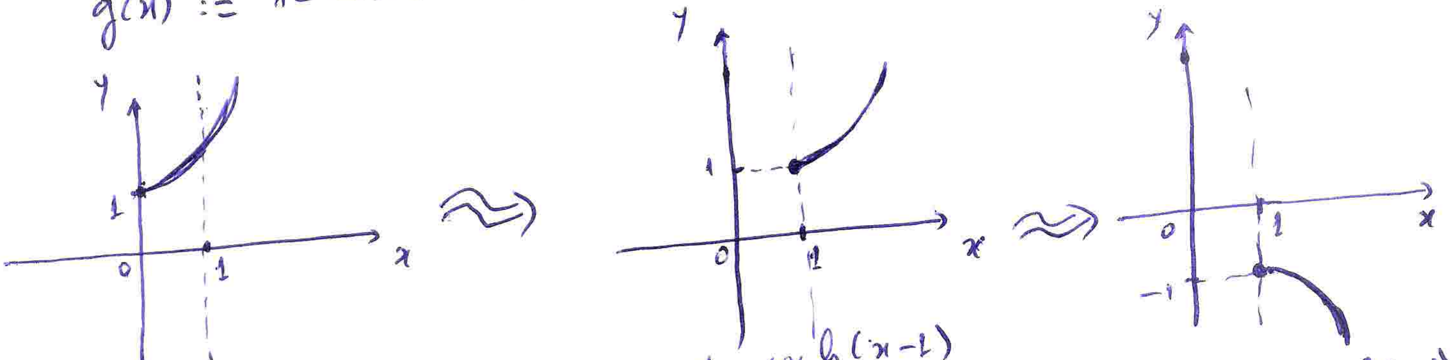
Primeiramente, façamos o gráfico de $x = g(y)$:

$$y = 1 + \operatorname{arccosh}(1-x) \iff y-1 = \operatorname{arccosh}(1-x)$$

$$\iff \cosh(y-1) = 1-x \iff x = 1 - \cosh(y-1).$$

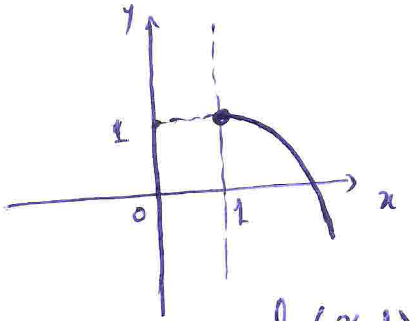
Assim

$g(x) := 1 - \cosh(x-1).$



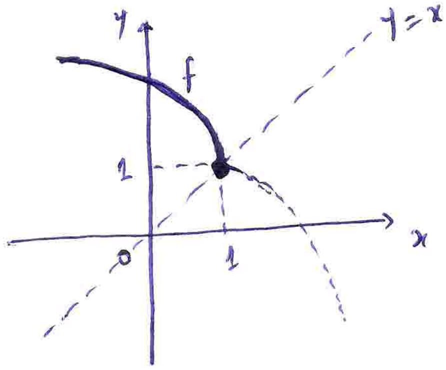
$y = \cosh x$ $\quad$ $y = \cosh(x-1)$ $\quad$ $y = -\cosh(x-1)$

[TOMAMOS APENAS UM RAMO POIS TEMOS DE TOMAR A INVERSA DEPOIS]



$y = 1 - \cosh(x-1)$

Agora, espelhando este gráfico em relação à reta $y = x$, obteremos o gráfico de $f(x) = 1 + \operatorname{arccosh}(1-x)$:



$D(f) = (-\infty, 1]$

$Im(f) = [1, +\infty).$

08)

(a) PROVAR QUE $\lim_{x\to 2} 3x-1 = 5$:

Dado $\varepsilon>0$, precisamos achar $\delta>0$ tal que,

$$\forall x:\ 0<|x-2|<\delta \Longrightarrow |(3x-1)-5|<\varepsilon.$$

Estimando $|f(x)-5|$:

$$|f(x)-5| = |(3x-1)-5| = |3x-6| = 3.|x-2| < \underbrace{3\delta}_{=:\varepsilon}.$$

Assim, chamando $\varepsilon = 3\delta$ vamos obter $\delta = \frac{\varepsilon}{3}$; e com isso, mostramos que,

$\forall \varepsilon>0$, $\exists\, \delta>0$ $\left(\delta = \frac{\varepsilon}{3}\right)$ tal que, $\forall x: 0<|x-2|<\delta$, implica em $|f(x)-5|<\varepsilon$; ou seja, provamos que $\lim_{x\to 2}(3x-1)=5$. $\square$.

