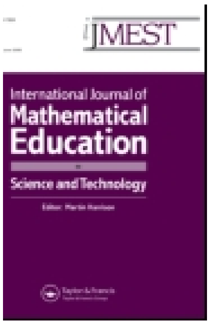

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A simple demonstration of zero factorial equals one

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When asked, a number of students answer zero factorial to be zero as a continuation to the answer of one factorial to be one. Any instructor would then seek a justification of zero factorial to be one from computing ${}_nC_n$ via the well- known combination formula. This article conveys a simple presentation of zero factorial to be one based on lower and upper bounds of n factorial. We have not seen this explanation covered in any algebra textbook.

Keywords: factorial; bound; limit

1. Introduction

Many students have wondered why 0! is defined to be 1. To motivate this definition, one such example is the formula for combination given by ${}_nC_k = \frac{n!}{k!(n-k)!}$. Before introducing this formula, students may be prompted with a question, for instance, to find the number of ways of arranging three distinct books on a shelf without any specific order. The students would naturally answer that there is only one way of choosing three books from the three distinct books. So, for the above combination formula to be consistent with this intuition, ${}_3C_3 = \frac{3!}{3!(3-3)!}$ must be one. This implies $(3-3)! = 0!$ is required to be 1. In this article, we offer a simple explanation of zero factorial to be one based on lower and upper bounds of n factorial.

2. Main result

For $n \geq 1$,the factorial of n is defined by

$$n! = (n)(n-1)\ldots(3)(2)(1). \quad (1)$$

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Since $1 \le n$, a lower bound of (1) follows

$$1^n = (1)(1)\dots(1)(1)(1) \le n! \\ \Rightarrow 1 \le n! \tag{2}$$

Similarly, an upper bound for (1) is given by

$$n! \le (n)(n)\dots(n)(n)(n) = n^n. \tag{3}$$

From (2) and (3), we arrive at

$$1 \le n! \le n^n. \tag{4}$$

Noting that the upper bound n^n in (4) is valid for $n > 0$, we take the limit as $n \to 0^+$ in (4) to write

$$\lim_{n\to 0^+} 1 \le \lim_{n\to 0^+} n! \le \lim_{n\to 0^+} n^n. \tag{5}$$

We observe that $\lim_{n\to 0^+} n^n$ in (5) is indeterminate, since $0^n = 0$ for any $n > 0$ but $n^0 = 1$ for any $n \ne 0$. It follows from (5) that

$$\lim_{n\to 0^+} 1 \le \lim_{n\to 0^+} n! \le \lim_{n\to 0^+} \left(e^{\ln n}\right)^n \\ \Rightarrow \lim_{n\to 0^+} 1 \le \lim_{n\to 0^+} n! \le \lim_{n\to 0^+} e^{n \ln n}. \tag{6}$$

The evaluation of $\lim_{n\to 0^+} e^{n \ln n}$ involves writing $\lim_{n\to 0^+} n \ln n$ as $\lim_{n\to 0^+} \frac{\ln n}{1/n}$, which is in the form of $\frac{\infty}{\infty}$. Hence, using l'Hospital's Rule, $\lim_{n\to 0^+} n \ln n = \lim_{n\to 0^+} \frac{\ln n}{1/n} = \lim_{n\to 0^+} \frac{1/n}{-1/n^2} = \lim_{n\to 0^+}(-n) = 0$. As a consequence, (6) yields

$$1 \le \lim_{n\to 0^+} n! \le e^0. \tag{7}$$

We let $0! = \lim_{n\to 0^+} n!$ in (7), so $1 \le 0! \le 1$, which implies, $0! = 1$.

3. Conclusion

This paper provided a justification of zero factorial to be one by establishing an upper and lower bound of n factorial. We believe that the explanation is simple and effective for the purpose of classroom teaching and adds a pedagogical value in the understanding of the concept.

Disclosure statement

No potential conflict of interest was reported by the authors.