(b) PROVAR QUE $\lim_{x\to 2} \frac{2x-1}{x+2} = \frac{3}{4}$.

Primeiramente, notamos que $x\neq -2$, pois senão teríamos divisão por zero.

$-2 \quad 0 \quad 2$

Assim, dado $\varepsilon>0$, precisamos achar $\delta>0$ [veja que $0<\delta<4$, pois senão a vizinhança de $x=2$ poderá conter o ponto $x=-2$, onde a função não está definida!], $0<\delta<4$, tal que,

$$\forall x\in D(f):\ 0<|x-2|<\delta \Longrightarrow \left|f(x)-\frac{3}{4}\right|<\varepsilon.$$

Vamos estimar $\left|f(x)-\frac{3}{4}\right|$:

$$\left|f(x)-\frac{3}{4}\right| = \left|\frac{2x-1}{x+2}-\frac{3}{4}\right| = \left|\frac{4(2x-1)-3(x+2)}{4(x+2)}\right| =$$

$$= \left|\frac{8x-4-3x-6}{4(x+2)}\right| = \left|\frac{5x-10}{4(x+2)}\right| = \frac{5}{4}\cdot\frac{|x-2|}{|x+2|}.$$

[segue...] →

Então,

$$\left|f(x) - \frac{3}{4}\right| = \frac{5}{4} \cdot \frac{|x-2|}{|x+2|},$$ e como x é tal que $0 < |x-2| < \delta$,

obtemos

$$\left|f(x) - \frac{3}{4}\right| = \frac{5}{4} \cdot \frac{|x-2|}{|x+2|} < \frac{5}{4} \cdot \frac{\delta}{|x+2|}. \qquad (*)$$

Vamos agora obter uma estimativa para o denominador $|x+2|$:

$$|x+2| = |x-2+4| = |4 + (x-2)| \geq 4 - |x-2|,$$

e como $|x-2| < \delta \Rightarrow -|x-2| > -\delta$, e daí

$$|x+2| \geq 4 - |x-2| > 4 - \delta \leadsto |x+2| > 4 - \delta > 0$$

pois $0 < \delta < 4$

e com isso a estimativa $(*)$ fica:

$$\left|f(x) - \frac{3}{4}\right| < \frac{5}{4} \cdot \frac{\delta}{|x+2|} < \underbrace{\frac{5}{4} \cdot \frac{\delta}{4-\delta}}_{=:\varepsilon}$$

[pois $|x+2| > 4-\delta$ implica que $\frac{1}{|x+2|} < \frac{1}{4-\delta}$]

Chamando $\varepsilon := \frac{5}{4} \cdot \frac{\delta}{4-\delta}$, temos que:

$$\frac{4}{5} \cdot \varepsilon(4-\delta) = \delta \Rightarrow 4\varepsilon(4-\delta) = 5\delta \Rightarrow \dots \Rightarrow \delta = \frac{16\varepsilon}{5+4\varepsilon}.$$

Portanto, $\exists \lim_{x \to 2} f(x) = \frac{3}{4}$. $\square$.

(e) $\lim_{x \to \frac{\pi}{6}} \operatorname{sen} x = \frac{1}{2}$.

Dado $\varepsilon > 0$, precisamos achar $\delta > 0$ tal que, $\forall x : 0 < |x - \frac{\pi}{6}| < \delta$, implique em $|\operatorname{sen} x - \frac{1}{2}| < \varepsilon$. Analisando:

$$\left|\operatorname{sen} x - \frac{1}{2}\right| = \left|\operatorname{sen} x - \operatorname{sen}\frac{\pi}{6}\right| = \left|2 \cdot \operatorname{sen}\frac{x-\frac{\pi}{6}}{2} \cdot \cos\frac{x+\frac{\pi}{6}}{2}\right| =$$

$$= 2 \cdot \left|\operatorname{sen}\frac{x-\frac{\pi}{6}}{2}\right| \cdot \left|\cos\frac{x+\frac{\pi}{6}}{2}\right| .$$

[USAMOS AQUI A FÓRMULA: $\operatorname{sen} p - \operatorname{sen} q = 2 \operatorname{sen}\frac{p-q}{2} \cdot \cos\frac{p+q}{2}$]

Com $|\cos\theta| \leq 1$, $\forall \theta$ e $|\operatorname{sen}\alpha| \leq |\alpha|$, podemos estimar:

$$\left|\operatorname{sen} x - \frac{1}{2}\right| = 2 \cdot \underbrace{\left|\operatorname{sen}\frac{x-\frac{\pi}{6}}{2}\right|}_{\leq \left|\frac{x-\frac{\pi}{6}}{2}\right|} \cdot \underbrace{\left|\cos\frac{x+\frac{\pi}{6}}{2}\right|}_{\leq 1} \leq$$

$$\leq 2 \cdot \left|\frac{x-\frac{\pi}{6}}{2}\right| \cdot 1 = \left|x - \frac{\pi}{6}\right| < \delta := \varepsilon$$

Assim, tomando $\delta = \varepsilon$ serve! $\square$.

09)

(a) $\lim_{x\to 2} \frac{x^2-3x+2}{6-3x} = \frac{0}{0}$ (INDETERMINAÇÃO)

$$\begin{array}{l|l} x^2-3x+2 & x-2 \\ \cline{2-2} -x^2+2x & x-1 \\ \cline{1-1} \quad -x+2 & \\ \quad +x-2 & \\ \cline{1-1} \qquad 0 & \end{array} \Bigg\} \; x^2-3x+2 = (x-2)\cdot(x-1)$$

Assim:

$$\lim_{x\to 2} \frac{x^2-3x+2}{6-3x} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x-1)}{-3\cancel{(x-2)}} = \lim_{x\to 2} \frac{x-1}{-3} =$$

$$= \frac{2-1}{-3} = -\frac{1}{3}. \;//$$

(b) $\lim_{x\to 1} \frac{x^4-1}{x-1} = \frac{0}{0}$ (INDETERMINAÇÃO)

Como $x^4-1 = (x-1)(x^3+x^2+x+1)$, temos: [OU FAZER A DIVISÃO DE POLINÔMIOS COMO NO ITEM (a)]

Assim, temos:

$$\lim_{x\to 1} \frac{x^4-1}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}\cdot(x^3+x^2+x+1)}{\cancel{x-1}} =$$

$$= \lim_{x\to 1} (x^3+x^2+x+1) = 4. \;//$$

(c) $\displaystyle\lim_{x\to -2} \frac{x^2+3x+2}{2x^2-3x-14} = \frac{0}{0}$ (INDETERMINAÇÃO).

$$\begin{array}{l|l} \cancel{x^2}+3x+2 & x+2 \\ \hline -\cancel{x^2}-2x & x+1 \\ \hline \quad x+2 & \\ \quad -x-2 & \\ \hline \quad 0 & \end{array} \Bigg\} \Rightarrow x^2+3x+2 = (x+2).(x+1)$$

$$\begin{array}{l|l} 2x^2-3x-14 & x+2 \\ \hline -\cancel{2x^2}-4x & 2x-7 \\ \hline \quad -7x-\cancel{14} & \\ \quad +7\cancel{x}+14 & \\ \hline \quad 0 & \end{array} \Bigg\} \Rightarrow 2x^2-3x-14 = (x+2).(2x-7)$$

Portanto,

$$\lim_{x\to -2} \frac{x^2+3x+2}{2x^2-3x-14} = \lim_{x\to -2} \frac{(\cancel{x+2})(x+1)}{(\cancel{x+2}).(2x-7)} = \lim_{x\to -2} \frac{x+1}{2x-7} = \frac{-2+1}{2(-2)-7} = +\frac{1}{11}$$

(d) $\displaystyle\lim_{x\to 2} \frac{x^5-32}{4-x^2} = \frac{0}{0}$ (INDETERMINAÇÃO).

$$\begin{array}{l|l} \cancel{x^5}-32 & x-2 \\ \hline -\cancel{x^5}+2x^4 & x^4+2x^3+4x^2+8x+16 \\ \hline 2\cancel{x^4}-32 & \\ -\cancel{2x^4}+4x^3 & \\ \hline 4\cancel{x^3}-32 & \\ -\cancel{4x^3}+8x^2 & \\ \hline 8\cancel{x^2}-32 & \\ -\cancel{8x^2}+16x & \\ \hline 16x-\cancel{32} & \\ -16\cancel{x}+32 & \\ \hline 0 & \end{array} \Bigg\} \Rightarrow x^5-32 = (x-2).(x^4+2x^3+4x^2+8x+16)$$

$$\begin{array}{l|l} -\cancel{x^2}+4 & x+2 \\ \hline +\cancel{x^2}+2x & -x+2 \\ \hline 2x+\cancel{4} & \\ -2\cancel{x}-4 & \\ \hline 0 & \end{array} \Bigg\} \Rightarrow 4-x^2 = (x+2)(-x+2)$$

Assim, obtemos:

$(\Rightarrow)$

$$\lim_{x\to 2} \frac{x^5-32}{4-x^2} = \lim_{x\to 2} \frac{\cancel{(x-2)}(x^4+2x^3+4x^2+8x+16)}{-(x+2)\cancel{(x-2)}} =$$

$$= \lim_{x\to 2} \frac{x^4+2x^3+4x^2+8x+16}{-(x+2)} = \frac{(2)^4+2.(2)^3+4.(2)^2+8.(2)+16}{-(2+2)} =$$

$$= -\frac{80}{4} = -20 \,//$$

(e) $$\lim_{x\to -1} \frac{\sqrt{x+3}-\sqrt{2}}{3x+3} = \frac{0}{0} \quad \text{(INDETERMINAÇÃO)}$$

$$\lim_{x\to -1} \frac{\sqrt{x+3}-\sqrt{2}}{3x+3} = \lim_{x\to -1} \frac{\sqrt{x+3}-\sqrt{2}}{3(x+1)} \cdot \frac{\sqrt{x+3}+\sqrt{2}}{\sqrt{x+3}+\sqrt{2}} =$$

$$= \lim_{x\to -1} \frac{(x+3)-2}{3(x+1).(\sqrt{x+3}+\sqrt{2})} = \lim_{x\to -1} \frac{\cancel{x+1}}{3\cancel{(x+1)}(\sqrt{x+3}+\sqrt{2})}$$

$$= \lim_{x\to -1} \frac{1}{\sqrt{x+3}+\sqrt{2}} = \frac{1}{\sqrt{2}+\sqrt{2}} = \frac{1}{2\sqrt{2}} \,//$$

(f) $$\lim_{x\to 3} \frac{\sqrt{x^2-5}-\sqrt{x+1}}{x^2-2x-3} = \frac{0}{0} \quad \text{(INDETERMINAÇÃO)}$$

$$\lim_{x\to 3} \frac{\sqrt{x^2-5}-\sqrt{x+1}}{x^2-2x-3} = \lim_{x\to 3} \frac{\sqrt{x^2-5}-\sqrt{x+1}}{x^2-2x-3} \cdot \frac{\sqrt{x^2-5}+\sqrt{x+1}}{\sqrt{x^2-5}+\sqrt{x+1}} =$$

$$= \lim_{x\to 3} \frac{x^2-5-(x+1)}{(x^2-2x-3).(\sqrt{x^2-5}+\sqrt{x+1})} = \lim_{x\to 3} \frac{x^2-x-6}{(x^2-2x-3)(\sqrt{x^2-5}+\sqrt{x+1})}$$

Vamos fatorar os polinômios do numerador e denominador:

$$\begin{array}{l|l} x^2 - x - 6 & x-3 \\ \cline{2-2} -x^2 + 3x & x+2 \\ \hline \quad 2x - 6 & \\ \quad -2x + 6 & \\ \hline \quad\quad 0 & \end{array} \Bigg\} \Longrightarrow x^2 - x - 6 = (x+2)(x-3)$$

$$\begin{array}{l|l} x^2 - 2x - 3 & x-3 \\ \cline{2-2} -x^2 + 3x & x+1 \\ \hline \quad x - 3 & \\ \quad -x + 3 & \\ \hline \quad\quad 0 & \end{array} \Bigg\} \Longrightarrow x^2 - 2x - 3 = (x-3)(x+1).$$

Assim, temos:

$$\lim_{x \to 3} \frac{x^2 - x - 6}{(x^2 - 2x - 3)(\sqrt{x^2-5} + \sqrt{x+1})} = \lim_{x \to 3} \frac{(x+2)(x-3)}{(x-3)(x+1)(\sqrt{x^2-5} + \sqrt{x+1})} =$$

$$= \lim_{x \to 3} \frac{x+2}{(x+1)(\sqrt{x^2-5} + \sqrt{x+1})} = \frac{3+2}{(3+1)(\sqrt{4} + \sqrt{4})} = \frac{5}{16} //$$

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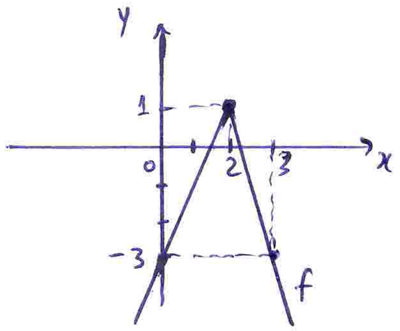
10) (a)



Precisamos calcular os limites laterais. Assim:

- $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2x-3) = 4 - 3 = 1.$

- $\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (9-4x) = 9 - 8 = 1.$

Logo, $\lim_{x \to 2^-} f(x) = 1 = \lim_{x \to 2^+} f(x)$; portanto, $\exists \lim_{x \to 2} f(x) = 1.$